Mechanical Properties

Sections 1.2.1-1.2.6, 1.2.8-1.2.9[†]

[†] Mamlouk, M.S., and Zaniewski, J.P. (2006). ***Materials for Civil and Construction Engineers***, 2nd ed., Prentice Hall

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What is a perfect/ideal material?

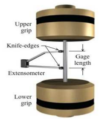
- Can resist the load
- Deforms but not excessively
- Can absorb energy
  - Absorb and recover?
  - Absorb and deform beyond recovery?

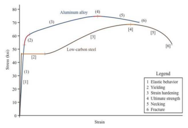
Introduction

- Need to understand and work within the characteristics as well as the limitations of the materials
- Different materials respond uniquely to applied loads

Introduction

- To determine the "mechanical" properties of materials we use simple laboratory tests.
  - Example: steel test
  - but fixed dimensions… what to do to eliminate the dependency on dimension



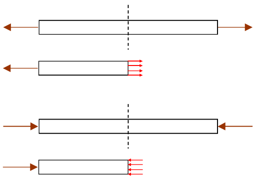


Stress

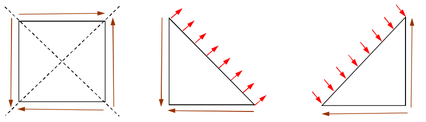
All solid materials deform under load but need to normalize the effect of dimension

- Stress is like the ``intensity`` of the force

Types of Stresses



- Normal (perpendicular to the surface - σ)
  - Tension
  - Compression
- Shear (in plane - τ)



Stress

- Normal stress

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

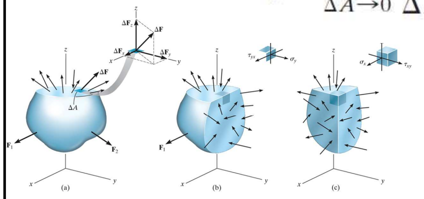


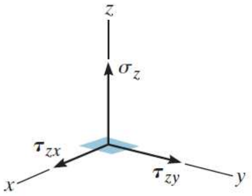
Figure: 01-10A-C

Stress

- Shear stress

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$



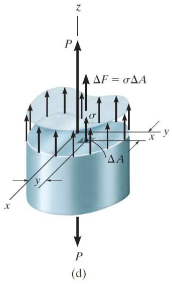
Stress

- Average normal stress

$$\int dF = \int_A \sigma \, dA$$

$$P = \sigma A$$

$$\boxed{\sigma = \frac{P}{A}}$$



Units of Stress

- SI Units

$$1 \text{ Pa} = \frac{\text{N}}{\text{m}^2} \qquad 1 \text{ MPa} = 10^6 \text{ Pa} = \frac{\text{N}}{\text{mm}^2}$$

- U.S. Customary Units

$$1 \frac{\text{lb}}{\text{ft}^2} \text{ (psf)} = 47.88 \text{ Pa}$$

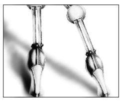
$$1 \frac{\text{lb}}{\text{in}^2} \text{ (psi)} = 6.895 \text{ kPa}$$

Stress *examples*

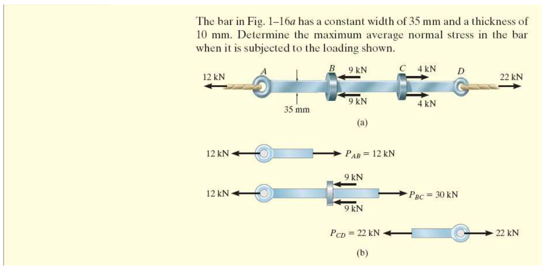
Bed of nails

High heel vs

elephant step



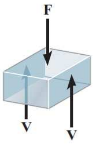
Stress *example*



The bar in Fig. 1–16*a* has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.

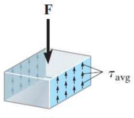
Stress

- Average shear stress



(b)

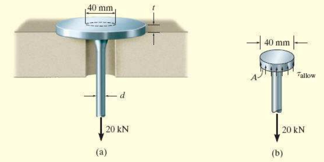
$$\tau_{avg} = \frac{V}{A}$$



(c)

Shear stress example

The suspender rod is supported at its end by a fixed-connected circular disk as shown in Fig. 1–33*a*. If the rod passes through a 40-mm-diameter hole, determine the minimum required diameter of the rod and the minimum thickness of the disk needed to support the 20-kN load. The allowable normal stress for the rod is σ_{allow} = 60 MPa, and the allowable shear stress for the disk is τ_{allow} = 35 MPa.

