

**GUIDE FOR WRITING
LABORATORY REPORTS**

**Department of Civil Engineering
University of Ottawa**

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1. INTRODUCTION

A laboratory report describes and explains the results of assigned laboratory experiments in a concise but complete manner. The writing of a laboratory report is regarded as an essential component in many of the courses you will take at the Department of Civil Engineering. Laboratory reports will not only give your instructor an indication of your understanding of the experiment carried out, but they will also help you develop skills in technical writing and engineering data presentation. As a future engineer, you will spend a significant amount of your time communicating your work through the use of oral presentations and written reports. It is for this reason that training and practice in report writing are important parts of your engineering education. It will provide you with the opportunity to develop or improve a skill that will be extremely valuable in your future career.

This manual is intended to help you in preparing and writing laboratory reports for civil engineering courses taken at the University of Ottawa. It outlines how the material in a laboratory report should be organized, describes the purpose of the different sections of the report, and introduces some standard conventions and rules for writing laboratory reports of professional quality. Remember that poor organization and presentation as well as improper grammar will affect significantly the quality of your report regardless of the technical content.

You are encouraged to follow the guidelines presented in this manual when preparing your laboratory reports; however, there are many other sources on the subject of report writing in Morisset Library or the internet that you can consult (e.g., Davis et al. 1982; Ebel et al. 1987; Handbook 2002; Writing 2002). You should also adapt these guidelines to the specific requirements given to you by your laboratory instructor in any given civil engineering course.

2. ORGANIZATION

In general, a laboratory report can be divided into the following sections:

- (i) Title Page
- (ii) Abstract
- (iii) Table of Contents
- (iv) Objective(s)
- (v) Theoretical Background
- (vi) Materials and Equipment
- (vii) Experimental Procedure
- (viii) Analysis of Data
- (ix) Discussion of Results
- (x) Conclusions and Recommendations
- (xi) References
- (xii) Appendices

Keep in mind that the previous list is general in nature. You do not need to include all of these sections in your reports; you should adapt the different sections to the requirements set in the specific course you are taking. The content of each of the sections listed in the preceding is described in the following pages.

2.1. Title Page

The following information should appear on the title page:

- Course code and name;
- A brief but informative title that describes the experiment;
- Your name;
- Date(s) the experiment was performed;
- Date the report was due;
- Name of the teaching assistant;
- Laboratory group number; and,
- Names of other group members *who were present* for the experiment.

A sample title page can be found in Appendix A.

2.2. Abstract

The abstract is a concise summary of your laboratory report. It should convey to the reader the following information: the objective of the experiment, the methodology employed, key findings and their significance, and major conclusions drawn. It is good practice to write it last, after the entire report has been completed.

The abstract should be one paragraph of approximately 100 to 200 words long, and it is placed on the second page directly following the title page.

2.3. Table of Contents

The table of contents lists the various sections used in the report with their starting page number (refer to the table of contents of this manual on page ii as an example). The table of contents is placed on the page directly following the abstract page. Make sure that the section headings appearing in the table of contents correspond to the actual section headings you have used in your report.

If tables and figures are included in the report, they should also be listed on separate pages entitled “List of Tables” and “List of Figures,” respectively. The list of tables lists all the tables that appear in the report in numerical order with their table caption and the page number where they are located. Likewise, the list of figures lists all the figures that appear in the report in numerical order with their figure caption and the page number where they appear.

Remember to number all pages of the report. Nowadays, page numbering can be done automatically with the use of any word-processing software (e.g., MS Word and WordPerfect). The page numbers should appear on the right margin of the page. It is standard practice to use lowercase Roman numerals to number the pages where the abstract, table of contents, list of tables, and list of figures appear. Although the title page is considered page i, it is not explicitly numbered, and thus page numbering starts with the abstract page as page ii. The rest of the report is numbered with Arabic numerals, starting with the “Objective” page as page 1.

2.4. Objective(s)

This section of the laboratory report briefly describes in paragraph form the purpose(s) of the experiment and the scope of the report. These are usually listed in the laboratory manual of the course you are taking. However, you should not just repeat what it is written in the laboratory manual, but rather show your own

understanding of why the experiment was carried out. The use of action verbs such as “investigate”, “determine”, “measure”, “plot”, or “compare” is an effective way of achieving this goal.

