Circle: DGD 1 (Frédéric) **SOLUTIONS**
DGD 2 (Yue)
DGD 3 (Andrew)

Marks: /10

MAT 1348A (Prof. T. Schmah) — Ninth Homework Assignment
Due Thursday April 7, 2016 by 11:00am

Instructions:

Print out this document and staple the pages. You may write on both sides of the paper or insert additional pages if necessary.

Submit a finished, presentable product. *Drafts and illegible papers will not be marked.* Show all relevant work to receive full credit. Submit the assignment to your TA in the DGD or in the *submission box labeled MAT 1348A* in the Department of Mathematics and Statistics. Circle the DGD you attend. Your marked paper will be returned to you in that DGD. Late assignments will not be accepted.

This is **not** a group assignment.

Important note on academic integrity:

Students are permitted, and indeed encouraged, to discuss homework problems with others, but are not permitted to help each other write the final solutions (unless the assignment is explicitly announced as a group assignment). Once you understand a solution, you must write it out entirely by yourself. For each question, any help from other people must be clearly acknowledged, as well as any sources used (e.g. textbooks, websites, videos), if that source contains a solution to a very similar question, or a new method or idea that you used that was not in the course materials. Failure to follow these rules constitutes plagiarism (academic fraud). Note that if one student copies from the other, both students have committed academic fraud. If we believe plagiarism has occurred, the students will receive:

- a mark of 0 for the current assignment if this is the first offence;
- a mark of 0 for the whole assignment component of the course if this is the second offence.

Students are advised to carefully examine the *University Guidelines on Academic Integrity* — see

`http://web5.uottawa.ca/mcs-smc/academicintegrity/home.php`

as well as the *Course Policy on Plagiarism* — see

`http://mysite.science.uottawa.ca/msajna//teaching/plagiarism_policy.html`

Please sign below to confirm that you have read and understood these regulations and policies, and you agree to act with academic integrity as defined therein. Only signed papers will have the mark recorded.

Student's signature:

1. What is the coefficient of

   (a) x^{10}

   (b) x^{11}

   in the expansion of $(2x - \frac{1}{x^2})^{16}$? *Fully evaluate your answers.* [5pts]

$$\left(2x - \frac{1}{x^2}\right)^{16} = \sum_{i=0}^{16} \binom{16}{i} (2x)^{16-i} (-x^{-2})^i$$

$$= \sum_{i=0}^{16} \binom{16}{i} 2^{16-i} (-1)^i x^{16-3i}$$

(a) coefficient of x^{10}:

we need $16-3i = 10$ so $i=2$

Answer: $\binom{16}{2} 2^{16-2} (-1)^2 = \frac{16 \cdot 15}{2} \cdot 2^{14} = \underline{\underline{1,966,080}}$

(b) coefficient of x^{11}:

we need $16-3i = 11$ but there is no such $i \in \mathbb{N}$.

Hence the coefficient of x^{11} is 0.

Answer: $\underline{\underline{0}}$

2. Use Mathematical Induction to prove that 6 divides $n^3 - n$ for all $n \in \mathbb{N}$. *Clearly define the proposition $P(n)$ to be proved. Explicitly state the induction hypothesis and indicate where it is used in the induction step.* [5pts]

Let $P(n)$: " $6 \mid n^3 - n$ ".

We must prove $P(n)$ is T for all $n \in \mathbb{N}$.

BI: to prove $P(0)$: " $6 \mid 0^3 - 0$ "

Since $0^3 - 0 = 0$ and $0 = 6 \cdot 0$, indeed $6 \mid 0$

and $P(0)$ is T.

IS: to prove $P(k) \to P(k+1)$ for all $k \geq 0$.

Fix $k \geq 0$, and assume $P(k)$ is T,

That is, assume IH : $6 \mid k^3 - k$.

show that $P(k+1)$ follows:

$$(k+1)^3 - (k+1) = k^3 + 3k^2 + 3k + 1 - k - 1$$

$$= (k^3 - k) + 3k(k+1)$$

Observe that $k(k+1)$ is even for all $k \in \mathbb{N}$

(because either k or $k+1$ is even).

Hence $6 \mid 3k(k+1)$. Also, $6 \mid (k^3 - k)$ by IH.

Hence $6 \mid ((k^3 - k) + 3k(k+1))$, i.e.

$6 \mid ((k+1)^3 - (k+1))$, and $P(k+1)$ follows.

Conclusion: Since $P(0)$ is T, and $P(k) \to P(k+1)$

is T for all $k \geq 0$, it follows that $P(n)$

is T for all $n \geq 0$.