Chapter 15

Introduction to Game Theory

- Game theory is a branch of mathematics used to find equilibrium in situations where the interaction of players is important.
- It is a good approach to understanding how firms in oligopolistic markets reach the outcomes they do.
- In this chapter, we'll become acquainted with the basic concepts.

Basic Definitions

- Players - entities like individuals or firms that make choices.
- Strategies - the choices made by the players (how much to produce, what prices to charge, etc.).
- Strategy combinations - a list of strategies for each player.
- Payoff - the outcome (profit, etc.) from selecting a strategy.

- The payoff for each player is dependent upon the strategy he/she selects and that selected by other players.
- The player has a best response function to other players' strategies.
- An equilibrium strategy combination arises if every player's strategy is a best response to the strategies of all other players.
- This is called a Nash equilibrium, after John Nash, 1950. We often call it a Cournot-Nash equilibrium because the concept is based on the work of Cournot a century earlier.

- We define:
- Cournot-Nash equilibrium: An equilibrium strategy combination where there is nothing any individual player can independently do that increases that player's payoff.
- Each player's own strategy maximizes that player's own payoff.

- A strategy better than all others, regardless of the actions of others, is a dominant strategy.
- If one strategy is worse than another for some player, regardless of the actions of other players, it is a dominated strategy.

Example

- This is a game between two players who can move in different directions.

- Each player gets a dollar payoff based on their move and the move the other player makes.
- The payoffs are written as:
- (player 1’s payoff, player 2’s payoff)

		Player 2		
		Left	Middle	Right
Player 1	Up	4, 3	5, 1	6, 2
	Middle	2, 1	8, 4	3, 6
	Down	3, 0	9, 6	2, 8

- So, if Player 1 moves up and Player 2 moves to the middle, Player 1 gets $5 and Player 2 gets $1 and so forth.
- The first number in each cell is always Player 1's payoff and the second number is always Player 2's payoff.
- The aim of each player is to maximize their payoff – to maximize the money they receive at the end of the game.

- Let’s start with Player 2:
- If she moved Right rather than Middle, she’d be better off no matter what Player 1 did (her payoff is higher all around).
- Her strategy of Middle is dominated by her strategy of Right and she can eliminate Middle as a strategy.

Player 2

Player 1	Left	Middle	Right
Up	4, 3	5, 1	6, 2
Middle	2, 1	8, 4	3, 6
Down	3, 0	9, 6	2, 8

Player 2 will always pick Right over Middle: her payoff is always higher.

Compare 2, 6 and 8 from Right to 1, 4 and 6 from Middle.

We can eliminate Middle from the game since Player 2 will never choose it.