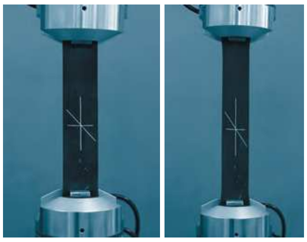


Strain

Strain

- All solid materials deform under load.
- Hence stress (force) will be accompanied by strain (deformation)
- Strain is deformation ***per unit length***
- strain is like deformation with the size factored out, so: strain = deformation / original length

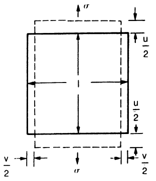
Strain



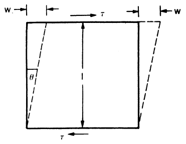
Note the before and after positions of on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Types of Strain

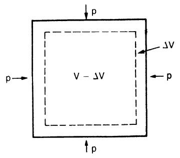
$$\varepsilon_n = \frac{u}{l} \qquad \gamma = \frac{w}{l} = \tan\theta \qquad \Delta = \frac{\Delta V}{V}$$



Tensile Strain
Change in length



Shear Strain
Change in angle



Dilation

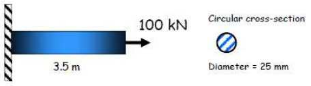
Units of Strain

- Dimensionless
- Often expressed as:
  - m/m (in/in)
  - %
  - Radians (Shear strain)

Example stress-strain

Example #1:

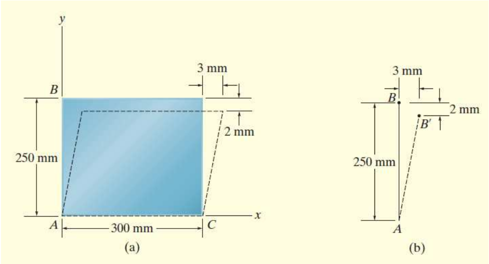
A bar having circular diameter D = 25 mm is subjected to a 100 kN tension force. If its initial length was 3.5 m and its final length is 3.515 m, find the stress and strain in the bar.



Example suspended bowling ball

- We want to suspend a 7.7 kg bowling ball from a 1.5 m long piece of steel. There are 2 options:
  - Piano wire of 0.40 mm diameter
  - Threaded rod of 3.0 mm diameter

Strain - example



Poisson's Ratio v

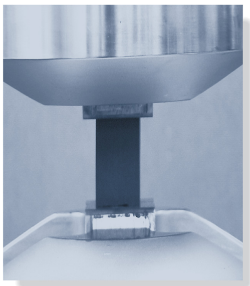


Figure: 03-20-17P
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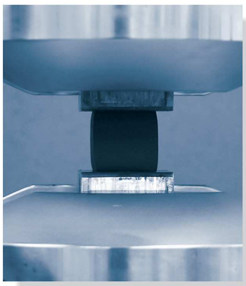

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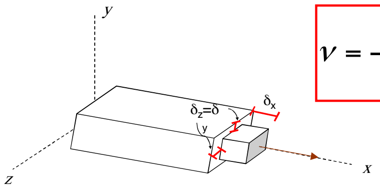
Poisson's Ratio v

- Relates lateral strain, ε_y, to axial strain, ε_x
- As material is stretched the cross section shrinks and vice versa for compression (***hence the negative sign***)



$$\boxed{\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}}$$

Poisson's Ratio (cont'd)

$$\boxed{\varepsilon_{transverse} = -\nu\,\varepsilon_{longitudinal}}$$

- For most construction materials, ν ranges from 0.15 to 0.4

Concrete	0.1-0.2
Steel	0.27-0.3
Wood	0.3-0.45

Stress-Strain Relationship

Figure: 03-00CO
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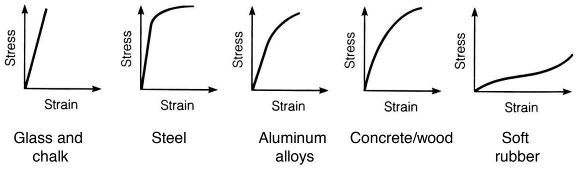
Stress-Strain Diagram

