Concordia University
Applied Ordinary Differential Equations, ENGR 213
Final Exam
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Only approved calculators with stickers are allowed.
Time allotted: Three hours.

1. (14%) Find the general solution of each of the following differential equations. In each case, you may leave the solution in implicit form.

   (a) $\dfrac{dy}{dx} = \dfrac{y^2+1}{x^2y+y}$

   (b) $\left(ye^{2xy} + \cos(x+2y)\right)\,dx + \left(xe^{2xy} + 2\cos(x+2y)\right)\,dy = 0$

2. (10%) Solve the initial value problem

$$t\,\frac{dy}{dt} + (t+1)\,y = t, \quad y(\ln 2) = 1,\ t > 0.$$

3. (10%) Solve the following differential equation by an appropriate substitution

$$(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0.$$

   You may leave the solution in implicit form.

4. (10%) The Space Shuttle lands in Kennedy Space Center. The spacecraft touches down at $t = 0$ with a velocity of 100 m/sec. The spacecraft chute is deployed at $t = 5$ sec. Between touch down and deployment of chute ($0 \le t \le 5$), the velocity of the spacecraft $V(t)$ (in m/sec) is governed by:

$$\frac{dV}{dt} = -0.0001V^2$$

   and after the deployment of chute by:

$$\frac{dV}{dt} = -0.002V^2$$

   Determine the spacecraft velocity at $t = 5$ sec and $t = 15$ sec.

5. (10%) Find the general solution of the following differential equation using the method of *variation of parameters*.

$$x^2y'' + 8xy' + 12y = e^x$$

   Hint. For a positive integer n

$$\int x^n e^{ax}dx = \frac{e^{ax}}{a}\left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \cdots \frac{(-1)^n n!}{a^n}\right)$$

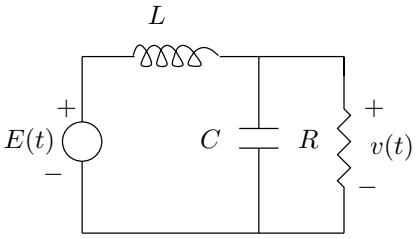
6. (12%) The following circuit consists of an inductor $L = \dfrac{1}{2}$ henry, a resistor $R = 1$ ohm, and a capacitor $C = \dfrac{1}{4}$ farad connected to an input voltage $E(t)$. The resistor voltage $v(t)$ satisfies the following ordinary differential equation.

$$LC\,\frac{d^2v}{dt^2} + \frac{L}{R}\,\frac{dv}{dt} + v(t) = E(t)$$

Assume that at $t = 0$ no energy is stored in the inductor and the capacitor, and hence, $v(0) = 0$ and $v'(0) = 0$. Furthermore, for $t \geq 0$, the input voltage is $E(t) = \sin 3t$ volts.

(a) Find the voltage $v(t)$ (for $t \geq 0$).

(b) Identify in $v(t)$, the transient terms and the steady state terms.



7. (10%)

(a) Find the general solution of the differential equation

$$(x^2 - 1)\,y'' + y = 0$$

by using the power series method about the ordinary point $x = 0$. Write out all terms in the solution up to, and including x^5.

(b) Find the singular points of the differential equation. What is the minimum radius of convergence of the power series solution obtained in (a)?

8. (12%) Solve the linear system either by the method of systematic elimination or by a matrix method

$$\begin{aligned} \frac{dx}{dt} &= 4y + \sin(t) \\ \frac{dy}{dt} &= -x + \cos(t)\,. \end{aligned}$$

9. (12%) Solve the initial value problem by using eigenvalues and eigenvectors

$$\begin{aligned} \frac{dx}{dt} &= y - x \\ \frac{dy}{dt} &= 2y \\ \frac{dz}{dt} &= y + z - x\,, \end{aligned}$$

$x(0) = 1,\ y(0) = 1,\ z(0) = 1.$