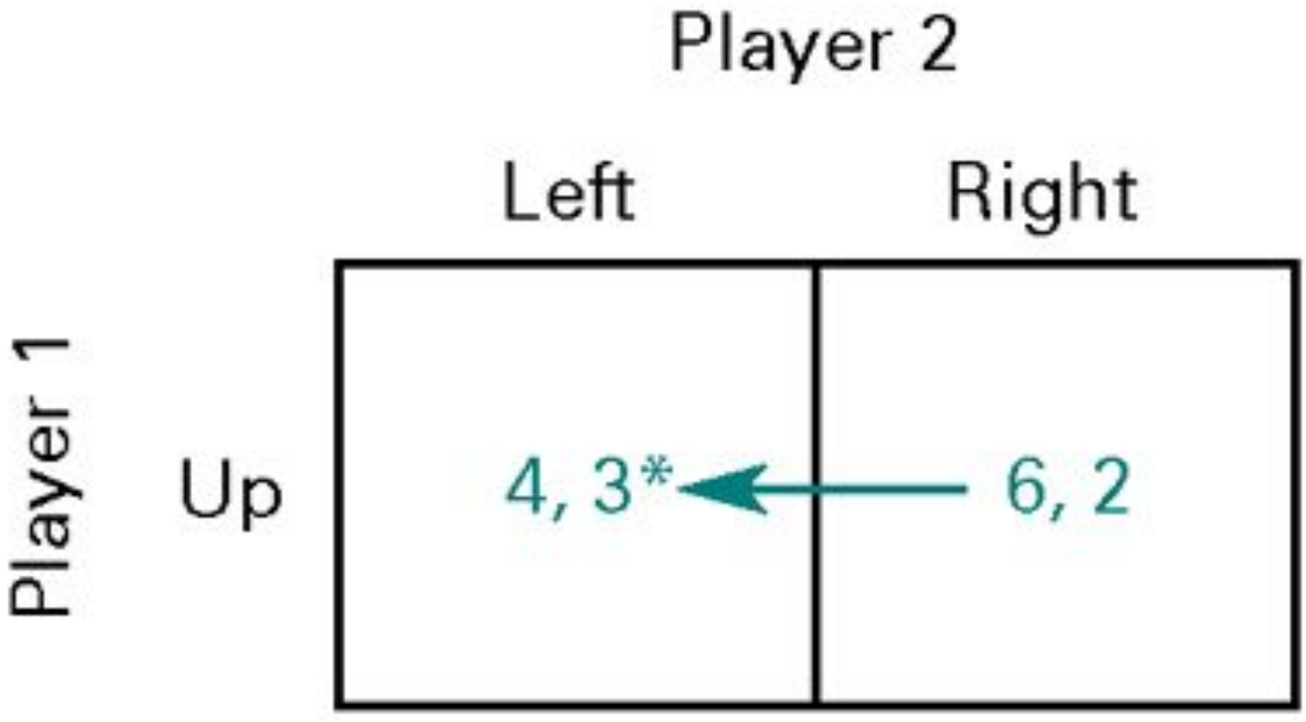
		Player 2	
		Left	Right
Player 1	Up	4, 3	6, 2
	Middle	2, 1	3, 6
	Down	3, 0	2, 8

Now consider Player 1. No matter what Player 2 does, Player 1 should move Up.

Compare his payoffs from Up of 4 and 6 to 2 and 3 for moving to the Middle or 3 and 2 for moving Down.

His strategies of Middle and Down are dominated by Up and they can be eliminated from the game.

For Player 1, Up is the dominant strategy.



Player 2 will move Left to end the game (Player 1 has no more moves left). Player 2's payoff is $3 and Player 1's payoff is $4.

The game results in a Nash equilibrium of (Up. Left).

The Prisoner's Dilemma

- Two bozos, Paris and Lindsay, after a late-night party decide that it would be fun to steal Brittney's car and take it for a joy ride.
- Brittney gets a glimpse of them as they pull out of her driveway, but she doesn't see them well enough to make a positive identification.
- Nonetheless, Paris and Lindsay are arrested for possession of a stolen vehicle the next morning, but there's not enough evidence to charge them with the theft of the car.

- The police decide to separate Paris and Lindsay, and they offer them each the same deal:
- “If you stay quiet and your friend finks on you, she’ll go free and you’ll get 9 months for theft.
- If you fink on her and she stays quiet, you go free and she’ll get 9 months.
- If you both stay quiet, you’ll both spend 1 month in jail for the possession of stolen property charge.
- If you both fink, it’s your word against hers, so you’ll each spend 6 months in jail.”

Payoffs are negative since jail time is bad.

Paris

Lindsay		Silent	Fink
	Silent	–1, –1	–9, 0
	Fink	0, –9	–6, –6

No matter what Lindsay does, Paris should fink: if Lindsay stays silent, Paris goes free and if Lindsay finks, Paris gets 6 months instead of 9 if she stayed silent. Lindsay’s strategy is the same.

- Both Paris and Lindsay have the same dominant strategy: fink.
- This is a Nash equilibrium of (fink, fink).

- But, the equilibrium is not Pareto-optimal:
- Both could have done better if they both stayed silent (only 1 month in jail instead of 6 months).

- The idea of a best response is critical to a Nash equilibrium.
- In a 2-player game like we have just seen, the strategy combination (S_1*, S_2*) is a Nash equilibrium if:
- 1. S_1* is player 1's best response to S_2*
- 2. S_2* is player 2's best response to S_1*

- The Prisoner's Dilemma is an example of a one-shot game.
- The only payoffs are those given in the game – there are no further consequences to either prisoner's decision.
- If a game is repeated, the outcome is usually one of cooperation among the players and a Pareto-optimal outcome.