- Defines how a material will respond to load without regard for material's physical size or shape
- Obtained from testing in tension or compression
- From σ-ε curve, we can find a number of important mechanical properties

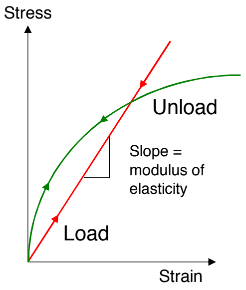
Stress-Strain Diagram

- σ – ε is usually linear in the low stress range but transforms into non-linear
- Important to be able to recognize material behavior



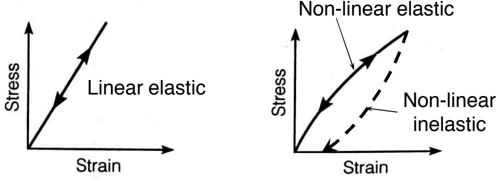
Elastic Behaviour

- Elastic deformation is recoverable
  - stretches bonds between atoms without rearranging them
- Instantaneous response to load



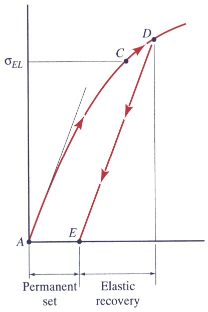
Linear and non-linear Behaviour

- A linear material has a straight line stress-strain graph
- In most material elastic deformation is linear (linear elastic)



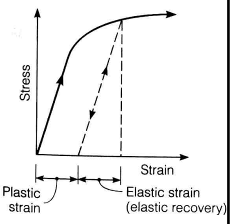
Plastic Behaviour

- Non-recoverable deformation
- Atomic bonds slip past each other and rearrange
- Permanent deformations (does not spring all the way back)



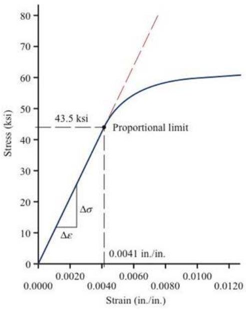
Elastoplastic Behaviour

- When unloaded, rebound parallel to the linear portion with some remaining plastic deformation
  - stretched bonds return, rearranged ones don't
- when reloaded, follows the rebound line and then original curve



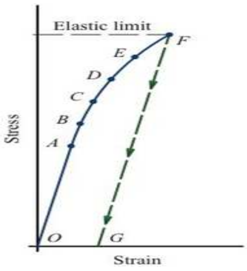
Proportionality limit

- The stress at which the stress-strain plot changes from **linear** to **non-linear**



Elastic Limit

- The stress at which the deformation will change from **elastic** to **plastic**
- The **elastic limit** is the largest stress that a material can withstand without permanent deformation



Elastic Limit

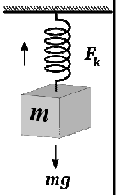
- How does ***Elastic limit*** differ from ***Proportional limit***?
  - For **most** common materials the elastic limit is indistinguishable to the proportional limit.
  - However some materials have non-linear elastic behavior in which case the elastic limit could be substantially greater than the proportional limit.

Hooke's Law

- Most engineering structures are designed to undergo relatively small deformations → ***linear elastic behaviour***

$$\boxed{\sigma = E\varepsilon}$$

- Robert Hooke (17th century, England)
  - found that the elongation of a spring was proportional to the applied force



Modulus of Elasticity

$$\boxed{E = \frac{\sigma}{\varepsilon} \quad \text{(Pa)}}$$

- Three moduli corresponding to 3 stresses:
  - Tension
  - Compression

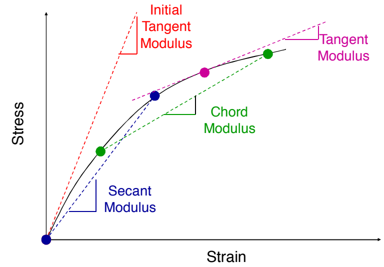
  } Young's modulus
  - Shear → Modulus of rigidity

Material	Modulus, E (GPa)
Aluminum	70
Concrete	14 - 40
Steel	200
Wood	6 - 15

$$\tau = G\gamma \qquad G = \frac{E}{2(1+\nu)} \text{(Pa)}$$

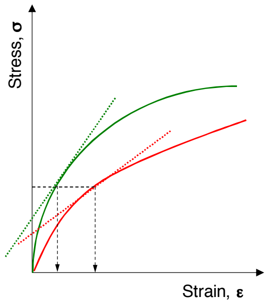
Non-linear response

- What if response is non-linear in elastic region?
- How do we find the slope (Modulus of Elasticity)?



