MAT1320A, Prof Nevins — practise sheet

This is a list of fresh problems, for those who have completed the exercises in Stewart; these exercises are of the type and style you can expect to see on midterm 1. If you can do all these, you will have no trouble on the midterm. Come to office hours to verify your solutions in case of doubt.

1. What is the domain and range of the following functions?

1. $f(x) = \frac{x+4}{x^2-9}$
2. $f(x) = \frac{\sqrt{x-3}}{x^2+2x+1}$
3. $f(x) = \log(x+3)$
4. $f(x) = \sin(\sqrt{2x^2-2})$
5. $f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}$
6. $f(t) = \sqrt[3]{10^t - 100}$
7. $f(t) = \sqrt{10^t - 100}$
8. $y(x) = \frac{1+x}{e^{\cos(x)}}$
9. $s(t) = \ln((x-2)^2 \cdot e^{3x})$

2. Solve for x.

1. $e^{6-4x} = 3$
2. $2^{x-5} = 3$
3. $\ln(x) + \ln(x-1) = 1$
4. $\frac{x-3}{x^2+2x+1} = 1$
5. $3\ln(x^2) + 2\ln(x^3) = 5$
6. $\cos(2x) = 5$
7. $e^{3x} + e^{6x} = 5$
8. $x^4 + 4x^2 + 2 = -2$
9. $\ln(7x+3) + \ln(2x) = 2$

3. Find a formula for the inverse of the following functions:

1. $f(x) = 1 + \sqrt{2+3x}$
2. $f(x) = \ln(x+3)$
3. $f(x) = \frac{1-e^{-x}}{1+e^{-x}}$
4. $f(x) = e^{2x} - 3e^x + 2$
5. $f(x) = \log_{10}(x+5)$
6. $f(x) = \ln(e^x - 3)$

4. What is the equation of the tangent line to graphs of the following functions at the given points?

1. $f(x) = 1 + \sqrt{4+3x}$ at $(7, f(7))$?
2. $f(x) = \log_3(x+3)$ at $(6, f(6))$?
3. $f(x) = x^2 + 2x + 1$ at $(2, 9)$?
4. $f(x) = \cos(2x)$ at $(\pi/2, -1)$?

5. Compute the following limits.

1. $\lim_{x\to 4} \frac{\ln(x)-\ln(4)}{x-4}$
2. $\lim_{x\to 5^+} \frac{x+1}{x-5}$
3. $\lim_{x\to -2} \frac{x+2}{x^3+8}$
4. $\lim_{x\to 0} \sqrt{9+h} - 3h$
5. $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$
6. $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$
7. $\lim_{x\to\infty}(\sqrt{9x^2+x} - 3x)$
8. $\lim_{t\to\infty} \frac{\sqrt{x+3x^2}}{4x-1}$
9. $\lim_{x\to\infty} \frac{1+x^6}{x^4+1}$
10. $\lim_{x\to-\infty} \frac{2x^2+x-1}{x^2+x-2}$
11. $\lim_{x\to-\infty} \frac{2x^3+x-1}{x^2+x-2}$
12. $\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$

6. Find the first and second derivatives of the following functions, using any method.

1. $f(x) = x^3 + 3x + 1$
2. $y(t) = \sqrt{3t^2 + \cos(t)}$
3. $f(z) = e^{z^2}\sqrt{z+3}$
4. $f(x) = \frac{3+x^2}{5x^2+1}$
5. $f(x) = \frac{\sqrt{3x+1}}{6x^2+1}$
6. $f(x) = \sin(3x^2e^x)$
7. $f(x) = \frac{\sin(3x)}{\cos(3x)}$
8. $f(x) = \frac{\sin(2x)+1}{\cos(4x)}$
9. $f(t) = \sqrt{\frac{3}{t} + 2t}$
10. $f(y) = y^{500} \cdot e^{y^2}$
11. $f(x) = 3^{\cos(5x^2)}$

7. Compute the derivatives of each of the following functions using the definition of the derivative. Then verify your answer using known methods.

1. $f(x) = \dfrac{1}{x^3}$
2. $y(t) = \dfrac{2t}{t+1}$
3. $f(z) = z^3 - 3z^2$
4. $f(x) = \dfrac{x+3}{5x^2+1}$

8. State the following theorems or definitions presented in class. Remember that the first part of each is the hypotheses, which is the conditions under which the conclusion is true. Compare with your class notes or the textbook.

1. Theorem: The limit laws
2. The squeeze theorem
3. Definition: A continuous function
4. The Intermediate Value Theorem
5. Definition: A differentiable function
6. Theorem: relationship of continuity and differentiability
7. Theorems: linearity of the derivative, product rule, quotient rule, chain rule