2.5. Theoretical Background

This section includes a concise description of the relevant theory needed to put the experiment into context. It also provides the necessary information that explains how you are going to analyse the experimental data. All the equations that you will be using throughout the report should be introduced here, and all their terms need to be clearly defined. Make sure you provide references to the literature used (see Section 2.11).

2.6. Materials and Equipment

This section can be a simple list of the materials and equipment used in the laboratory experiment. Although this information most probably appears in your laboratory manual, it is good practice to include it again in your report. If needed to clarify the experimental procedure carried out, you can provide a simple drawing of the experimental set-up.

2.7. Experimental Procedure

In this section, you should describe in paragraph form the procedure used to carry out the experiment, step-by-step as it happened. Experimental procedures usually follow a standardized test; if this is the case in your laboratory experiment, make sure you reference to the appropriate standard. However, be sure to document any departure from the procedure described in the standard. The reader should be able to reproduce your results by following the process described in your report. If a test is not standardized, then you should fully describe the methodology employed in the laboratory.

2.8. Analysis of Data

This section presents in written form data obtained either directly from the experiment or from manipulation of the former. In general, only the most important results required to meet the objectives of the experiment should be included here. These results usually include some of the raw data and sample calculations. When reporting the latter, make sure you include the details of the calculations and reference the appropriate formulae in the “Theoretical Background” section. You should leave the remainder of the raw data and calculations for the appendices (see more in Section 2.12). Remember that you should not be interpreting the results at this point, as this is left for the “Discussion of Results” section (to be described next).

Engineering data is usually best illustrated in tabular or graphical form. When inserting a table or figure in your report, make sure you number them sequentially and title them. Also ensure that you reference them in the written discussion. This will assure that the information contained in the report is clear. Tables and figures usually appear after they have been mentioned in the text portion of your results. More guidelines on how to format tables and figures can be found in Sections 3.3 and 3.4, respectively.

2.9. Discussion of Results

In this section, you have to state the meaning and implication of your results. You need to demonstrate your understanding of the experiment beyond its mere completion. You can do this by analyzing and interpreting the data presented in the previous section. At the end, you should be able to answer the question “What is the significance of the results?” You can achieve this by (Handbook, 2002):

- Analysing experimental error, and highlighting its sources (refer to Appendix B.4);
- Comparing your results with expected behaviour, and explaining any differences. You can do this by calculating the percentage error between the two:

$$\% \text{ error} = \frac{T - E}{T} \times 100 \quad (1)$$

where T is the theoretical or expected value and E is the experimental value. This value by itself is meaningless unless you also analyse the experimental error and compare the two;

- Explaining your results in light of the theory you have learned in class;
- Relating your results to the objectives of the laboratory experiment; and/or,
- Analyzing the strengths and limitations of the experiment.

Make sure that your discussion is based on the results that you have obtained in your laboratory experiment. This section should generally take less than one page in length.

2.10. Conclusions and Recommendations

In this section, summarize in few words the key findings of the laboratory experiment as well as their relevance. Do not introduce any new information here, and make sure that your conclusions are solely based on your actual test results and not on general knowledge. You can also include final comments and suggestions about any amendments and/or improvements you find necessary to the laboratory work. What did you learn from this exercise?

2.11. References

The “References” section is a complete and accurate list of all the literature sources you have consulted during the preparation of the report. One of them will be your laboratory manual. Make sure that a source that is included in the list of references is also referred to at the appropriate location(s) in the report. Likewise, list any source cited in the text in the “References” section.

There are several standard bibliographic formats used to cite literature sources in a text. You can find a compilation of several citation and style guides at <http://juno.concordia.ca/help/howto/citations.html>. In civil engineering, it is common to use the author-date method, in which a source is cited in the text by the last names of the authors followed by the publication year. The “References” section is prepared by listing all sources alphabetically by last name of the first author. It is suggested that you get familiar with one of the standard bibliographic formats available and be consistent throughout your report. A starting point can be the format followed by either the American Society of Civil Engineers Journals (<http://www.pubs.asce.org/authors/index.html#ref>) or the Canadian Journal of Civil Engineering (http://pubs.nrc-cnrc.gc.ca/rp/rppdf/cjce_instruct_e.pdf).