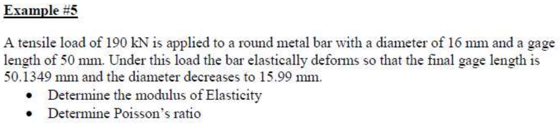
Stiffness

- Relative measure of deformability of a material under load
- Measured by the rate of stress with respect to strain
  - slope of σ-ε curve



Example – MOE

- A series of specimens are being tested in tension for possible use as a structural component. A <u>strain gauge</u> on one of the specimens measures a strain of 0.00032 at a time when the load cell on the test frame indicates a load of 5.2 kN. At the start of the test, the strain gauge reads 0.00001 while the load cell reads 0.102 kN. The test specimen has a cross-section of 3×25 mm and a length of 152 mm. Calculate the **<u>elastic modulus</u>**.

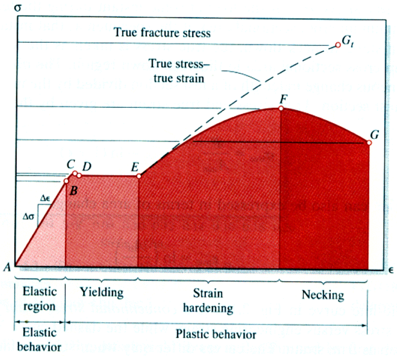


<u>Example #5</u>

A tensile load of 190 kN is applied to a round metal bar with a diameter of 16 mm and a gage length of 50 mm. Under this load the bar elastically deforms so that the final gage length is 50.1349 mm and the diameter decreases to 15.99 mm.

- Determine the modulus of Elasticity
- Determine Poisson's ratio

Example: Steel σ-ε Curve (low carbon)

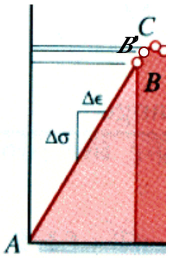


Elastic Region

- σ_B → proportional limit

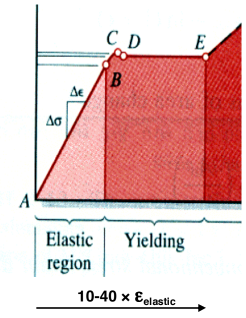
  (max. stress below which σ/ε is constant)
- $\sigma_{B'}$ → elastic limit

  (max. stress under which material remains elastic)
- B & B' usually assumed to be the same



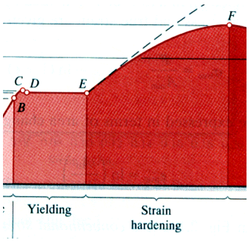
Yielding

- σ_C → yield stress or yield point σ_Y
- Often 2 values define σ_Y:
  - upper yield point, C
  - lower yield point, D
- D-E yield plateau → perfectly plastic material no increase in load



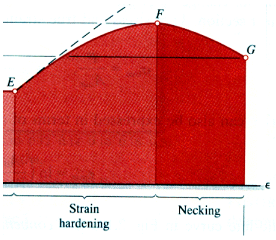
Strain Hardening

- Stress begins to increase at E
- σ_F → ultimate stress σ_u
- E-F → strain hardening zone

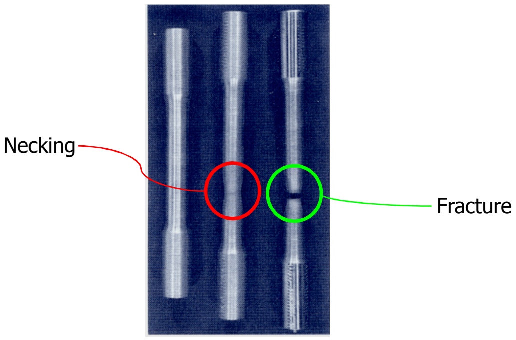


Necking

- At F the stress begins to drop as the specimen begins to "neck down."
- Cross-sectional area begins to decrease in a *localized* region
- σ_G → fracture stress σ_f