Coordination Games

- Often situations may have no equilibrium or they may have multiple equilibria.
- In these situations, other forms of behaviour must arise for a solution to be found.

- Consider two academic authors, Peyton and Eli.
- They write papers together, and often email each other with new pages they have written.
- Peyton prefers to use Word and Eli prefers Word Perfect.

- Peyton and Eli have discussed which program to use, but have yet to agree.
- Suppose that they don’t know which the other has used until they actually receive their work.
- The figures represent how much better/worse each author is under the various strategies measured in more/less papers written.
- Note that if they use different programs, they don’t merge nicely and they get mad at each other – a disaster.
- Here are their payoffs from using the programs:

Eli

		Word	Word Perfect
Peyton	Word	2, 1	- 2, -2
	Word Perfect	-2, -2	1, 2

Here, it's a disaster for both to use different programs. If Eli uses Word Perfect, Peyton is better off using it, too. If Peyton uses Word, Eli is better off using Word.

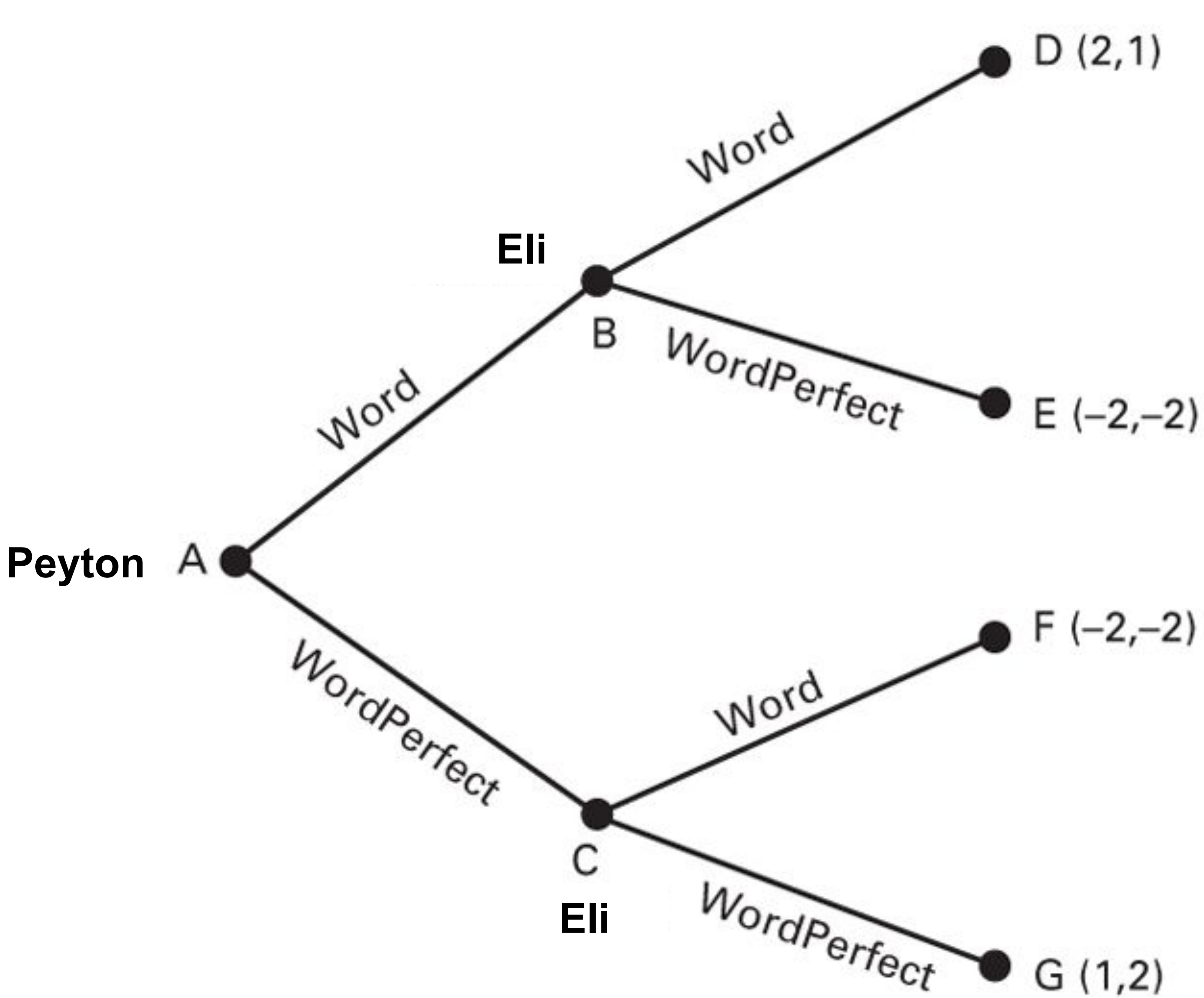
Thus, there are 2 equilibria: (Word, Word) and (Word Perfect, Word Perfect).

