



uOttawa

MAT 1341 F – DGD 2
Test 1 (Diagnostic Test)

September 26, 2016 Duration: 80 minutes

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Famil _____
 First _____
 Stu _____

ϑ	$\sin(\vartheta)$	$\cos(\vartheta)$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0

	Answer
1	F
2	D
3	B
4	E
5	C
6	C
7	C
8	D
9	A
10	D
11	B
12	F
Σ	11

PLEASE READ THESE INSTRUCTIONS CAREFULLY

1. You have 80 minutes to complete this exam.
2. This is a closed book exam, and no notes of any kind are permitted. The use of calculators, cell phones, or similar devices is not permitted. All implanted cyber devices not necessary for life-support must be disabled at the beginning of the exam.
3. Read each question carefully – you will save yourself time and unnecessary grief later on.
4. All questions are multiple choice, are worth 1 point each and no part marks will be given. Please record your answers in the spaces on this page provided next to the question numbers above.
5. Where it is possible to check your work, do so.
6. Good luck! Bonne chance!

1. A Cartesian equation of the plane which contains the point (2, 4, 3) and which is perpendicular to the planes with Cartesian equations $x + 2y - z = 1$ and $3x - 4y = 2$ is:

- A. $4x - 3y + 10z = -50$
- B. $4x + 3y - 10z = 50$
- C. $4x - 3y + 10z = 50$
- D. $-4x + 3y + 10z = 50$
- E. $4x + 3y + 10z = -50$
- F. $4x + 3y + 10z = 50$

$$(1, 2, -1) \times (3, -4, 0)$$

$$\begin{vmatrix} 1 & 2 & -1 & 1 & 2 & -1 \\ 3 & -4 & 0 & 3 & -4 & 0 \end{vmatrix}$$

$$(+4, +3, +10)$$

$$+4(2) + 3(4) + 10(3) =$$

$$8 + 12 + 30 = 50$$

2. An equation of the plane passing through the points (2, 1, -1) and (3, 2, 1) and parallel to the y -axis is:

- A. $x + y - z = 4$
- B. $2y - z = 5$
- C. $2x - y = 5$
- D. $2x - z = 5$
- E. $2x + z = 5$
- F. $2y - z = -5$

$$(3, 2, 1) - (2, 1, -1)$$

$$(1, 1, 2)$$

$$(1, 1, 2) \times (0, 1, 0)$$

$$\begin{vmatrix} 1 & 1 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{vmatrix}$$

$$(-2, 0, 1)$$

3. An equation of the plane containing the point $(1, -1, 2)$ and the line with parametric equations $x = 4, y = -1 + 2t, z = 2 + t$ is:

A. $x + y - 2z = -5$

B. $y - 2z = -5$

C. $y + 2z = -5$

D. $y - 2z = 5$

E. $x + y + 2z = 5$

F. $y + 2z = 5$

$\checkmark (0, 2, 1)$ $(4, -1, 2)$ $\checkmark (3, 2, 1)$

$(0, 2, 1) \times (3, 2, 1)$

$\begin{array}{r|l} 0 & z & 1 & 0 & z & 1 \\ 3 & 2 & 1 & 3 & 2 & 1 \end{array}$

$(0, 3, -6)$

4. Parametric equations for the line containing $(3, -1, 4)$ and $(-1, 5, 1)$ are:

A. Such a line does not exist.

B. $x = 1 - 2t, y = -5 + 4t, z = 1; t \in \mathbf{R}$.

C. $x = -1 - t, y = 5 - 6t, z = 1 + 3t; t \in \mathbf{R}$.

D. $x = 3 + 4t, y = -1 - 6t, z = 4 + t; t \in \mathbf{R}$.

E. $x = 3 + 4t, y = -1 - 6t, z = 4 + 3t; t \in \mathbf{R}$.

F. $x = -1 + 4t, y = 5 + 6t, z = 2 + 3t; t \in \mathbf{R}$.

$4, -6, 3$

$(-1, 5, 1) - (3, -1, 4)$

$(-4, 4, -3)$

5. Find a Cartesian (scalar) equation for the plane with vector parametric equation

$$v = (0, 0, -2) + s(1, 1, 2) + t(2, -4, 1); s, t \in \mathbf{R}.$$

- A. $x + y + 2z = -4$
- B. $2x - 4y + z = -2$
- C. $3x + y - 2z = 4$
- D. $3x - y - 2z = -4$
- E. $9x + 2y + 5z = -6$
- F. $9x - 2y + 5z = -1$

$$(1, 1, 2) \times (2, -4, 1)$$

$$\begin{vmatrix} 1 & 1 & 2 \\ 2 & -4 & 1 \end{vmatrix}$$

$$(9, 3, -6)$$

$$(3, 1, -2)$$

* 6. The set of all vectors in \mathbf{R}^3 which are perpendicular to both $(-1, 1, 5)$ and $(2, 1, 2)$.

