ECON 320 - SUGGESTED ANSWERS TO ASSIGNMENT ONE

B. Lapham
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Question 2

2.A. We know that a household which seeks to maximize its utility subject to its time constraint and its budget constraint will choose consumption of apples and time devoted to leisure to satisfy the following two equations (in an interior solution with $c > 0$ and $0 < l < 1$):

$$MRS(c,l) \equiv \frac{MU_l(c,l)}{MU_c(c,l)} = w$$

$$c + wl = w.$$

The first equation reflects that the household will choose c and l at a point where the slope of its indifference curve ($-MRS(c,l)$) equals the slope of its overall constraint (-w). The second equation reflects the time constraint and the budget constraint combined (the overall constraint).

For the preferences given in this question, these equations become

$$MRS(c,l) \equiv \frac{MU_l(c,l)}{MU_c(c,l))} = \frac{\left(\frac{8}{3}\right)(l)^{-\left(\frac{2}{3}\right)}}{\left(\frac{2}{3}\right)(c)^{-\left(\frac{2}{3}\right)}} = w \tag{2.1}$$

$$c + wl = w. \tag{2.2}$$

We can simplify equation (2.1) as follows:

$$\frac{4(c)^{\frac{2}{3}}}{(l)^{\frac{2}{3}}} = w,$$

and rearranging this gives

$$c = w^{\frac{3}{2}}\left(\frac{1}{4}\right)^{\frac{3}{2}} l = \left(\frac{w^{\frac{3}{2}}}{8}\right) l. \tag{2.3}$$

Substituting this into the overall constraint given by equation (2.2) gives:

$$\left(\frac{w^{\frac{3}{2}}}{8}\right) l + wl = w.$$

Now, dividing all terms by w and pulling out the common l on the left-hand side gives

$$l\left[\left(\frac{w^{\frac{1}{2}}}{8}\right)+1\right]=1.$$

Solving this equation for l gives us the household's leisure demand function:

$$l^D(w)=\frac{1}{\frac{w^{\frac{1}{2}}}{8}+1}=\frac{8}{w^{\frac{1}{2}}+8} \tag{2.4}$$

Substituting this into equation (2.3) gives the household's demand for apples:

$$c^D(w)=\left(\frac{w^{\frac{3}{2}}}{8}\right)\left[\frac{8}{w^{\frac{1}{2}}+8}\right]=\frac{w^{\frac{3}{2}}}{w^{\frac{1}{2}}+8}. \tag{2.5}$$

Substituting $l^D(w)$ into the time constraint gives labour supply:

$$n^S(w)=1-l^D(w)=\frac{w^{\frac{1}{2}}}{w^{\frac{1}{2}}+8}. \tag{2.6}$$

Now, we should always check that our solution is actually an interior one with $c^D(w)>0$, $0<l^D(w)<1$ and $0<n^S(w)<1$. Clearly this is the case here.

2.B. There are different ways to approach this question. We could take the derivative of $n^S(w)$ with respect to w and see if it is positive, negative or zero. Alternatively, we can examine the leisure demand function given by equation (2.4) and note that it is clearly decreasing in w. Now since $n^S(w)$ equals $1-l^D(w)$, then this means that labour supply must be increasing in w. Now an increase in w is an increase in the relative price of leisure and this will cause the household to decrease its leisure because leisure is now relatively more expensive (and, therefore, increase its labour supply) due to the substitution effect. However, an increase in w will also make the household wealthier and will cause it to increase its leisure because leisure is a normal good (and, therefore, decrease its labour supply) due to the wealth effect. Since, the overall effect for these preferences is an increase in labour supply due to an increase in w, it must be the case that the substitution effect is greater than the wealth effect.

2.C. An increase in the coefficient on c in the utility function increases the utility the household receives from apple consumption relative to the utility the household receives from leisure. Thus, we would expect the household to demand more apples and demand less leisure when the coefficient increases. Furthermore, to afford more apples, the household has to work harder and, therefore, supply more labour to increase its apple consumption. So, in the end, we expect that an increase in the coefficient on c in the utility function will increase both the household's labour supply and its demand for apples.