- Because there are two equilibria, we can’t tell what the outcome will actually be.
- There is a coordination problem here.
- The players have to somehow settle on one of the equilibria.
- There is no definitive method of solving coordination games, and actual outcomes often depend upon laws, social customs or pre-emptive moves by players before the game.
- In our example, maybe Peyton is the higher ranking academic, so he gets to make the decision.
- In some cases there simply is no equilibrium.

Sequential Coordination Games

- Let’s assume that instead of Peyton and Eli writing simultaneously, Peyton gets to go first (he gets to make the first move).
- Now let’s model the game by using a game tree.
- This is called the extensive form of a game.
- It involves branches, nodes and payoffs.

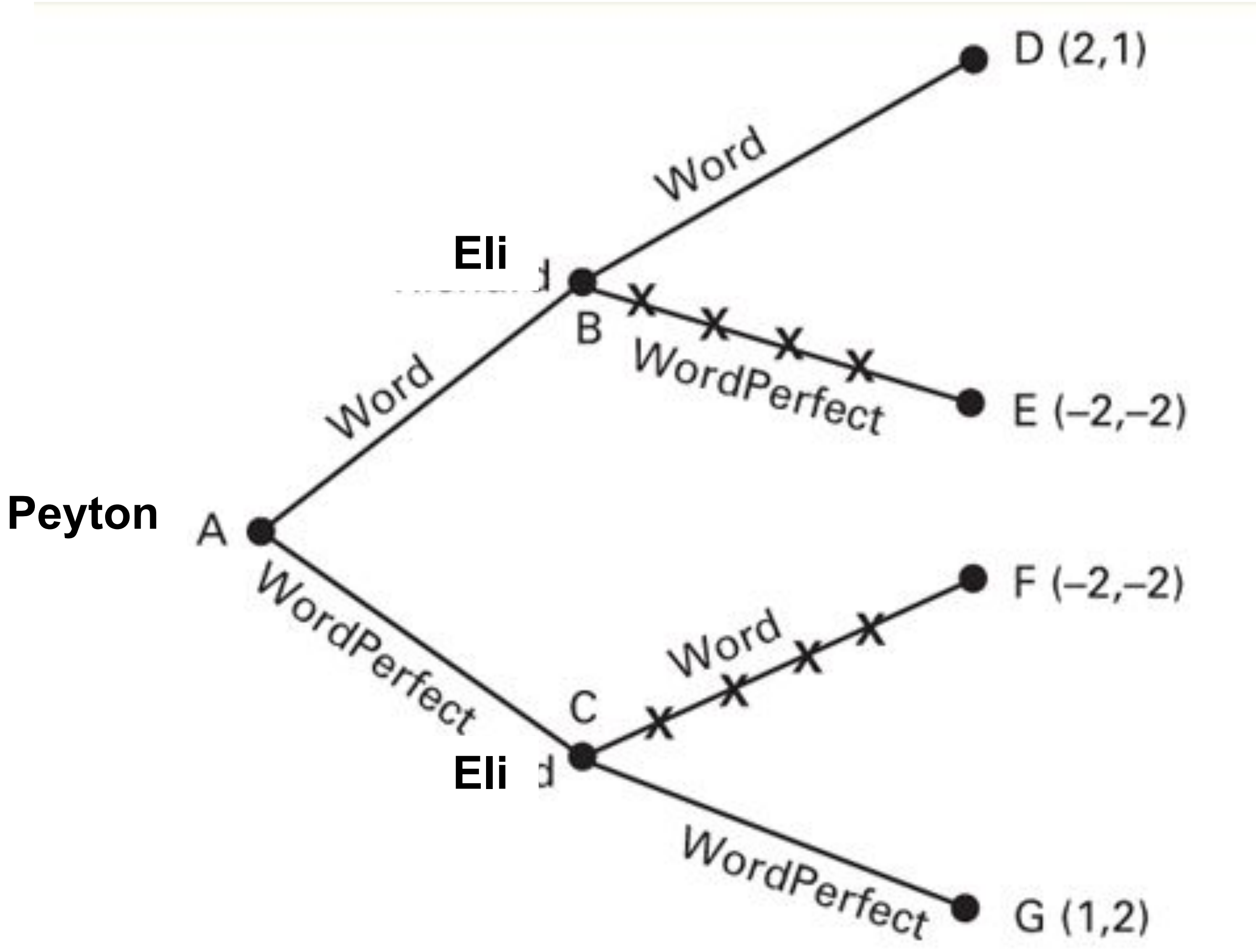
- Nodes are points where either decisions are made or payoffs are received.
- Terminal nodes are the final nodes on an extensive form game, representing the payoffs for reaching that node.