2.12. Appendices

Any additional information that has not been incorporated in the report itself, such as raw data, calculations, graphs or tables, should be included in the appendices. Create separate appendices for different types of information. Appendices are usually designated by a specific letter, e.g., Appendix A, and they should be listed in the table of contents. Make sure you refer to each appendix at least once in your report.

3. FORMATTING

Laboratory reports are expected to be typed with word-processing software. If you do not have independent access to a computer, there are several computing laboratories available throughout campus. For a comprehensive list, please visit <http://www.ccs.uottawa.ca/students/labs/index.html>. The university has a license for both MS Word and WordPerfect.

3.1. Body Text

The text of your report should be doubled spaced with appropriate margins all around (usually 1 to 1.5 in.). If you are going to bind your report, it is better to leave at least 1.5 in on the left margin. For the text font face and size, use Times New Roman and 12 pt, respectively.

3.2. Section Headings

Use separate headings for each section, and allow space between sections. Headings are usually in bold type. Whatever format you choose for the headings, you should be consistent throughout the report.

3.3. Tables

Tables are used in the body of the report to display significant data or key findings. Raw data from the experiment should be tabulated in the appendices. When using a variable in a table, make sure it is clearly identified by a symbol or name. Likewise, units should always be clearly noted if necessary. Numerical data should be reported to the relevant accuracy (see more on this in Appendix B). Remember that tables must be referenced in the text and should appear after they have been cited.

Tables are identified by their captions, which are placed above them and sequentially numbered throughout the report. They are usually typed in bold face. Make sure that the caption is descriptive enough of the information being displayed. Also, provide on each column a descriptive heading of the data tabulated. For clarity purposes, do not include vertical lines in your table, only horizontal lines above and below the column headings and at the bottom of the table (see example in Table 1).

The concrete cores were tested for their compressive strength f'_c at 7, 14, and 28 days. The results obtained by the different laboratory groups are shown in Table 1. Each lab group had a concrete mix of different water-to-cement ratios w/c .

Table 1: Results for the concrete compressive strength at different ages

Lab Group	w/c	f'_c (MPa)		
		7 days	14 days	28 days
A1	0.49	35	38	41
A2	0.49	38	39	42
A3	0.54	34	36	39
A4	0.50	39	41	46
B1	0.97	11	13	15
B2	0.63	25	26	29
B3	0.54	31	33	39
B4	0.53	31	34	37

3.4. Figures

Figures include any graphs, flow diagrams, drawings, or pictures you might want to include in your report. Like tables, figures should be numbered sequentially throughout the report, and their caption

should be placed directly below them. Make sure that figure captions include a descriptive title of what the figure represents. Also ensure that you reference each figure in the written portion of your report.

Most of the figures that you will be including in your report will be graphs that illustrate trends on your experimental data. The graphs in your report should be as clear as possible. You will achieve this by considering the following:

- Determine a suitable scale for each axis, and label it with the quantity it represents and the units of measurement in brackets; and,
- Use different symbols for various experimental data sets plotted on the same graph, and include a legend or label identifying them (theoretical results are usually displayed by lines).

Figure 1 is an example of how a graph can illustrate experimental data in your laboratory report.

