

CONCORDIA UNIVERSITY
 Faculty of Engineering and Computer Science
 Department of Engineering

FINAL EXAMINATION ENGR 213, 13. 4. 2007

Sections: F (Dr. A. B. Keviczky), G (Dr. S. T. Ali), J (Dr. A. Stancu)

ENGR 213 Course Coordinator: Dr. Georges Vatisstas

Written by: A. B. Keviczky

Answer each question! Calculators Permitted!

(10)#1...Solve the following two ordinary differential equations:

$$(A) \frac{dy}{dx} = \frac{y^2 e^x}{1 + e^{2x}}, \quad (B) \frac{dy}{dx} - \frac{2}{x}y = x^2 \cos x \text{ with } y(\pi) = 4.$$

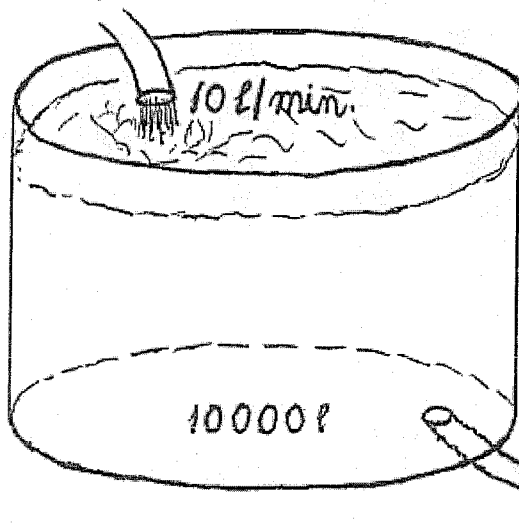
(10)#2...Solve the following ordinary differential equations by means of a suitable integrating factor $\mu = \mu(x)$ or $\mu = \mu(y)$, which shall make it exact:

$$6xy dx + (4y + 9x^2)dy = 0. \text{ Note: } \frac{\mu'(x)}{\mu(x)} = \frac{M_y - N_x}{N} \text{ or } \frac{\mu'(y)}{\mu(y)} = \frac{N_x - M_y}{M}.$$

(10)#3...Solve the following two ordinary differential equations by means of a suitable substitution:

$$(A) \frac{dy}{dx} = \frac{x + 3y}{3x + y}, \quad (B) \frac{dy}{dx} + y = 8xy^4.$$

(10)#4...A cylindrical tank is filled with 10000 liters of brine containing 500 kg of



salt initially. Entering from the top of the tank, at a rate of 10 l/min, is a saline solution containing 0.3 kg/l. From the well-stirred tank 10 l/min is discharged at the bottom.

- (A) How much salt is contained in the tank at time t (in minutes)?
 (B) What does this tell you about about the amount of salt in the tank as $t \rightarrow \infty$?

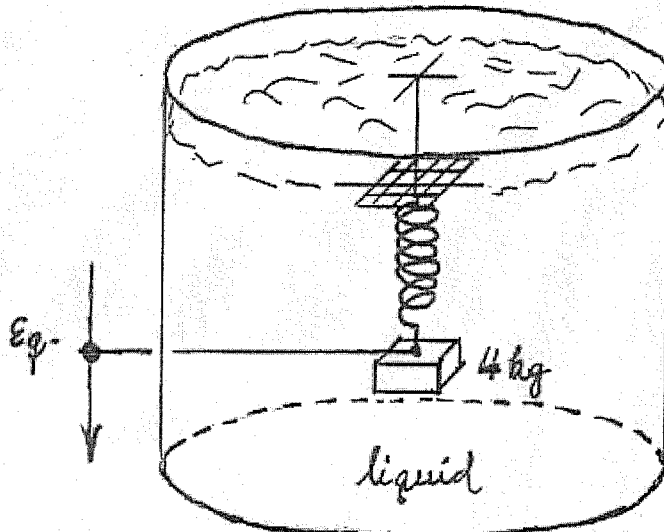
(10)#5... Solve the following linear homogeneous second order ordinary differential equations with the given initial conditions:

$$x^2 y'' - 6xy' - 18y = 0 \quad \text{satisfying: } y(1) = 2 \text{ and } y'(1) = 16.$$

(10)#6... Find the general solutions of the following linear inhomogeneous second order ordinary differential equation by means of variation of parameter:

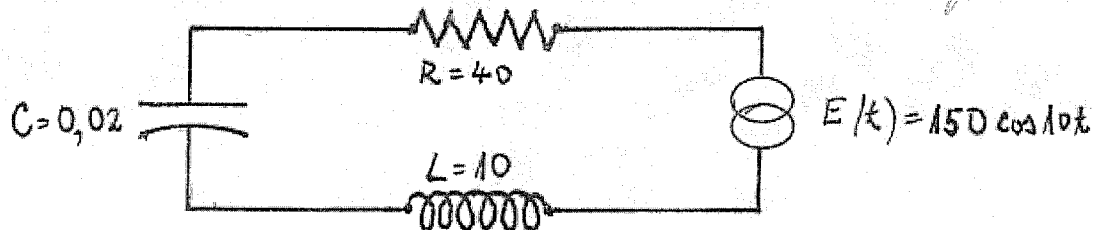
$$y'' - 4y = x^{-1} e^{2x}.$$

(10)#7... A mass of 4 kg is attached to a spring with spring constant 36 N/m



and the entire system is immersed in a liquid whose damping is 24 times the velocity. Find the equation of motion of this spring system if it is released 2 m below the equilibrium position with an upward velocity of 12 m/sec., where m stands for the metric unit of meter. system.

(10)#8... Given the L-R-C circuit (inductor, resistor, capacitor)



with charge $Q = Q(t)$ satisfying the linear second order ordinary differential equation

$$L \frac{d^2 Q}{dt^2} + R \frac{dQ}{dt} + \frac{1}{C} Q = E(t) \quad \text{where: } R = 40 \text{ ohms, } L = 10 \text{ henries, } C = 0.02 \text{ farads}$$

and an electromotive force $E(t) = 150 \cos 10t$ volts. Charge Q further satisfies the initial conditions $Q(0) = 1500$ volts and $Q'(0) = 0$.

(A) Solve for the charge $Q(t)$.

(B) Indicate in $Q(t)$ the transient and steady state terms.

(10)#9...Find the general solution of the linear system of ordinary differential equations

$$\begin{aligned} (d/dt + 1)x + (d/dt - 1)y &= 2 \\ 3x + (d/dt + 2)y &= -1 \end{aligned}$$

where d/dt is the derivative with respect to time t .

(10)#10. Find ^{two} linearly independent power series solutions $\phi_1(x)$ and $\phi_2(x)$ of the second order ordinary differential equation

$$y'' + 2xy' + y = 0,$$

assuming that the solution has the power series form $y = y(x) = \sum_{n=0}^{\infty} A_n x^n$.

Calculate only the first three non-zero terms of each of the functions $\phi_1(x)$ and $\phi_2(x)$.