2.D. When the weather is good, we have $z = 36$ and since $w^* = z$, we have $w^* = 36$. Substituting this into the leisure demand function (2.4), the consumption demand function (2.5) and the labour supply function (2.6) gives the following:

$$l^* \approx 0.5714 \qquad c^* \approx 15.4286 \qquad n^* \approx 0.4286$$

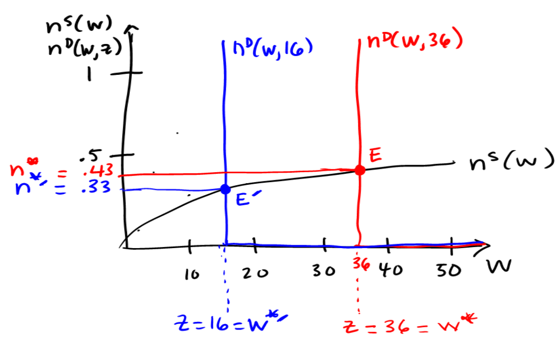
Now since apple consumption must equal apple output, we can also derive output of apples as $y^* = c^* \approx 14.4286$. We could also obtain this from the technology: $y^* = 36n^* \approx (36)(.4286) = 14.4286$.

When the weather is bad, we have $z = 16$ and since $w^* = z$, we have $w^* = 16$. Substituting this into (2.4)-(2.6) gives the following:

$$l^* \approx 0.6667 \qquad c^* = y^* \approx 5.3333 \qquad n^* \approx 0.3333.$$

2.E.

w	$\frac{w^{1/2}}{w^{1/2}+8}$
1	.11
2	.15
5	.21
10	.28
20	.36
30	.40
40	.44
50	.47



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2.F. By examining our answers to (2.D.), we see that when z is relatively high (equal to 36) that aggregate output, the real wage, aggregate consumption, and aggregate hours are all relatively high compared to when z is relatively low (equal to 16). Hence all the variables in question are high when output is high and low when output is low – that is they move in the same direction as output over the business cycle. Thus, all of the variables in question are *procyclical.*

Question 3

3.A. A household which seeks to maximize its utility subject to its time constraint and its budget constraint will choose consumption of the final good and time devoted to leisure to satisfy the following two equations (in an interior solution with $c > 0$ and $0 < l < 1$):

$$MRS(c,l) \equiv \frac{MU_l(c,l)}{MU_c(c,l)} = w$$

$$c + wl = w.$$

For the preferences given in this question, these equations become

$$MRS(c,l) \equiv \frac{MU_l(c,l)}{MU_c(c,l))} = \left(\frac{\frac{8}{l}}{\frac{2}{c}}\right) = w \tag{4.1}$$

$$c + wl = w. \tag{4.2}$$

We can simplify equation (4.1) as follows:

$$4\left(\frac{c}{l}\right) = w.$$

Rearranging gives

$$c = \frac{wl}{4}. \tag{4.3}$$

Substituting this into the overall constraint given by equation (4.2) gives:

$$\frac{wl}{4} + wl = w.$$

Now, dividing all terms by w and pulling out the common l on the left-hand side gives

$$l\left[\frac{1}{4} + 1\right] = 1.$$

Solving this equation for l gives us the household's leisure demand function:

$$l^D(w) = \frac{4}{5} \tag{4.4}$$

Substituting $l^D(w)$ into the time constraint gives labour supply function:

$$n^S(w) = 1 - l^D(w) = 1 - \frac{4}{5} = \frac{1}{5}.$$

We see that labour supply is independent of w, that is the household will always spend one-fifth of its time working, regardless of the level of the real wage. This must imply that the substitution effect and the wealth effect are of the same magnitude and directly offset one another.