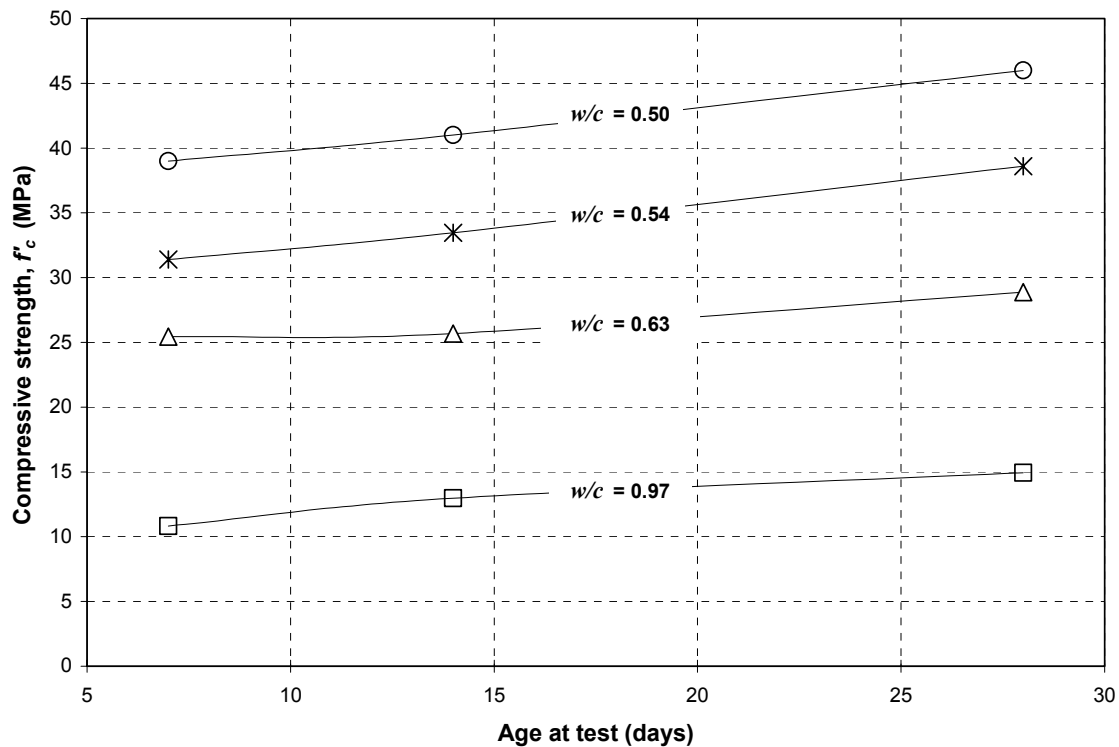


Figure 1: Relationship between concrete compressive strength f_c and age for different water-to-cement ratios w/c

3.5. Equations

All equations appearing in the main body of the report should be created using the equation editor tool of your word-processing software. Following there is an example of how you might introduce an equation in your report.

The normal stress σ is calculated according to Eq. (2):

$$\sigma = \frac{P}{A} \quad (2)$$

where P is the applied axial load (N) and A is the cross-sectional area of the specimen (mm^2). Note that σ is given in MPa in Eq. (2).

Make sure you define all the terms that appear in an equation. However, as long as notation is consistent throughout the report, you only need to define a variable the first time it is introduced. For example, if P appears again in another equation, there is no need to define it for a second time, since it was already introduced in Eq. (2). Note how equations are cited within the text (the word “equation” should be spelled out if it is to appear at the beginning of a sentence).

All the equations appearing in the report should be numbered sequentially. It is standard practice to enclose equation numbers in parentheses and place them flush with the right margin of the page (as in the previous example).

4. LANGUAGE USE

Laboratory reports will be judged not only on their technical content, but also on the clarity and ease of understanding of the written text, the choice of wording, and grammatical correctness. When writing your laboratory report, you should allow time to *edit* and *proofread* your report *before* submission. As you edit your report, delete unnecessary words, rewrite unclear sentences, clean up grammatical errors, and look out for misspelled words. Some of things you should avoid when writing your report are listed in the following:

- Be as concise as possible, as it will make your report easier to read;
- Avoid excessive repetition when possible;
- Write in complete sentences with a subject and verb, and avoid run-on sentences;
- Do not begin a sentence with an abbreviation, such as “Fig. 1 illustrates ...” Write the entire word instead;
- Be consistent with verb tenses: use past tense for events or actions that are complete (e.g., laboratory procedures) and present tense for facts that were and are still true (e.g., laboratory results);
- Do not use personal pronouns (i.e., I, me, we, our, etc.). When writing a technical report, it is preferred to use third passive voice construction (e.g., “The equipment was set up ...”);
- Use plural subjects with plural verbs (e.g., the word “data” is a plural noun; its singular is “datum”);
- Use the spell-checker tool in your word-processing software; and,
- Do your final editing *visually*. Many common word confusions such as “there/their/they're” or “to/too/two”, and misspelled technical terms are not corrected by the spell checker. Visual editing is very important in catching all of these mistakes.

More information on the correct use of the English language as well as rules on proper grammar and correct punctuation can be found in Alley (1996).

5. ORAL PRESENTATIONS

Occasionally you may be asked to present your work orally. Although many aspects in oral presentations are the same as written reports, you should be aware that there are several differences too. When presenting your laboratory work orally, you should follow the same logical structure as in a written report. However, here you should think more carefully how the information is going to be delivered, since you will only have a limited amount of time to convey your work to the audience effectively.