Necking (cont'd)



Engineering Stress & Strain

- Based on original dimensions A_o & L_o
- At small strains, these values are adequate descriptions of the deformation

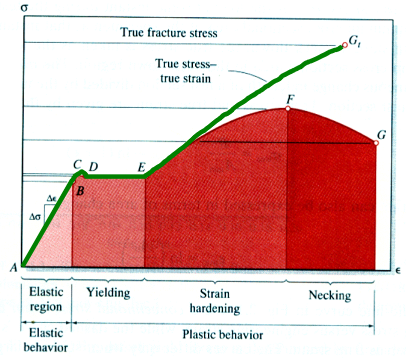
True Stress & Strain

- Since area changes as specimen deforms, engineering stress is not a correct measure of stress at higher strains
- True stress & strain represent *actual* area and incremental change in length

$$\sigma_t = \frac{P}{A}$$

$$\varepsilon_t = \int_{L_0}^{L} \frac{1}{L} dL = \ln \frac{L}{L_0}$$

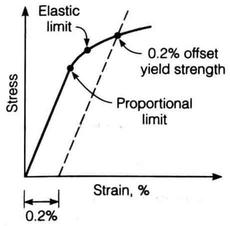
Steel True σ- True ε Curve



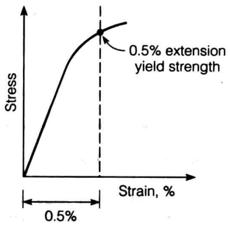
Yield Point – Offset Method

- What if there's no clear transition point?

Offset method



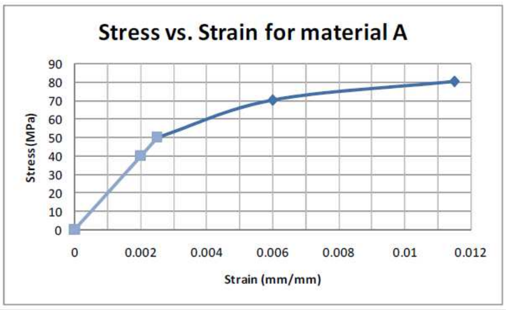
Extension method



Example # 6

A tension test is performed on a metal specimen upto rupture. The stress-strain curve shown below is obtained. Determine the following:

- Modulus of Elasticity
- Yield stress at an offset strain of 0.002



Ductile, Brittle Materials

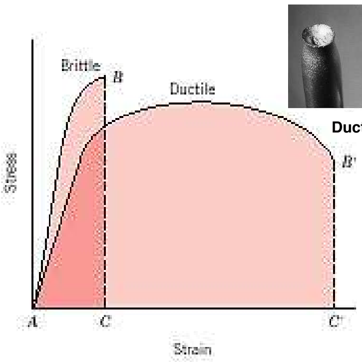
Ductile materials

- Undergo large strains before fracture
- Chosen for design of structures because:
  - they are capable of absorbing shock or energy
  - if overloaded, they exhibit large deformation before failing (giving warnings)

Brittle materials

- Materials that exhibit little or no yielding before failure

Ductile vs. Brittle Materials



Ductile

Brittle

Why manufacture a brittle material?

- Usually higher strength
- Tool manufacturing

How can a ductile material absorb energy?

How can a ductile material absorb energy?

How can a ductile material absorb energy?

Yielding failure of a helicopter engine shaft and bearing components

How can a ductile material absorb energy?

Ductile Fracture of 2 1/2 Inch Hose Fitting

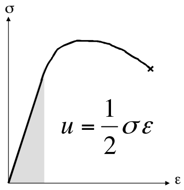
Brittle failure

Fracture of an Aluminum Crank Arm.

