Concordia University
Applied Ordinary Differential Equations, ENGR 213
Final Exam
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Evaluation out of 100. Only admissible calculators are allowed.
Time allotted: Three hours.

1. (15) (a) Find the general solution of the differential equation

$$\frac{dy}{dx} = \left(\frac{2y+3}{4x+5}\right)^2 .$$

You may leave the solution in implicit form.

(b) Solve the initial value problem

$$x\frac{dy}{dx} + y = e^x , \qquad y(1) = 2$$

2. (10) Find the general solution (explicit or implicit) of the equation

$$(y^2 \cos\, x - 3x^2\, y - 2x)\, dx + (2y\, \sin\, x - x^3 + \ln y)\, dy = 0 .$$

3. (10) Find the general solution of the equation using an appropriate substitution:

$$\frac{dy}{dx} = \tan^2(x+y) .$$

You may leave the solution in implicit form.

4. (10) The Space Shuttle lands in Kennedy Space Center. The spacecraft touches down at $t = 0$ with a velocity of 100 m/sec. The spacecraft chute is deployed at $t = 4$ sec. Between touch down and deployment of chute ($0 \le t \le 4$), the velocity of the spacecraft $V(t)$ (in m/sec) is governed by:

$$\frac{dV}{dt} = 0$$

and after the deployment of chute by:

$$\frac{dV}{dt} = -0.002V^2$$

Determine when the spacecraft velocity reaches 20 m/sec.

5. (10) Find the general solution of the following differential equations using the method of *undetermined coefficients*.

(a) $y'' + 6y' + 8y = \sin 3x$
(b) $y'' + 10y' + 25y = e^x$

6. (10) Find the general solution of the differential equation

$$2x^2y'' + 5xy' + y = x^2 - x$$

by *variation of parameters*.

7. (12) Solve the following system of differential equations by any method you wish (systematic elimination, undetermined coefficients, variation of parameters, or diagonalization).

$$\begin{aligned} \frac{dx}{dt} &= 2x + 3y - e^{2t} \\ \frac{dy}{dt} &= -x - 2y + e^{2t}. \end{aligned}$$

8. (11) Find the power series solution about the ordinary point $x = 0$ for the initial value problem

$$y'' - 3xy' - y = 0, \quad y(0) = 1,\ y'(0) = 0.$$

It suffices to give only the constant term and those of x, x^2 and x^3.

9. (12) Given the LRC-circuit with $L = \dfrac{5}{3}$ henries, $R = 10$ ohms, $C = \dfrac{1}{30}$ farads, and $E(t) = 50\cos t$ volts, the charge $q(t)$ satisfies the linear second order ordinary differential equation

$$L\,\frac{d^2q}{dt^2} + R\,\frac{dq}{dt} + \frac{1}{C}\,q = E(t).$$

   (a) Find the charge $q(t)$ if $q(0) = 100$ coulombs and $q'(0) = 0$ amperes.

   (b) Identify in $q(t)$ the transient terms and, respectively, the steady state terms. Is the circuit overdamped, underdamped, or critically damped?

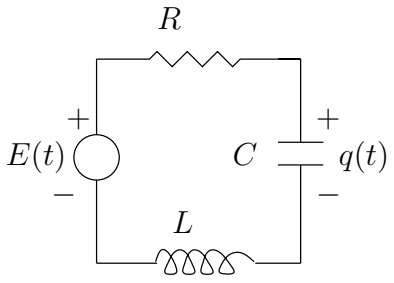

Figure 1: Problem 9.