

Concordia University

Course				Number	Sections
ENGR				233	P, Q
Examination	Date	Time	Total Marks	Pages	
Final	April 2008	3 hours	100	2	

Course Coordinator

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Instructors

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Special Instructions: use of calculators and outside materials is NOT permitted.

Each problem is worth 10 marks unless stated otherwise.

Problem 1. For the vector field

$$\vec{F}(x, y, z) = x^2y\mathbf{i} + xy^2\mathbf{j} + 2xyz\mathbf{k}$$

compute **if possible** the following quantities. If it is not possible **explain why not**.

(a) $\text{div}(\text{curl } \vec{F}(x, y, z))$, (b) $\text{curl}(\text{div } \vec{F}(x, y, z))$, (c) $\text{grad}(\text{div } \vec{F}(x, y, z))$, (d) $\text{div}(\text{grad } \vec{F}(x, y, z))$

Problem 2. Find the equation of the tangent plane of the surface defined by

$$z^3 - xyz = 1$$

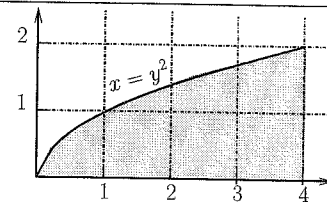
at the point $(4, \frac{1}{2}, -1)$.

Problem 3.

Evaluate the following integral by reversing the order of integration

$$\int_0^2 \int_{y^2}^4 e^{\sqrt{x^3}} dx dy$$

[Hint: the following substitution may be of help: $u = x^{\frac{3}{2}}$]



Problem 4. Find the rate of change at the point $(2, 1, 3)$ of the following function $f(x, y, z) = \frac{xy}{z^2}$ along the directions given by unit vectors parallel to

(a) $\mathbf{i}$; (b) $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

Problem 5. Using **Stokes' theorem**, compute the flux of the curl of the vector field

$$\vec{F}(x, y, z) = 6yz\mathbf{i} - 24x\mathbf{j} + yze^{x^2 + \arctan(z)}\mathbf{k}$$

across the surface S (oriented upwards) of the paraboloid $z = y^2 + x^2$, $z \leq 4$, with boundary the circle $z = 4$, $x^2 + y^2 = 4$.

Problem 6. Find the mass $M = \iiint_{\mathcal{R}} \rho(x, y, z) dV$ of the solid in the first octant (namely $x \geq 0, y \geq 0, z \geq 0$) bounded by the coordinate planes and the graph of $x + y + z = 1$ if the density is given by $\rho = x + 2y$.

Problem 7.

Evaluate the work done by the **conservative** force

$$\vec{F}(x, y) = ye^{xy}\mathbf{i} + (xe^{xy} + 2y)\mathbf{j}$$

along any path that joins the starting point $(0, 0)$ and ending point $(1, 2)$. **You must use the potential function.**

Problem 8. Find the curvature $\kappa(t)$ and the components of the acceleration $a_N(t), a_T(t)$ for the curve described by

$$\mathbf{r}(t) = (t + 1)\mathbf{i} + (t^2 - t)\mathbf{j} + e^{-t}\mathbf{k}$$

Problem 9. Use **Green's theorem** to compute the line-integral

$$\oint_C y^2 dx + x dy$$

where C is the boundary of the region determined by the graphs of $x = 0$, $x^2 + y^2 = 4$ and with $x \geq 0$.

Problem 10. Use the **Divergence Theorem** to evaluate the outward flux $\iint_S \vec{F} \cdot \vec{n} dS$ of the given vector field across the surface specified

$$\vec{F}(x, y, z) = x^3\mathbf{i} + (y^3 + xz)\mathbf{j} + (z^3 + z^2)\mathbf{k}$$

$$x^2 + y^2 + z^2 = a^2, \quad a > 0.$$
