(A)

MAT 1320 A   Fall 2015   October 7th, 8:30   Prof. Desjardins

TEST #1

Max = 15

Name: $\boxed{\text{Solutions}}$

Student Number: ______________________________

• Time: 80 min.

• Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability.

• Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.

• Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite. Graphs and sketches may be drawn in pencil.

• Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

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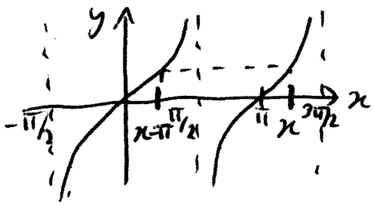
Signature: ______________________________

(A)

1. [1 point] Simplify: $16^{\log_4 x}$.

$$16^{\log_4 x} = (4^2)^{\log_4 x} = (4^{\log_4 x})^2 = \boxed{x^2}$$

2. [1 point] If $\pi/2 < x < 3\pi/2$, what is $\arctan(\tan(x))$?



$$\arctan(\tan(x)) = \boxed{x - \pi}$$

3. [1 point] If $f(x)$ is continuous at $x = a$, must it be differentiable there?

(A) YES (B) NO

4. [1 point] List the three common types of discontinuities.

$\boxed{\text{hole, jump, vertical asymptote}}$

5. [1 point] If $f(x) = \ln(6x + 7)$, what is $f^{-1}(x)$?

if $y = \ln(6x+7)$

$e^y = 6x+7$

$6x = e^y - 7$

$x = \frac{1}{6}(e^y - 7)$

$\therefore \boxed{f^{-1}(x) = \frac{1}{6}(e^x - 7)}$

6. [2 points] Find the limit $\lim_{x \to 1} \dfrac{\sqrt{x+3} - 2}{x - 1}$.

$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \lim_{x\to 1} \left(\frac{\sqrt{x+3}-2}{x-1}\right)\left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right)$$

$$= \lim_{x\to 1} \frac{(x+3)-4}{(x-1)(\sqrt{x+3}+2)}$$

$$= \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)}$$

$$= \lim \frac{1}{\ } = \boxed{1/4}$$

(A)

7. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x+1}$. Then verify your answer with the Quotient Rule.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{(x+h)(x+1) - x(x+h+1)}{(x+1)(x+h+1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{\cancel{x^2} + \cancel{xh} + \cancel{x} + h - \cancel{x^2} - \cancel{xh} - \cancel{x}}{(x+1)(x+h+1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{h}{(x+1)(x+h+1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{(x+1)(x+h+1)}$$

$$= \boxed{\frac{1}{(x+1)^2}}$$

check: $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{(1)(x+1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$$

A

8. [5 points] Find the first derivatives of the following functions.

(a) $f(x) = e^{2x}\cos x$

$$f'(x) = 2e^{2x}\cos x - e^{2x}\sin x$$
$$= \boxed{e^{2x}(2\cos x - \sin x)}$$

(b) $g(t) = \sqrt{4t^2 + 6t - 2}$

$$g'(t) = \frac{1}{2}(4t^2+6t-2)^{-1/2}(8t+6)$$
$$= \boxed{\frac{4t+3}{\sqrt{4t^2+6t-2}}}$$

(c) $\varphi(\theta) = \tan(3e^{\theta})$

$$\varphi'(\theta) = \sec^2(3e^{\theta})\,(3e^{\theta})$$
$$= \boxed{3e^{\theta}\sec^2(3e^{\theta})}$$

(d) $p(x) = \dfrac{3x^2 + 5x}{x + 1}$

$$p'(x) = \frac{(6x+5)(x+1) - (3x^2+5x)(1)}{(x+1)^2}$$
$$= \boxed{\frac{3x^2+6x+5}{(x+1)^2}}$$

(e) $y = x^3 e^{\sin x}$

$$\frac{dy}{dx} = 3x^2 e^{\sin x} + x^3 e^{\sin x}\cos x$$
$$= \boxed{x^2 e^{\sin x}(3 + x\cos x)}$$