In an oral presentation, you should keep the content restricted to what is relevant. What is it that you want your audience to remember from your work? Try to avoid any experimental details, unless they are necessary to understand your results. When presenting your results, choose those that are most significant. A few well-chosen examples are better than an exhaustive list of details. You should also keep in mind that the level of the presentation should be tailored to the type of audience you are going to have. In your case, your audience most probably will be your classmates. However, if this is not the case, try to find out the background of your audience beforehand, so that you can adjust the degree of technical information accordingly.

You should make use of visual aids to enhance your presentation. There are various tools available for this purpose: overhead transparencies, PowerPoint presentations, poster boards, etc. Whatever tool you use, your presentation should always follow the same format: an introduction to yourself and the topic of your presentation; an outline of what is coming; a background on the subject of the presentation; a summary of main results; and, a list of concluding remarks/recommendations. Try to keep the graphics you use as simple as possible. An effective slide or transparency focuses the audience on what is being said. Do not overcrowd your slides with too much text or data, since this will detract your audience from what you are saying. Also make sure that any text, figures and tables you include in your visual aids are legible from at least 3 m and highly contrasting with the background.

The trick to deliver a good presentation is to practice ahead of time. You will only have a limited amount of time to present your work. It is normal at the beginning to overestimate the amount of information you can convey in an allocated period of time. To avoid this, prepare one slide or less per minute of presentation as a general rule. Remember to also budget some time for questions at the end and be prepared to answer them. On the day of the presentation, make sure you arrive a little bit before to check the room and any technology you will be using, so you will not have any surprises. During the presentation, speak clearly and with confidence projecting your voice to the audience.

As with any other skill, the more you practice the better speaker you will become. Since conveying technical information orally will be an important aspect of your future career, the sooner you start now the better off you will be later on.

6. PLAGIARISM

Because of space and time constraints, laboratory experiments are usually carried out in groups of students. As a result, each member of the group will follow the same experimental procedure and obtain the same set of experimental data. However, it is expected that each individual student will write his/her own report and submit it as his/her own work.

Students are expected to familiarize themselves with the University of Ottawa's policy on plagiarism, which is described at <http://www.uottawa.ca/plagiarism.pdf>. This policy will be strictly enforced in all courses at the university. Failure to comply with the university's policy on plagiarism might result in severe academic sanctions, including withdrawal from the university.

7. ACADEMIC WRITING HELP CENTRE

Free one-on-one assistance is available at the Academic Writing Help Centre (AWHC) for students who would like to improve their writing. The AWHC is located at 110 University Street (room 119), near the Stanton/Marchand residences. Further information can be found at their web page: <http://www.sass.uottawa.ca/writing/>. You are encouraged to use this service when writing your laboratory reports. You should also go over their online writing kit, which can be found at <http://www.sass.uottawa.ca/writing/kit/>.

8. REFERENCES

- Alley, M. (1996). *The craft of scientific writing*, 3rd edition, Springer-Verlag, New York, N.Y.
- Davis, H.E., Troxell, G.E., and Hauck, G.F.W. (1982). "Presentation of results." *The testing of engineering materials*, 4th edition, McGraw-Hill, New York, N.Y., 56-67.
- Ebel, H.F., Bliefert, C., and Russey, W.E. (1987). *The art of scientific writing: from student reports to professional publications in chemistry and related fields*, VCH, Weinheim, Germany.
- "Handbook: Laboratory reports." (2002). <<http://www.ecf.utoronto.ca/~writing/handbook-lab.html>> (August, 2003).
- Taylor, J.R. (1982). *An introduction to error analysis: the study of uncertainties in physical measurements*, University Science Books, Mill Valley, CA.

APPENDIX A: TITLE PAGE SAMPLE

CVG 2141
Civil Engineering Materials

Lab 1: Sieve Analysis

Your Name

Lab Date:
Report Date:
T.A.:
Lab Group:
Lab Partners:

APPENDIX B: UNITS AND NUMBERS

B.1. SI Units

The International System of Units (SI) is constructed from seven base units for independent physical quantities and two supplementary units for angles, as shown in Table 2. The supplementary units are actually non-dimensional units, but they are sometimes included in the dimensional expression of a unit for clarity purposes.