Bright: Brittle fracture. Dark: Fatigue fracture

Resilience

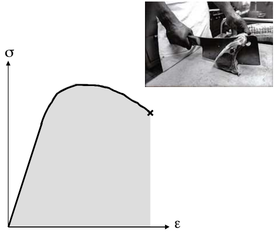
- Ability to absorb energy without any permanent damage



Modulus of resilience

Toughness

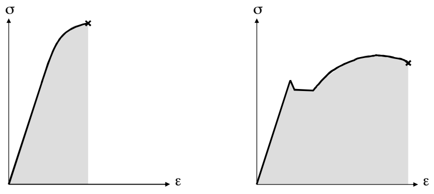
- Ability to absorb energy before fracture



Modulus of toughness

Example: Steel

- Often increase in material strength is achieved at the cost of reduced ductility



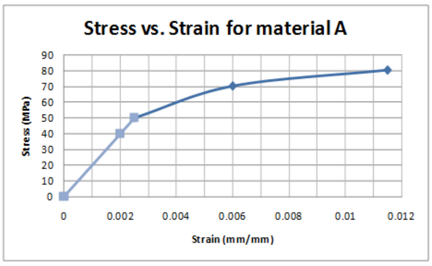
High-carbon steel
high strength, low toughness, brittle

Low-carbon steel
lower strength, higher toughness, ductile

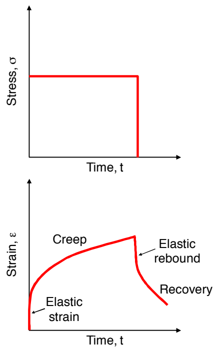
Exemple # 7

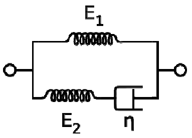
For the same graph shown in example #6, determine the following:

- Modulus of resilience
- Toughness
- Poisson's ratio if at a stress of 40 Mpa the lateral strain is -0.00057
- The permanent strain at 75 MPa

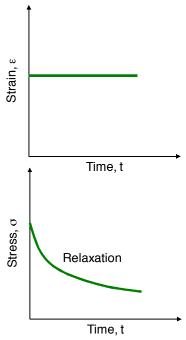


Time-Dependent Response





Time-Dependent Response

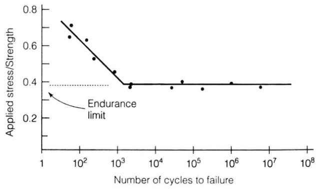


Fatigue

- Gradual reduction in a material's strength due to repeated ***cycles*** of stress or strain
- Fracture occurs at a stress level lower than material's yield stress
  - connections or supports for bridges
  - railroad wheels & axles
  - steam or gas turbine blades, …

Fatigue

- As increase level of stress, can withstand less # of cycles before failure If stress is below "endurance limit" can withstand ∞ cycles



Good to know when dealing with experiments

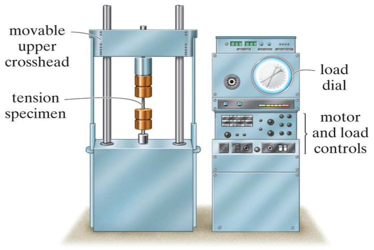


Figure: 03-02

Material Variability

- All materials have variability
  - Some materials are more uniform than others (Steel vs. concrete vs. wood)
- Three sources of variance:
  - Material
  - Sampling (reduce by using good sampling techniques)
  - Testing (reduce by reducing experimental errors)

Sampling

Proper sampling must ensure that a random and representative sample is taken from the population (e.g., stockpile, lot, etc.)

- Random: equal chance of being selected
- Representative: perfect average of the entire stockpile

Sample size:

- depends on materials variability & tolerance level of results
- more variability dictates a larger sample

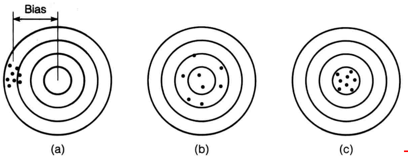
Experimental error

Caused by 3 factors:

- Procedural errors: Are often undiscovered
- Machine errors (bias): If known and constant can be easily corrected
- Human errors: Minimize by repetition, double-checking, etc. Always do more than one test

Material Variability

- Precision: measure many times and get same result
- Bias: tendency to deviate in one direction from true value
- Accuracy: close to true value; no bias

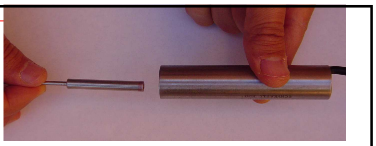


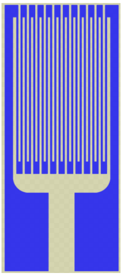
Laboratory measuring devices

Direct
- Ruler, dial gauge, calipers
- Physical & material properties are usually measured (time, deformation, force, etc.)

