

1. (1 point) It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the **order of the equation** – (what is the highest number of derivatives involved) and whether or not the equation is **linear**. Linearity is important because the structure of the the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

[?] 1. $(1+y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$

[?] 2. $\frac{d^2y}{dt^2} + \sin(t+y) = \sin t$

[?] 3. $t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin t$

[?] 4. $\frac{dy}{dt} + ty^2 = 0$

Correct Answers:

- 2NONLINEAR
- 2NONLINEAR
- 2LINEAR
- 1NONLINEAR

2. (1 point) In problems below, (a) identify the independent variable and the dependent variable of each equation (use 't' for the independent variable if an independent variable is not given explicitly); (b) give the order of each differential equation (enter '1' for first order, '2' for second order and so on; do not include the quotes); and (c) state whether the equation is linear or nonlinear. If your answer to (c) is nonlinear, make sure that you can explain why this is true.

equation	(a) independent	(a) dependent	(b) order	(c) linear/nonlinear
$y' = y - x^2$	—	—	—	[?/Nonlinear]
$xy' = 2y$	—	—	—	[?/Linear]
$x'' + 5x = e^{-x}$	—	—	—	[?/Linear]

Correct Answers:

- x
- y
- 1
- linear
- x
- y
- 1
- linear

- t
- x
- 2
- nonlinear

3. (1 point) Determine the order of the given differential equation and state whether the equation is linear or nonlinear.

$$(\sin \theta)y''' - (\cos \theta)y' = 6$$

(a) The order of this differential equation is ____.

(b) The equation is [Choose/Linear/Nonlinear].

Correct Answers:

- 3
- Linear

4. (1 point) Determine the order of the given differential equation and state whether the equation is linear or nonlinear.

$$\frac{d^5u}{dr^5} + \frac{du}{dr} + 8u = \cos(r+u)$$

(a) The order of this differential equation is ____.

(b) The equation is [Choose/Linear/Nonlinear].

Correct Answers:

- 5
- Nonlinear

5. (1 point) Find all values of k for which the function $y = \sin(kt)$ satisfies the differential equation $y'' + 5y = 0$. Separate your answers by commas. Hint: There are more than 2 values of k

Correct Answers:

- 2.23606797749979, -2.23606797749979, 0

6. (1 point) Match the following differential equations with their solutions. The symbols A, B, C in the solutions stand for arbitrary constants.

You must get all of the answers correct to receive credit.

- 1. $\frac{d^2y}{dx^2} + 25y = 0$
- 2. $\frac{dy}{dx} = \frac{-2xy}{x^2 - 5y^2}$
- 3. $\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 36y = 0$
- 4. $\frac{dy}{dx} = 10xy$

5. $\frac{dy}{dx} + 18x^2y = 18x^2$

- A. $y = A \cos(5x) + B \sin(5x)$
- B. $y = Ce^{-6x^3} + 1$
- C. $y = Ae^{-6x} + Bxe^{-6x}$
- D. $3yx^2 - 5y^3 = C$
- E. $y = Ae^{5x^2}$

Correct Answers:

- A
- D
- C
- E
- B

7. (1 point) Which of the following functions are solutions of the differential equation $y'' + 10y' + 25y = 0$?

- A. $y(x) = -5x$
- B. $y(x) = x^2e^{-5x}$
- C. $y(x) = -5xe^{+5x}$
- D. $y(x) = e^{-5x}$
- E. $y(x) = e^{+5x}$
- F. $y(x) = 0$
- G. $y(x) = xe^{-5x}$

Correct Answers:

- DFG

8. (1 point) If $y = e^{2t}$ is a solution to the differential equation

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + ky = 0,$$

find the value of the constant k and the general solution to this equation.

$k =$ _____

$y =$ _____

(Use constants $A, B, \text{etc.}$, for any constants in your solution formula.)

Correct Answers:

- 8
- $A * e^{(2 * t)} + B * e^{(4 * t)}$

9. (1 point) The functions

$$y = x^2 + \frac{c}{x^2}$$

are all solutions of equation:

$$xy' + 2y = 4x^2, (x > 0).$$

Find the constant c which produces a solution which also satisfies the initial condition $y(9) = 5$.

$c =$ _____

Correct Answers:

- -6156

10. (1 point) The solution of a certain differential equation is of the form

$$y(t) = a \exp(5t) + b \exp(6t),$$

where a and b are constants.

The solution has initial conditions $y(0) = 4$ and $y'(0) = 4$.

Find the solution by using the initial conditions to get linear equations for a and b .

$y(t) =$ _____

Correct Answers:

- $20 * (e^{(5 * t)}) + -16 * (e^{(6 * t)})$

11. (1 point) Is the piecewise-defined function

$$y = \begin{cases} -x^2, & \text{if } x < 0 \\ x^2, & \text{if } x \geq 0 \end{cases}$$

a solution of the differential equation $xy' - 2y = 0$ on $(-\infty, \infty)$?

You have only one attempt for credit.

[Choose/Yes/No]

Correct Answers:

- Yes

12. (1 point) Is the family of functions $P = \frac{c_1 e^t}{1 + c_1 e^t}$ a so-

lution of the differential equation $\frac{dP}{dt} = P(1 - P)$? Assume an appropriate interval I of definition for the solution.

You have only one attempt for credit.

[Choose/Yes/No]

Correct Answers:

- Yes