(B)

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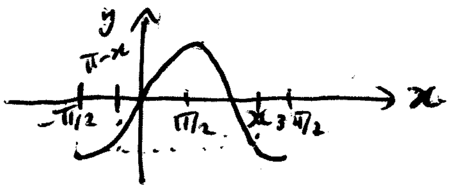
(B)

1. [1 point] Simplify: $27^{\log_3 x}$.

$$27^{\log_3 x} = (3^3)^{\log_3 x} = (3^{\log_3 x})^3 = \boxed{x^3}$$

2. [1 point] If $\pi/2 < x < 3\pi/2$, what is $\arcsin(\sin(x))$?



$$\arcsin(\sin(x)) = \boxed{\pi - x}$$

3. [1 point] If $f(x)$ is continuous at $x = a$, must it be differentiable there?

(A) NO (B) YES

4. [1 point] List the three common types of discontinuities.

$\boxed{\text{hole, jump, vertical asymptote}}$

5. [1 point] If $f(x) = \ln(2x+3)$, what is $f^{-1}(x)$?

if $y = \ln(2x+3)$

$e^y = 2x+3$

$2x = e^y - 3$

$x = \frac{1}{2}(e^y - 3)$

$\therefore \boxed{f^{-1}(x) = \frac{1}{2}(e^x - 3)}$

6. [2 points] Find the limit $\lim\limits_{x \to 2} \dfrac{\sqrt{x+2} - 2}{x - 2}$.

$$\lim_{x\to 2} \frac{\sqrt{x+2} - 2}{x-2} = \lim_{x \to 2} \left(\frac{\sqrt{x+2} - 2}{x-2}\right)\left(\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}\right)$$

$$= \lim_{x\to 2} \frac{(x+2) - 4}{(x-2)(\sqrt{x+2}+2)}$$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)(\sqrt{x+2}+2)}$$

$$= \lim \frac{1}{\quad} = \boxed{1/4}$$

(B)

7. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x+2}$. Then verify your answer with the Quotient Rule.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x+h}{x+h+2} - \frac{x}{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{(x+h)(x+2) - x(x+h+2)}{(x+2)(x+h+2)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{\cancel{x^2} + \cancel{xh} + \cancel{2x} + 2h - \cancel{x^2} - \cancel{xh} - \cancel{2x}}{(x+2)(x+h+2)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2h}{(x+2)(x+h+2)}\right)$$

$$= \lim_{h\to 0} \frac{2}{(x+2)(x+h+2)}$$

$$= \boxed{\frac{2}{(x+2)^2}}$$

check: $\dfrac{d}{dx}\left(\dfrac{x}{x+2}\right) = \dfrac{(1)(x+2) - x(1)}{(x+2)^2} = \dfrac{2}{(x+2)^2}$

(B)

8. [5 points] Find the first derivatives of the following functions.

(a) $f(x) = e^{2x}\sin x$

$$f'(x) = 2e^{2x}\sin x + e^{2x}\cos x$$
$$= \boxed{e^{2x}(2\sin x + \cos x)}$$

(b) $g(t) = \sqrt{3t^2 + 5t - 1}$

$$g'(t) = \frac{1}{2}(3t^2+5t-1)^{-1/2}(6t+5)$$
$$= \boxed{\frac{6t+5}{2\sqrt{3t^2+5t-1}}}$$

(c) $\varphi(\theta) = \sec(2e^{\theta})$

$$\varphi'(\theta) = \sec(2e^{\theta})\tan(2e^{\theta})(2e^{\theta})$$
$$= \boxed{2e^{\theta}\sec(2e^{\theta})\tan(2e^{\theta})}$$

(d) $p(x) = \dfrac{2x^2+4x}{x+3}$

$$p'(x) = \frac{(4x+4)(x+3) - (2x^2+4x)(1)}{(x+3)^2}$$
$$= \boxed{\frac{2x^2+12x+12}{(x+3)^2}}$$