- A. $\{(3, -12, 3)\}$
- B. $\{(t+3, -12, t+3) \mid t \in \mathbf{R}\}$
- C. $\{(t, -4t, t) \mid t \in \mathbf{R}\}$
- D. $\{(-t, 0, t) \mid t \in \mathbf{R}\}$
- E. $\{(0, 0, 0)\}$
- F. $\{(3, 12, 3)\}$

$$(-1, 1, 5) \times (2, 1, 2)$$

$$\begin{vmatrix} -1 & 1 & 5 \\ 2 & 1 & 2 \end{vmatrix}$$

$$(-3, 12, -3)$$

$$(1, -4, 1)$$

7. The angle between the vectors $\overset{u}{(0, 3, -3)}$ and $\overset{v}{(-2, 2, -1)}$ is:

- A. $\pi/6$
- B. $\pi/2$
- C. $\pi/4$
- D. $\pi/3$
- E. $\pi/5$
- F. $\pi/7$

$$\cos \theta = \frac{u \cdot v}{|u| \cdot |v|}$$

$$= \frac{6 + 3}{\sqrt{18} \cdot \sqrt{9}}$$

$$\frac{9}{3\sqrt{18}} = \frac{3}{\sqrt{18}}$$

$$\sqrt{18} = \sqrt{2} \cdot 3$$

$$\frac{1}{\sqrt{2}}$$

8. The orthogonal projection $\text{proj}_u v$ of $v = (-5, 1, 8)$ along $u = (3, 0, 3)$ is:

- ~~x~~ A. $(1, -\frac{1}{5}, -\frac{8}{5})$
- ~~x~~ B. $(-1, \frac{1}{5}, \frac{8}{5})$
- C. $(-\frac{3}{2}, 0, -\frac{3}{2})$
- D. $(\frac{3}{2}, 0, \frac{3}{2})$
- ~~x~~ E. $(5, -1, -8)$
- ~~x~~ F. $(-5, 1, -8)$

$$\text{proj}_u v = \frac{-15^2 + 0 + 24^2}{|u|^2} \cdot (3, 0, 3)$$

$$\sqrt{18}$$

* 9. The volume of the parallelepiped with a vertex at the origin and edges given by the vectors $u = (1, 1, 2)$, $v = (0, 2, 5)$ and $w = (1, 0, 1)$ is:

- A. 3
- B. 7
- C. 9
- D. 10
- E. 11
- F. 14

$$V = |u \times v| \cdot w$$

$$\begin{vmatrix} 1 & 2 & 1 & 1 \\ 0 & 2 & 5 & 0 \\ 2 & 5 & 0 & 2 \end{vmatrix}$$

$$(1, -5, 2) \cdot (1, 0, 1)$$

$$1 + 0 + 2$$

* 10. Find the area of the triangle with vertices $A = (-1, 5, 0)$, $B = (1, 0, 4)$ and $C = (1, 4, 0)$.

- A. 1
- B. 2
- C. 3
- × D. 4
- E. 5
- ✓ → F. 6

$$\frac{1}{2} (AB) \times (AC)$$

$$\vec{AB} = (2, -5, 4) \quad \vec{BA} = (-2, 5, -4)$$

$$\vec{AC} = (2, -1, 0) \quad \vec{CA} = (-2, 1, 0)$$

$$\vec{BC} = (0, -4, 4)$$

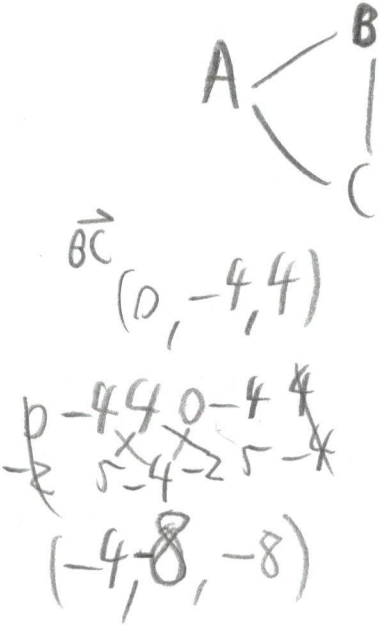
$$\begin{vmatrix} 2 & -5 & 4 & 2 & -5 & 4 \\ 2 & -1 & 0 & 2 & -1 & 0 \end{vmatrix}$$