3.B. The consumer who seeks to maximize her utility subject to her overall constraint when dividends are positive will choose consumption of the final good and leisure to satisfy the following two equations:

$$MRS(c, l) \equiv \frac{MU_l(c, l)}{MU_l(c, l)} = w$$

$$c + wl = w + d.$$

For the preferences given in this question, these equations become

$$MRS(c, l) = 4\left(\frac{c}{l}\right) = w$$

$$c + wl = w + d.$$

If we solve these two equations, we derive consumption demand and leisure demand for this consumer as follows:

$$c^D(w, d) = \frac{w + d}{5} \qquad\qquad l^D(w, d) = \left(\frac{4}{5}\right)\left(\frac{w + d}{w}\right). \tag{4.5}$$

Finally, the time constraint allows us to derive the consumer's labour supply:

$$n^S(w, d) = 1 - l^D(w, d) = 1 - \left(\frac{4}{5}\right)\left(\frac{w + d}{w}\right) = \frac{w - 4d}{5w}. \tag{4.6}$$

This time, to determine if labour supply is increasing or decreasing in w, we differentiate $n^S(w)$ with respect to w:

$$\frac{\partial n^S(w)}{\partial w} = \frac{5w - 5(w-4d)}{(5w)^2} = \frac{20d}{25w^2} = \frac{4d}{5w^2} > 0.$$

Thus, an increase in w increases the household's labour supply. We can see directly that an increase in dividends, d, decreases household's labour supply.

3.C. We know that the firm will demand labour at the point where labour is paid its value marginal product. That is, where the real wage equals the derivative of the production function with respect to labour input (with $k = 8$):

$$\left(\frac{2}{3}\right) z(2)n^{-\left(\frac{1}{3}\right)} = w.$$

Solving this equation gives the firm's labour demand function:

$$n^D(w,z) = \frac{64z^3}{27w^3}.$$

3.D. We can solve for the firm's supply function for output by substituting in their labour demand function into their technology:

$$y^S(w,z) = z(2)\left(n^D(w,z)\right)^{\frac{2}{3}} = 2z\left(\frac{64z^3}{27w^3}\right)^{\frac{2}{3}} = \frac{32z^3}{9w^2}.$$

From here, we can calculate the dividend function:

$$d(w,z) \equiv y^S(w,z) - wn^D(w,z) = \frac{32z^3}{9w^2} - w\left(\frac{64z^3}{27w^3}\right) = \frac{32z^3}{27w^2}.$$

We can derive labour supply as a function of w and z (denote this function as $\hat{n}^S(w,z)$) by substituting this dividend function into the labour supply function we derived in (3.B.):

$$\hat{n}^S(w,z) \equiv n^S(w, d(w,z)) = \frac{w - 4\left(\frac{32z^3}{27w^2}\right)}{5w} = \frac{1}{5} - \frac{128z^3}{135w^3}.$$

3.E. Now, if we set labour supply equal to labour demand, we can find the equilibrium real wage, w^* that clears the labour market:

$$\hat{n}^S(w^*, z) = n^D(w^*, z),$$

or

$$\frac{1}{5} - \frac{128z^3}{135w^{*3}} = \frac{64z^3}{27w^{*3}}.$$

Solving this for the equilibrium real wage, w^*, gives

$$w^* \approx 2.5506z.$$

Substituting this into either the labour demand or the labour supply function gives equilibrium labour input (here we use the labour demand function):

$$n^* = \frac{64z^3}{27(2.5506z)^3} \approx .1429$$

Examining these expressions, we see that an increase in z will increase the equilibrium wage but will not change labour input (equilibrium labour input is independent of z). Thus, since $y^* = zn^*$, we know that an increase in z will increase y^*. So we can conclude that when z is high, output is high and the real wage is high implying that the real wage is procyclical. In contrast labour input is the same for every level of productivity which means it is constant over the business cycle which means that hours are acyclical in this economy.

So we see that in this model with a technology for final goods production which is not linear in n, we have positive dividends, a downward sloping labour demand function, and an equilibrium real wage that is positively related to productivity, z, but not equal to z. If we compare this to Question 2, where the technology for final goods production is linear in n, we see that in that question we have zero dividends, an L-shaped labour demand function, and an equilibrium real wage that equals productivity, z.