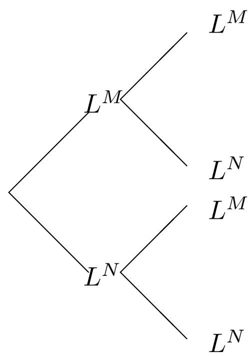
MAT 2379, Introduction to biostatistics

Solution to Assignment 1
Due date: Friday September 30, 2016 at 3:00 p.m.

$Total = 100$ marks

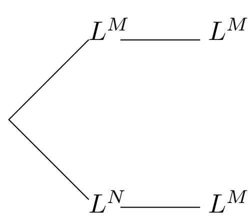
Problem 2.1. (15 marks) (a)



Female Gamete	Male Gamete $\frac{1}{2}L^M$	$\frac{1}{2}L^N$
$\frac{1}{2}L^M$	$\frac{1}{4}L^M L^M$ (type M)	$\frac{1}{4}L^M L^N$ (type MN)
$\frac{1}{2}L^N$	$\frac{1}{4}L^M L^N$ (type MN)	$\frac{1}{4}L^N L^N$ (type N)

The offspring can have type M blood with probability 1/4, type N blood with probability 1/4, and type MN blood with probability 1/2.

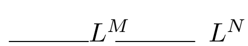
(b)



Female Gamete	Male Gamete $\frac{1}{2}L^M$	$\frac{1}{2}L^N$
L^M	$\frac{1}{2}L^M L^M$ (type M)	$\frac{1}{2}L^M L^N$ (type MN)

The offspring can type M or type MN blood, each with probability 1/2.

(c)



Female Gamete	Male Gamete L^N
L^M	$L^M L^N$ (type MN)

The offspring has type MN blood with probability 1.

Problem 3.5. (10 marks) Let A be the event that the tomato has an increased resistance to pests and B the event that the tomato has a longer shelf life. We know that $P(A) = 0.75$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$.
(a) $P(A \cap B') = P(A) - P(A \cap B) = 0.75 - 0.3$.
(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.75 + 0.5 - 0.3 = 0.95$.
(c) $P(A' \cap B) = P(B) - P(A \cap B) = 0.5 - 0.3 = 0.2$.
(d) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.95 = 0.05$.

Problem 4.3. (10 marks) (a) Let F be the event that a randomly chosen man has prostate cancer. Let A be the event that his PSA level is greater than 10, B be the event that his PSA level is between 4 and 10, and C be the event that his PSA level is smaller than 4. We know that $P(F|A) = 0.67$, $P(F|B) = 0.25$ and $P(F|C) = 0.05$. By the total probability rule,

$$\begin{aligned} P(F) &= P(F|A)P(A) + P(F|B)P(B) + P(F|C)P(C) \\ &= (0.67)(0.15) + (0.25)(0.10) + (0.05)(0.75) \\ &= 0.1005 + 0.025 + 0.0375 = 0.163. \end{aligned}$$

(b) By the Bayes' rule,

$$P(A|F) = \frac{P(F \cap A)}{P(F)} = \frac{P(F|A)P(A)}{P(F)} = \frac{0.1005}{0.163} = 0.62.$$

Problem 4.4. (15 marks) We observed

	Diseased	Non-diseased
Test +	197	8
Test −	3	92
Total	200	100

(a) The false positive rate is $P(T + |U-) = 8/100 = 0.08$. The false negative rate is $P(T - |U+) = 3/200 = 0.015$.
(b) The sensitivity is $P(T + |U+) = 197/200 = 0.985$.
The specificity is $P(T - |U-) = 92/100 = 0.92$.
(c) The positive predictive value is

$$\begin{aligned} \mathsf{PPV} = P(U + |T+) &= \frac{P(T + |U+)P(U+)}{P(T + |U+)P(U+) + P(T + |U-)P(U-)} \\ &= \frac{(197/200)(0.15)}{(197/200)(0.15) + (8/100)(0.85)} = 0.685. \end{aligned}$$

The negative predictive value is

$$\begin{aligned} \mathsf{NPV} = P(U - |T-) &= \frac{P(T - |U-)P(U-)}{P(T - |U-)P(U-) + P(T - |U+)P(U+)} \\ &= \frac{(92/100)(0.85)}{(92/100)(0.85) + (3/200)(0.15)} = 0.9971. \end{aligned}$$