The dots A, B and C are decision nodes where a player has to decide which strategy to choose.
Dots D, E, F and G are terminal nodes indicating the payoffs.
For example, if Peyton chooses Word and Eli chooses Word, Peyton gets 2 and Eli gets 1 as payoffs.

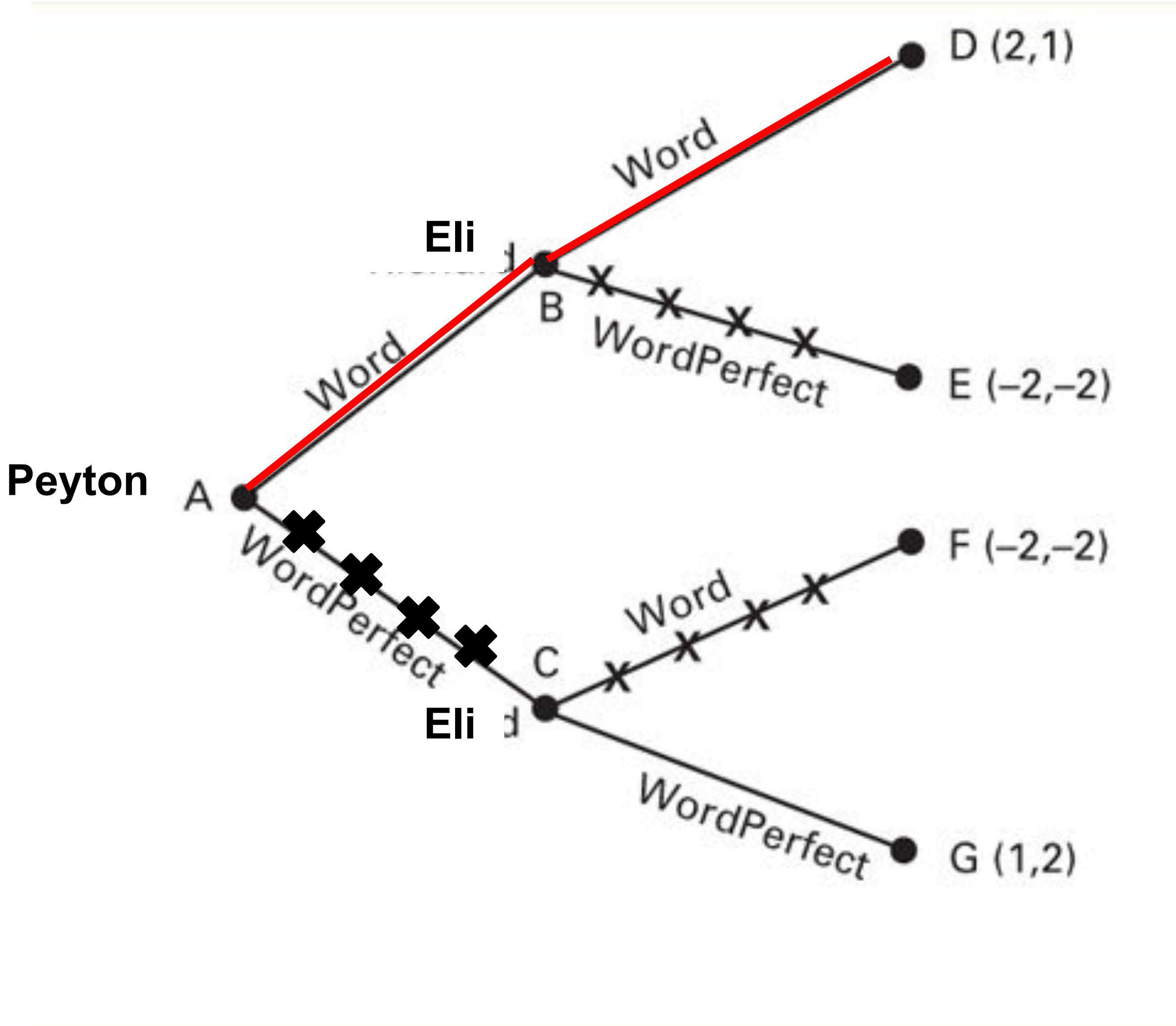
- To find the equilibrium of the game, we start at the end – at the payoffs – and work through the tree backwards.