$$(4, 8, 12)$$

$$(4, 8, 8)$$

$$\begin{vmatrix} 5 & 0 & -1 & 5 & 0 & -1 \\ 0 & 4 & 1 & 0 & 4 & 1 \end{vmatrix}$$

$$(20, -4, -5)$$



*11. Find

- A. $1/2$
- B. $\sqrt{2}/2$
- C. $\sqrt{8/11}$
- D. $8/25$
- E. $3/2$
- F. $\sqrt{14/11}$

Handwritten work for problem 11:

$\frac{\sqrt{4}}{\sqrt{20}} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$
 $\frac{\sqrt{8}}{\sqrt{11}} = \frac{2\sqrt{2}}{\sqrt{11}}$
 $\left| \frac{3-i}{2-4i} \right| = \frac{\sqrt{10}}{\sqrt{20}} = \frac{\sqrt{10}}{2\sqrt{5}} = \frac{\sqrt{2}}{2}$
 $\frac{2+4i}{2+4i} = 1$
 $\frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$
 $\frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$
 $\frac{4}{6} = \frac{2}{3}$
 $\frac{6+12i-2i+4}{4+16} = \frac{10+10i}{20} = \frac{1}{2} + \frac{1}{2}i$
 $\frac{1}{2} + \frac{1}{2}i$
 $\frac{2}{\sqrt{20}} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$
 $\frac{2\sqrt{20}}{20} = \frac{2\sqrt{20}}{20} = \frac{\sqrt{20}}{10} = \frac{2\sqrt{5}}{10} = \frac{\sqrt{5}}{5}$
 $\frac{\sqrt{5}}{5}$
 $\frac{2\sqrt{5}}{10} = \frac{\sqrt{5}}{5}$
 $\tan \theta = \frac{1}{3}$
 $\theta = \frac{\pi}{6}$
 $\tan \theta = \frac{-4}{-2} = 2$
 $\frac{2}{\sqrt{20}} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$
 $\frac{\sqrt{5}}{5}$
 $\frac{2\sqrt{5}}{10} = \frac{\sqrt{5}}{5}$

12. Find the polar form of

$$\frac{5 + 5\sqrt{3}i}{\sqrt{2} - \sqrt{2}i}$$

- *A. $5(\cos(\frac{5\pi}{12}) - i \sin(\frac{5\pi}{12})) = 5e^{-i\frac{5\pi}{12}}$
- B. $5(\cos(\frac{11\pi}{12}) + i \sin(\frac{11\pi}{12})) = 5e^{i\frac{11\pi}{12}}$
- *C. $5(\cos(\frac{7\pi}{12}) - i \sin(\frac{7\pi}{12})) = 5e^{-i\frac{7\pi}{12}}$
- D. $5(\cos(\frac{5\pi}{12}) + i \sin(\frac{5\pi}{12})) = 5e^{i\frac{5\pi}{12}}$
- *E. $5(\cos(\frac{11\pi}{12}) - i \sin(\frac{11\pi}{12})) = 5e^{-i\frac{11\pi}{12}}$
- F. $5(\cos(\frac{7\pi}{12}) + i \sin(\frac{7\pi}{12})) = 5e^{i\frac{7\pi}{12}}$

Handwritten work for problem 12:

$\tan \theta = \frac{5\sqrt{3}}{5} = \sqrt{3}$
 $\theta_1 = \frac{\pi}{3} = \frac{4\pi}{12}$
 $\tan \theta = -1$
 $\theta_2 = \frac{\pi}{4} = \frac{3\pi}{12}$
 $\theta_1 - \theta_2 = \frac{4\pi}{12} - (-\frac{3\pi}{12}) = \frac{7\pi}{12}$
 $\frac{7\pi}{12}$
 $\frac{2\pi}{12} = \frac{7\pi}{12}$