ENGG2120 Fall 2011

Chapter 19: Assignment with-Answers

1. Estimate the energy required to raise the temperature from 20 to 100°C for 2 kg of following materials: aluminum, steel, soda–lime glass, and high-density polyethylene.

Given

$Al - C_P = 900\ \text{J/kg-K}$,

$Steel - C_P = 486\ \text{J/kg-K}$,

$Glass - C_P = 840\ \text{J/kg-K}$,

$HDPE - C_P = 1850\ \text{J/kg-K}$

Answer:

The energy *E*, required to raise the temperature of a given mass of material is product of specific heat (C_p), mass of material (m) and the temperature change (ΔT)

$$E = c_p m \Delta T$$

The $\Delta T = 100^oC - 20^oC = 80^oC$ (= 80 K), while the mass is 2 kg, and the specific heat of material is given in the question. Thus,

$$E(\text{aluminum}) = (900\ \text{J/kg-K})(2\ \text{kg})(80\ \text{K}) = 1.44 \times 10^5\ \text{J}$$

$$E(\text{steel}) = (486\ \text{J/kg-K})(2\ \text{kg})(80\ \text{K}) = 7.78 \times 10^4\ \text{J}$$

$$E(\text{glass}) = (840\ \text{J/kg-K})(2\ \text{kg})(80\ \text{K}) = 1.34 \times 10^5\ \text{J}$$

$$E(\text{HDPE}) = (1850\ \text{J/kg-K})(2\ \text{kg})(80\ \text{K}) = 2.96 \times 10^5\ \text{J}$$

2. Briefly describe the heat capacity and its significance.

Answer:

Heat capacity is a property that is indicative of a material's ability to absorb heat from the external surroundings; it represents the amount of energy required to produce a unit temperature rise.

3. Write down the units of specific heat

Answer: J/Kg K

4. State True or False

‘C_p is greater than C_v’

Answer: TRUE

5. For aluminum, the heat capacity at constant volume C_v at 30 K is 0.81 J/mol-K and the Debye temperature is 375 K. Estimate the specific heat (a) at 50 K and (b) at 425 K.

Answer:

(a) For aluminum, C_v at 50 K may be approximated by equation $C_v = AT^3$ as this temperature is significantly below the Debye temperature (375 K). The value of C_v at 30 K is given, and thus, we may compute the constant A as

$$A = \frac{C_v}{T^3} = \frac{0.81 \text{ J/mol-K}}{(30 \text{ K})^3} = 3.00 \times 10^{-5} \text{ J/mol-K}^4$$

Therefore, at 50 K

$$C_v = AT^3 = \left(3.00 \times 10^{-5} \text{ J/mol-K}^4\right)(50 \text{ K})^3 = 3.75 \text{ J/mol-K}$$

and

$$c_v = (3.75 \text{ J/mol-K})(1 \text{ mol}/26.98 \text{ g})(1000 \text{ g/kg}) = 139 \text{ J/kg-K}$$

(b) Since 425 K is above the Debye temperature, a good approximation for C_v is

$$C_v = 3R$$
$$= (3)(8.31 \text{ J/mol-K}) = 24.9 \text{ J/mol-K}$$

And, converting this to specific heat

$$c_v = (24.9 \text{ J/mol-K})(1 \text{ mol}/26.98 \text{ g})(1000 \text{ g/kg}) = 923 \text{ J/kg-K}$$

6. The constant A in Equation $C_V = AT^3$ is $12\pi^4 R/5\theta_D^3$, where R is the gas constant and θ_D is the Debye temperature (K). Estimate θ_D for copper, given that the specific heat is 0.78 J/kg-K at 10 K.