(e) $y = x^4 e^{\cos x}$

$$\frac{dy}{dx} = 4x^3 e^{\cos x} + x^4 e^{\cos x}(-\sin x)$$
$$= \boxed{x^3 e^{\cos x}(4 - x\sin x)}$$

C

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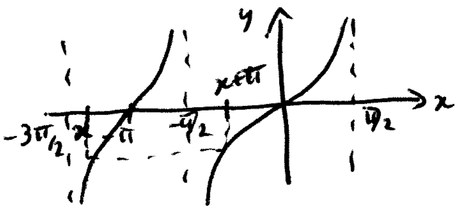
Signature: ____________________

C

1. [1 point] Simplify: $64^{\log_4 x}$.

$$64^{\log_4 x} = (4^3)^{\log_4 x} = (4^{\log_4 x})^3 = \boxed{x^3}$$

2. [1 point] If $-3\pi/2 < x < -\pi/2$, what is $\arctan(\tan(x))$?



$$\arctan(\tan(x)) = \boxed{x+\pi}$$

3. [1 point] If $f(x)$ is continuous at $x = a$, must it be differentiable there?

(A) YES (B) NO

4. [1 point] List the three common types of discontinuities.

$\boxed{\text{hole, jump, vertical asymptote}}$

5. [1 point] If $f(x) = \ln(4x+5)$, what is $f^{-1}(x)$?

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$e^y = 4x+5$

$4x = e^y - 5$

$x = \frac{1}{4}(e^y - 5)$

$\therefore \boxed{f^{-1}(x) = \frac{1}{4}(e^x - 5)}$

6. [2 points] Find the limit $\lim\limits_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$.

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \left(\frac{\sqrt{x+1}-2}{x-3}\right)\left(\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}\right)$$

$$= \lim_{x\to 3} \frac{(x+1)-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)}$$

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C

7. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x-1}$. Then verify your answer with the Quotient Rule.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

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$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{(x+h)(x-1) - x(x+h-1)}{(x-1)(x+h-1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{\cancel{x^2} + \cancel{xh} - \cancel{x} - h - \cancel{x^2} - \cancel{xh} + \cancel{x}}{(x-1)(x+h-1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{-h}{(x-1)(x+h-1)}\right)$$

$$= \lim_{h\to 0} \frac{-1}{(x-1)(x+h-1)}$$

$$= \boxed{\frac{-1}{(x-1)^2}}$$

Check: $\dfrac{d}{dx}\left(\dfrac{x}{x-1}\right) = \dfrac{(1)(x-1) - x(1)}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

c

8. [5 points] Find the first derivatives of the following functions.

(a) $f(x) = e^{3x}\cos x$

$$f'(x) = 3e^{3x}\cos x - e^{3x}\sin x$$
$$= \boxed{e^{3x}(3\cos x - \sin x)}$$

(b) $g(t) = \sqrt{5t^2 + 8t + 2}$

$$g'(t) = \frac{1}{2}(5t^2+8t+2)^{-1/2}(10t+8)$$
$$= \boxed{\frac{5t+4}{\sqrt{5t^2+8t+2}}}$$

(c) $\varphi(\theta) = \cot(2e^{\theta})$

$$\varphi'(\theta) = -\csc^2(2e^{\theta})(2e^{\theta})$$
$$= \boxed{-2e^{\theta}\csc^2(2e^{\theta})}$$

(d) $p(x) = \dfrac{3x^2 + 7x}{x + 2}$

$$p'(x) = \frac{(6x+7)(x+2) - (3x^2+7x)(1)}{(x+2)^2}$$
$$= \boxed{\frac{3x^2 + 12x + 14}{(x+2)^2}}$$

(e) $y = x^3\, e^{\cos x}$

$$\frac{dy}{dx} = 3x^2 e^{\cos x} + x^3 e^{\cos x}(-\sin x)$$
$$= \boxed{x^2 e^{\cos x}(3 - x\sin x)}$$