Table 2: Base and supplementary units in the SI system

	Physical Quantity	Dimension Symbol	Unit Name	Unit Symbol
Base Units	Mass	M	kilogram	Kg
	Length	L	metre	M
	Time	T	second	S
	Temperature	Θ	Kelvin	K
	Amount of substance	N	mole	Mol
	Electric current	I	ampere	A
	Luminous intensity	J	candela	Cd
Supplementary Units	Plane angle	α	radian	Rad
	Solid angle	Ω	steradian	Sr

The units of every measurement in the SI system are derived from one or more of the seven base units listed in Table 2. Table 3 lists some common derived units used in civil engineering. Refer to your textbook for a comprehensive list of the most common derived SI units used in your class.

Table 3: Some common derived SI units

Physical Quantity	Dimension	Unit Name	Unit Symbol
Acceleration	LT^{-2}	metre per second squared	m/s^2
Area	L^2	Square metre	m^2
Density	M/L^3	kilogram per cubic metre	kg/m^3
Energy	ML^2T^{-2}	Joule	J
Force	MLT^{-2}	Newton	N
Frequency	T^{-1}	Hertz	Hz
Power	ML^2T^{-3}	Watt	W
Pressure	$ML^{-1}T^{-2}$	Pascal	Pa
Velocity	LT^{-1}	metre per second	m/s
Volume	L^3	cubic metre	m^3

Multiples and submultiples of SI units are created by attaching prefixes to the units. Standard prefixes in the SI system are listed in Table 4. The use of a prefix avoids unusually large or small numbers. As a general rule, prefixes should be used to keep numbers between 0.1 and 1000. For example, a stress value of 40×10^6 Pa should be reported as 40 MPa. Try to avoid the use of “hecto”, “deka”, “deci” or “centi” unless you are reporting areas and volumes.

Table 4: Prefixes in the SI system

Prefix	Symbol	Multiplication Factor
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	H	10^2
deka	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

The units of the values reported in your laboratory report should be in the SI system. However, you might encounter in some of your labs apparatuses that are still in Imperial units. If this is your case, report recorded data in the “as measured” units, but calculations and tables and graphs displaying results should be in the SI system. You can always report the other units in parentheses, in dual-unit tables or graphs, or in a separate appendix.

When converting units from one system to another, make sure you retain sufficient number of digits to reflect the accuracy of the original quantity. However, converted quantities should not imply a greater degree of accuracy than that of the original value. A review on how to determine significant digits is presented in Appendix B.2. For your convenience, you can perform unit conversions online at the following web address <http://www.onlineconversion.com>.

B.2. Significant Digits

The significant digits of a quantity are the *meaningful* digits in it. In determining the number of significant digits, the following rules apply:

- Any digit from 1 to 9 is significant. For example, 323 and 7.59 both have three significant digits; and
- Any zero not used to show the position of the decimal point is significant. For example, 2.30 and 4023 have respectively three and four significant digits. However, 0.00476 only has three significant digits; this is easier to see if it is written in scientific notation as 4.76×10^{-3} .

When reporting numbers in your laboratory report, you should make sure they are written with the proper number of significant figures. For this, you have to make a distinction between numbers representing measured values and those obtained as a result of a calculation.

B.2.1. Measured values

The accuracy of the data measured in the lab depends on the accuracy of the equipment used to make the measurement. The number of significant digits in a measurement is equal to the number of digits that are known with some degree of confidence plus the last digit in the measured value, which is an estimate or approximation. For example, if the length of a sample x was measured as 12.7 mm using a ruler with a 1-mm scale, the true value of x probably lies in the interval of $12.6 \text{ mm} < x < 12.8 \text{ mm}$, because the last digit was obtained by visual interpolation and the range of reading error in reality was approximately $\pm 1/10$ of a scale unit. In this case, the length $x = 12.7 \text{ mm}$ has three significant digits.