Answer:

For copper, we want to compute the Debye temperature θ_D, given the expression for *A* in equation $C_V = AT^3$ and the heat capacity at 10 K. First of all, let us determine the magnitude of *A*, as

$$A = \frac{C_V}{T^3}$$

$$= \frac{(0.78\ \text{J/mol-K})(1\ \text{kg}/1000\ \text{g})(63.55\ \text{g/mol})}{(10\ \text{K})^3}$$

$$= 4.96 \times 10^{-5}\ \text{J/mol-K}^4$$

As stipulated in the problem statement

$$A = \frac{12\pi^4 R}{5\theta_D^3}$$

Or, solving for θ_D

$$\theta_D = \left(\frac{12\pi^4 R}{5A}\right)^{1/3}$$

$$= \left[\frac{(12)(\pi)^4(8.31\ \text{J/mol-K})}{(5)(4.96 \times 10^{-5}\ \text{J/mol-K}^4)}\right]^{1/3} = 340\ \text{K}$$

7. An aluminum wire of 10 m length is cooled from 38 to -1°C. Compute the change in length. (Given $\alpha_l = 23.6 \times 10^{-6}\ (^\circ\text{C})^{-1}$)

Answer:

In order to determine the change in length of the aluminum wire, we must employ a rearranged form of equation $\frac{\Delta l}{l} = \alpha_l \Delta T$. Value of α_l, given by $23.6 \times 10^{-6}\ (^\circ\text{C})^{-1}$

$$\Delta l = l_0 \alpha_l \Delta T = l_0 \alpha_l (T_f - T_0)$$

$$= (10\ \text{m})\left[23.6 \times 10^{-6} (^\circ\text{C})^{-1}\right](-1^\circ\text{C} - 38^\circ\text{C})$$

$$= -8.7 \times 10^{-3}\ \text{m} = -8.7\ \text{mm}$$

8. A metal rod of 0.1 m elongates 0.2 mm on heating from 20 to 100°C. Determine the value of the linear coefficient of thermal expansion.

Answer:

The linear coefficient of thermal expansion for this material may be determined using a rearranged form of equation $\frac{\Delta l}{l} = \alpha_l \Delta T$ as

$$\alpha_l = \frac{\Delta l}{l_0 \Delta T} = \frac{\Delta l}{l_0 (T_f - T_0)} = \frac{0.2 \times 10^{-3}\ \text{m}}{(0.1\ \text{m})(100°\text{C} - 20°\text{C})}$$

$$= 27.5 \times 10^{-6}\ (°\text{C})^{-1}$$

9. Above the Debye temperature heat capacity will be ______________________ of temperature.

Answer: Independent

10. Define phonon.

Answer: A single quantum of vibrational energy is called a phonon

11. What is the Value of C_v at Zero K?

Answer: **0**

12. Define co-efficient of thermal expansion

Answer:
Most solid materials expand upon heating and contract when cooled. The change in length with temperature for solid materials may be expressed as follows

$$l_f - l_o / l_o = \alpha_l (T_f - T_o)$$

α_l is called co-efficient of thermal expansion.

13. State True or False.

"Bond energy and bond length curve is asymmetric"

Answer: True

14. Why polymers have higher co-efficient of linear expansion (α_l) than metals and ceramics?

Answer: Polymers have higher α_l because of weak secondary bonds.

15. Define thermal conductivity. Write down the expression and units of thermal conductivity.

Answer: "Thermal conductivity is the ability of a material to transfer heat".

$$q = -k\frac{dT}{dx}$$

Where,
q=heat flux (J/m^2-s)
k=thermal conductivity (J/m-K-s)

16. Assuming steady state heat flow, calculate the heat flux through a 10 mm thick steel sheet, if the temperatures at the two faces are 300 and 100°C. (Given thermal conductivity of steel K= 51.9 W/m-K)

Answer:

The steady-state heat flux through the plate may be computed using equation $q = -k\frac{\Delta T}{\Delta x}$; the thermal conductivity for steel is given by 51.9 W/m-K.
Therefore,

$$q = -k\frac{\Delta T}{\Delta x}$$

$$= -(51.9\ \mathrm{W/m\text{-}K})\left[\frac{(100+273K)-(300+273K)}{10\times10^{-3}m}\right]$$

$$= 1.04 \times 10^{6}\ \mathrm{W/m^2}$$

17. Briefly explain why the thermal conductivities are lower for non-crystalline ceramics than crystalline ceramics.

Answer:

Non-crystalline ceramics have lower thermal conductivities than crystalline ceramics because phonon scattering is much more effective when the atomic structure is highly disordered and irregular.

18. Give two reasons for the generation of thermal stress in materials.

Answer: Uneven heating/cooling, mismatch in thermal expansion.