Solutions

School of Mathematics and Statistics
Carleton University
Math. 2004A, Fall 2015

TEST 1

ONLY NON-PROGRAMMABLE and NON-GRAPHING Calculators permitted, 1 or more blank sheets permitted for roughs

Print Name :

Student Number:

Tutorial Section (A1, A4, ...):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. [**2 marks**] Which of the following vectors is orthogonal to (1, -1, 2)?
   (a) $(2, 0, 1)$, (b) $(-1, 0, 1/2)$, (c) $(0, 1, 3)$, (d) $(2, 1, 0)$.
2. [**2 marks**] Find a vector orthogonal to the two vectors $(2, 0, 1)$ and $(0, 3, 5)$.
   (a) $2\mathbf{i} + 10\mathbf{j} + \mathbf{k}$, (b) $-3\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$, (c) $\mathbf{i} - 10\mathbf{j} + 2\mathbf{k}$, (d) $-3\mathbf{i} - 10\mathbf{j} + 6\mathbf{k}$.
3. [**2 marks**] Find the equation of the plane Π through the three points $P(1, 0, -1)$, $Q(2, 2, 1)$ and $R(4, 1, 2)$.
   (a) $4x + 3y - 5z = 9$, (b) $-4x + 3y - 5z = -9$, (c) $4x - 3y + 5z = 6$, (d) $4x + 3y + 5z = 10$.
4. [**2 marks**] Which of the following represents a line through the point $(-1, 0, 1)$ that is parallel to the vector $(2, -1, 0)$?
   (a) $x = -1, y = t, z = 1$, (b) $x = -t, y = 1, z = 1 - 2t$, (c) $x = -1 - 2t, y = t, z = 1$, (d) $x = -2 - t, y = 1, z = t$.
5. [**2 marks**] The lines defined by $x = t, y = -2t, z = -t$ where $-\infty < t < \infty$, and $\frac{x-2}{-1} = \frac{y-1}{1} = \frac{z-3}{-3}$ are orthogonal to each other.
   (a) TRUE, (b) FALSE.

PART II: Show all work here and give details.
No additional pages will be accepted

6. [**10 marks**] Find the equation of the plane passing through the two points $P(2, -1, 4)$ and $Q(-1, 3, 0)$ that is perpendicular to the plane $3x - 4y + 5z - 1 = 0$.

The vector $\bar{n} = (3, -4, 5)$ is orthogonal/normal/perpendicular to the given plane.
The vector $\overline{PQ}$ (or $\overline{QP}$) given by $(-3, 4, -4)$ (or $(3, -4, 4)$) lies on the given plane.
So the vector $\overline{PQ} \times \bar{n}$ (or $\overline{QP} \times \bar{n}$) is a normal to the desired plane.

Now $\overline{PQ} \times \bar{n} = \begin{vmatrix} i & j & k \\ -3 & 4 & -4 \\ 3 & -4 & 5 \end{vmatrix} = (4, 3, 0)$ (or $4\hat{i} + 3\hat{j} + 0\hat{k}$)

the plane is of the form $4x + 3y = D$.

To find D, use either one of the points $(-1,3,0)$ or $(2,-1,4)$. Say $(2,-1,4)$. Then

$$D = 4(2)+3(-1)+0\cdot 4 = 8-3 = 5$$

$\therefore$ $D=5$ $\qquad \therefore$ Plane has eq'n: $\boxed{4x+3y=5}$

Check? Both $(-1,3,0)$ and $(2,-1,4)$ are ON $4x+3y=5$.
Also $(4,3,0) \perp (3,-4,5)$. ($\therefore$ Checks out)

7. **[10 marks]** Find the *symmetric equations* of the line of intersection of the two planes Π_1, Π_2 defined by $-2x+3y-4z=-3$ and $2x+8y-4z=14$ respectively.

The normals $\vec{n}_1, \vec{n}_2$ of Π_1, Π_2 must both be perpendicular to the direction vector of the required line, say $\vec{v}$.
So can choose $\vec{v} = \vec{n}_1 \times \vec{n}_2$

$$= \begin{vmatrix} i & j & k \\ -2 & 3 & -4 \\ 2 & 8 & -4 \end{vmatrix} = 20\vec{i} - 16\vec{j} - 22\vec{k}$$
$$= (20,-16,-22)$$

(or $(10,-8,-11)$ is also OK)

Once we have $\vec{v}$ all we need is a point on $\mathcal{L}$. To do this set $z=0$ (i.e., see where the line intersects the xy-plane).
Then $\boxed{-2x+3y=-3}$ and $\boxed{2x+8y=14}$

Solving these simultaneously we get $x=3$, $y=1$.
$\therefore$ $\boxed{(3,1,0) \text{ is on } \mathcal{L}}$ $\therefore$ symmetric eq'n is given by

$$\boxed{\frac{x-3}{20} = \frac{y-1}{-16} = \frac{z-0}{-22}}$$

Parametric Equations are
$x = 3+20t$
$y = 1-16t$
$z = 0-22t = -22t$