B.2.2. Computed values

The accuracy of your computed results depends on the accuracy of the data used in performing the calculations as well as the accuracy of the computations performed. When using numbers of different degrees of accuracy and precision in your calculations, the following rules apply:

- When adding or subtracting numbers, the result should have as many decimal places as the number with the fewest decimal places. For example,

$$5.3 \text{ m} + 4.935 \text{ m} + 5.21 \text{ m} = 15.445 \rightarrow 15.4 \text{ m}$$

- When multiplying or dividing numbers, the number of significant digits in the result is the same as the least number of significant digits in any of the numbers used in the calculation. For example,

$$\begin{aligned} 352.3 \times 2.1 &= 739.83 && \rightarrow 7.4 \times 10^2 \\ 2339.3 \times 35.4 &= 82811.22 && \rightarrow 82.8 \times 10^3 \end{aligned}$$

B.3. Rounding Off Digits

Rounding is the process of discarding insignificant digits and keeping only significant ones. Assuming that a number is to be rounded to three significant digits, the following rules apply:

- (i) If the fourth digit is less than 5, the third digit is left unchanged:

$$\begin{aligned} 37.44 &\rightarrow 37.4 \\ 673,289 &\rightarrow 673,000 \end{aligned}$$

- (ii) If the fourth digit is greater than 5, or equal to 5 and followed by at least a non-zero digit, the third digit is increased by one:

$$\begin{aligned} 18.3682 &\rightarrow 18.4 \\ 3.245002 &\rightarrow 3.25 \end{aligned}$$

- (iii) If the fourth digit is equal to 5 and the following digits are zeroes, the third digit is left unchanged if it is even or increased by one if it is odd:

$$\begin{aligned} 26.25 &\rightarrow 26.2 \\ 26.35 &\rightarrow 26.4 \end{aligned}$$

B.4. Error Analysis

All laboratory measurements are subject to an amount of error or uncertainty, which reflects limitations in the techniques used to make them. For a measurement to be meaningful, it cannot consist of the measured value alone. An indication of how accurate the result is must also be included. Thus, the result of any measurement in the lab has two essential components: (1) a numerical value (in a specified system of units) giving the best possible estimate of the quantity measured, and (2) the degree of uncertainty associated with this estimated value. For example, measuring the length of a sample with a 1-mm scale ruler would yield a result such as 12.7 ± 0.1 mm.

An error is the difference between a quantity and its estimated or measured value. It is not the difference between a measurement and an accepted value published in the literature. Such accepted values are just measurements made at other instances, and they also have errors associated with them. An error is neither the result of a mistake made by the person conducting the experiment; this type of error can be caught and should be corrected. Measurement errors, then, have to do with the uncertainty in measurements. If a measurement is repeated, the values obtained will differ, and none of the results can be preferred over the others. Although it is not possible to do anything about such error, it can be characterized. For instance, repeated measurements may cluster tightly together or they may spread widely. The resulting pattern can be analyzed systematically.

A concise discussion on error analysis follows. For a more thorough treatment of the subject refer to Taylor (1982).

B.4.1. Data description

This section presents a brief summary of some of the descriptive measures you can use to characterize the data that you collect in a laboratory experiment.

Central tendency measures

The most commonly used central tendency descriptors are:

1. Average or mean value
2. Median value
3. Mode value

For a data set with n observations of equal weight, the average value \bar{X} is given by:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n x_i \quad (3)$$

where x_i is a sample point. The median value x_m is defined as the point that divides the data into two equal parts: 50% of the data are above x_m and 50% are below x_m . The mode value x_d is defined as the point with the highest frequency of occurrence of x .

Dispersion measures

Dispersion measures describe the level of scatter in the data about the central tendency location. The most commonly used dispersion measures are:

1. Variance
2. Standard deviation
3. Coefficient of variation

For a data set with n observations of equal weight, the variance σ^2 is defined as:

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{X})^2 \quad (4)$$

where the units of σ^2 are the square of the units of the variable x . The standard deviation σ is defined as the square root of the variance, i.e.,

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{X})^2} \quad (5)$$

The standard deviation has the same units as x . The coefficient of variation (COV or δ) is defined as:

$$\text{COV} = \frac{\sigma}{\bar{X}} \quad (6)$$

Note that the coefficient of variation is a dimensionless quantity.

B.4.2. Error classification

Measurement errors can be divided into two classes: systematic and random. Systematic errors can be caused by an imperfection in the equipment being used or from mistakes the individual makes while taking the measurement. They tend to shift all measurements in a systematic way so that their mean value is displaced.

Random errors result from the limitations in the equipment or techniques used to make a measurement. These are errors that fluctuate from one measurement to the next, and they yield results distributed about some mean value. They occur for a variety of reasons such as lack of sensitivity in the testing equipment, noise, and/or imprecise definition of the type of measurement.