(d) The positive predictive value is

$$\mathsf{PPV} = P(U + |T+) = \frac{P(T + |U+)P(U+)}{P(T + |U+)P(U+) + P(T + |U-)P(U-)}$$

$$= \frac{(197/200)(0.01)}{(197/200)(0.01) + (8/100)(0.99)} = 0.1106.$$

The negative predictive value is

$$\begin{aligned} \mathsf{NPV} = P(U-|T-) &= \frac{P(T-|U-)P(U-)}{P(T-|U-)P(U-) + P(T-|U+)P(U+)} \\ &= \frac{(92/100)(0.99)}{(92/100)(0.99) + (3/200)(0.01)} = 0.9998. \end{aligned}$$

Problem 5.1. (15 marks) Let A_1, A_2, A_3, A_4 be the event that the donor's blood type is O, A, B, respectively AB. Let B_1, B_2, B_3, B_4 be the event that the blood type of the receiving individual is O, A, B, respectively AB. The event A_i is independent of B_j, for any $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$. The event that the transfusion is successful can be written as the following union of disjoint events:

$$C = (A_1 \cap B_1) \cup (A_1 \cap B_2) \cup (A_1 \cap B_3) \cup (A_1 \cap B_4) \cup (A_2 \cap B_2) \cup (A_2 \cap B_4) \cup$$
$$(A_3 \cap B_3) \cup (A_3 \cap B_4) \cup (A_4 \cap B_4).$$

Hence,

$$\begin{aligned} P(C) &= P(A_1)P(B_1) + P(A_1)P(B_2) + P(A_1)P(B_3) + P(A_1)P(B_4) + P(A_2)P(B_2) + \\ &\quad P(A_2)P(B_4) + P(A_3)P(B_3) + P(A_3)P(B_4) + P(A_4)P(B_4) \\ &= (0.46)(0.46) + (0.46)(0.42) + (0.46)(0.09) + (0.46)(0.03) + (0.42)(0.42) + \\ &\quad (0.42)(0.03) + (0.09)(0.09) + (0.09)(0.03) + (0.03)(0.03) \\ &= 0.6607. \end{aligned}$$

The probability that the transfusion is not successful is:

$$P(C') = 1 - P(C) = 1 - 0.6607 = 0.3393.$$

Problem 8.5. (10 marks) A random person is chosen in Africa. Let A be the event that the person had access to safe drinking water, and B the event that the person suffers from waterborne disease. We know that $P(A) = 0.45$, $P(B|A) = 0.32$ and $P(B|A') = 0.88$.

(a) By the total probability rule,

$$P(B) = P(B|A)P(A) + P(B|A')P(A') = (0.32)(0.45) + (0.88)(0.55) = 0.144 + 0.484 = 0.628$$

The incidence rate of waterborne diseases in Africa is 62.8%.

(b) By the Bayes' rule,

$$P(A'|B) = \frac{P(B|A')P(A')}{P(B)} = \frac{0.484}{0.628} = 0.77$$

Problem 8.7. (15 marks) (a) Let A_i be the event that the i-th selected fly is black, for $i = 1, 2$. Using the total probability rule:

$$\begin{aligned} P(A_2) &= P(A_2|A_1)P(A_1) + P(A_2|A_1')P(A_1') \\ &= \frac{4}{24} \cdot \frac{5}{25} + \frac{5}{24} \cdot \frac{20}{25} = \frac{1}{5} \end{aligned}$$

Since $P(A_2|A_1) = 4/24 = 1/6 \neq 1/5 = P(A_2)$, A_1 and A_2 are not independent.

(b) As in part (a), we have:

$$\begin{aligned} P(A_2) &= P(A_2|A_1)P(A_1) + P(A_2|A_1')P(A_1') \\ &= \frac{1,999}{9,999} \cdot \frac{2,000}{10,000} + \frac{2,000}{9,999} \cdot \frac{8,000}{10,000} = \frac{1}{5} \end{aligned}$$

Since $P(A_2|A_1) = 0.2 \approx 1,999/9,999 = P(A_2)$, A_1 and A_2 are (almost) independent.

Problem 8.14. (10 marks) Let A be the event that the donor is HIV positive and that B is the event that the donor is positive for herpes. We know that $P(A) = 0.01$, $P(B) = 0.02$, and $P(A' \cap B) + P(A \cap B') = 0.015$. We first calculate $P(A \cap B)$. Note that

$$P(A) + P(B) = P(A' \cap B) + P(A \cap B') + 2\,P(A \cap B).$$

Hence, $P(A \cap B) = (0.03 - 0.015)/2 = 0.0075$. Next, we calculate

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.01 + 0.02 - 0.0075 = 0.0225.$$

The desired probability is

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.0225 = 0.9775.$$