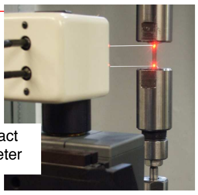
Indirect
- LVDT, strain gauge, load cell
- measuring changes in electric voltage and relating to deformation, stress, or strain
- must be calibrated

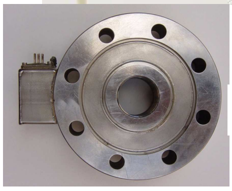

Dial Gauge


LVDT


Strain Gauge


Extensometer


Non-Contact Extensometer

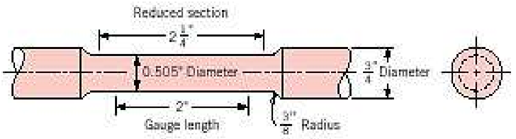

Load Cell

Tension Tests of Metals

Tests

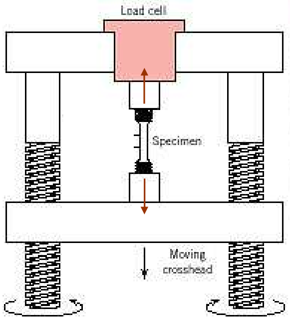
- Tension test
  (ASTM E8 & E111)

Tensile Test Specimen

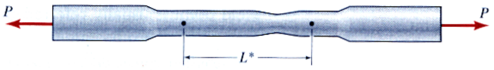


Dimensions are standardised to ensure reproducible results between laboratories

Tensile Test Frame



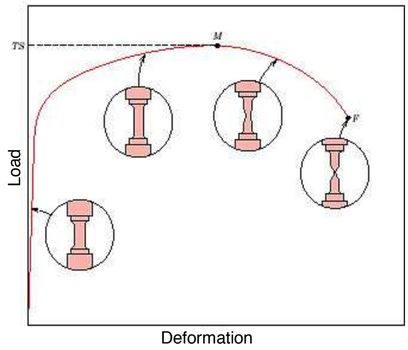
Deformed Specimen



$$\Delta L = L^* - L_0$$

Original gauge length is deformed to L^*. The load and the elongation are carefully measured.

Tensile Load-Deformation Curve

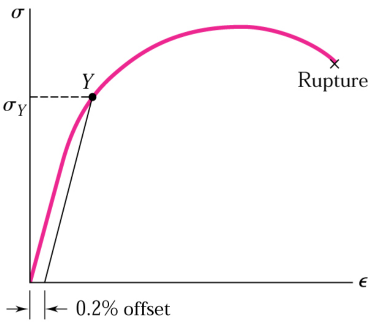


Stress-Strain Diagram

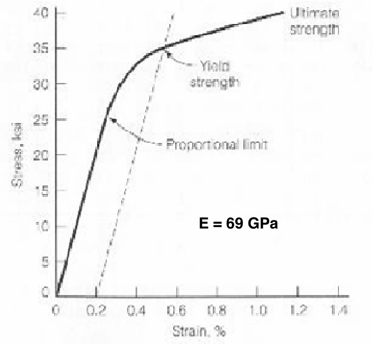
- From σ-ε curve, we can find a number of important mechanical properties

$$\sigma = \frac{P}{A_0} \qquad \varepsilon = \frac{L - L_0}{L_0}$$

Yield Strength – Offset Method



Aluminium Stress-Strain Curve



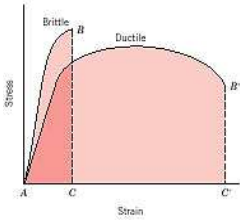
E = 69 GPa

Ductility

- Amount of inelastic deformation prior to rupture

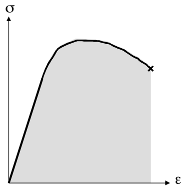
Ductile

Brittle

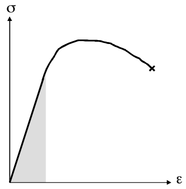


Stress/Strain curves for ductile and brittle metals

Toughness - Resilience



Modulus of toughness



Modulus of resilience

End of lecture
Mechanical Properties