Whereas some systematic errors can be substantially eliminated, random errors are inevitable and nothing can be done about them. If the average or mean value of your measurements is calculated, some of the random variations resulting from each measurement could be expected to cancel out with others in the sum. This is therefore the best way to deal with random errors; repeat the measurement as many times as possible, and use the average as the best estimate of the true value of your quantity x .

Many times measurements are quoted with two errors: the first error quoted is usually the random error, and the second one is the systematic error. If only one error is quoted, then the errors from all sources are combined together (see Section B.4.4. on error propagation).

B.4.3. Error measurement

Errors in measurements can be characterized by calculating the dispersion of the measured values, i.e., the deviation from the average or mean value. A brief presentation of different ways of doing this follows.

Relative error

The relative error is defined as:

$$\delta x = \frac{\Delta x}{x} = \frac{x_o - x}{x} = \frac{x_o}{x} - 1 \quad (7)$$

where Δx is the absolute error, x is the true value of a quantity, and x_o is the measured or inferred value of the same quantity. The percentage error is 100% times the relative error.

Maximum error

The maximum error is defined as:

$$\Delta x_{\max} = \frac{x_{\max} - x_{\min}}{2} \quad (8)$$

where x_{\max} and x_{\min} are the maximum and minimum values of the data set. Virtually no measurements should fall outside $\bar{x} \pm \Delta x_{\max}$.

Probable error

The probable error of a single measurement in a series of n measurements of equal weight is computed from:

$$\Delta x_{\text{prob}} = 0.6745 \sigma \quad (9)$$

where σ is the standard deviation for the n number of observations. The probable error is not the error most likely to occur, but rather the error that specifies the range $\bar{x} \pm \Delta x_{\text{prob}}$ where 50% of the measured values fall.

Average deviation

The average deviation is the average of the deviations from the mean, i.e.,

$$\Delta x_{\text{avg}} = \frac{\sum_{i=1}^n |x_i - \bar{X}|}{n} \quad (10)$$

Standard deviation

The standard deviation, as given in Eq. (5), is also a useful and significant measure of dispersion to characterize measurement errors.

Standard error of the mean

The standard error of the mean $\sigma_{\bar{x}}$ is calculated from:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad (11)$$

where σ is the standard deviation of the data set and n is the number of observations. If there were 20 measurements of the same quantity, according to Eq. (11) the error on the mean itself would be 4.47 times smaller than the error on each measurement. The number to report for this series of n measurements of quantity x would be $\bar{x} \pm \sigma_{\bar{x}}$. Note that the standard error of the mean varies inversely with the square root of the number of observations n . For example, taking four times the number of observations n only doubles the precision.

B.4.4. Error propagation

In many cases, the result of an experiment will not be measured directly. Instead, it will be calculated from the combination of several independently measured quantities, each of which has a mean value and an error. This section introduces techniques to determine the error in the final result of such an experiment.

If the final result is $u = f(x, y, z, \dots)$, where x, y, z, \dots , are independently measured quantities, and R_x, R_y, R_z, \dots , are their associated errors, the error of quantity u, R_u , is obtained from:

$$R_u^2 = \left(\frac{\partial u}{\partial x} R_x \right)^2 + \left(\frac{\partial u}{\partial y} R_y \right)^2 + \left(\frac{\partial u}{\partial z} R_z \right)^2 + \dots \quad (12)$$

Equation (12) can be applied to several specific cases where the function f is known, as shown in Table 5:

Table 5: Error R_u for several functions f

Calculation	Function f	Error R_u
Addition and subtraction	$u = x + y - z$	$R_u = \sqrt{R_x^2 + R_y^2 + R_z^2}$
Multiplication and division	$u = \frac{xy}{z}$	$R_u = u \sqrt{\left(\frac{R_x}{x} \right)^2 + \left(\frac{R_y}{y} \right)^2 + \left(\frac{R_z}{z} \right)^2}$
Exponentiation (assuming no uncertainty in y)	$u = x^y$	$R_u = uy \frac{R_x}{x}$
Logarithm	$u = \log x$	$R_u = 0.434 \frac{R_x}{x}$
Natural logarithm	$u = \ln x$	$R_u = \frac{R_x}{x}$