- If Peyton chooses Word, Eli will never choose Word Perfect (gives him a negative payoff).
- If Peyton chooses Word Perfect, Eli will never choose Word (gives him a negative payoff).
- We can cross those two branches off the tree.



D (2,1)
Word
Eli
B
WordPerfect
E (–2,–2)
Word
Peyton
A
WordPerfect
F (–2,–2)
Word
C
Eli
WordPerfect
G (1,2)

- Now, Peyton can see that if he chooses Word, Eli will choose Word and Peyton’s payoff will be 2.
- If he chooses Word Perfect, Eli will choose Word Perfect and Peyton’s payoff will be 1.
- Peyton will never choose Word Perfect so we can cross out that branch.
- So, we can see the equilibrium path – Peyton chooses Word and so does Eli.


D (2,1)
Word
Eli
B
WordPerfect
E (–2,–2)
Word
Peyton
A
WordPerfect
F (–2,–2)
Word
C
Eli
WordPerfect
G (1,2)

- This equilibrium is called a subgame perfect Nash equilibrium.
- Every strategy must be payoff maximizing for each player at all decision nodes.

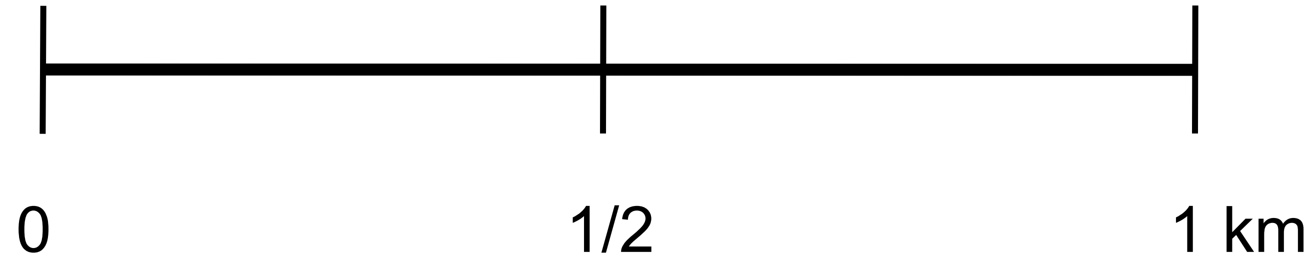
Hotelling's Location Model (Ch 1)

- Often, where firms locate is as important as how much output they produce.
- If a firm's strategy is a specific location, a Nash equilibrium will be a set of locations for each firm such that no firm could gain by unilaterally deviating form its Nash eqm location strategy.

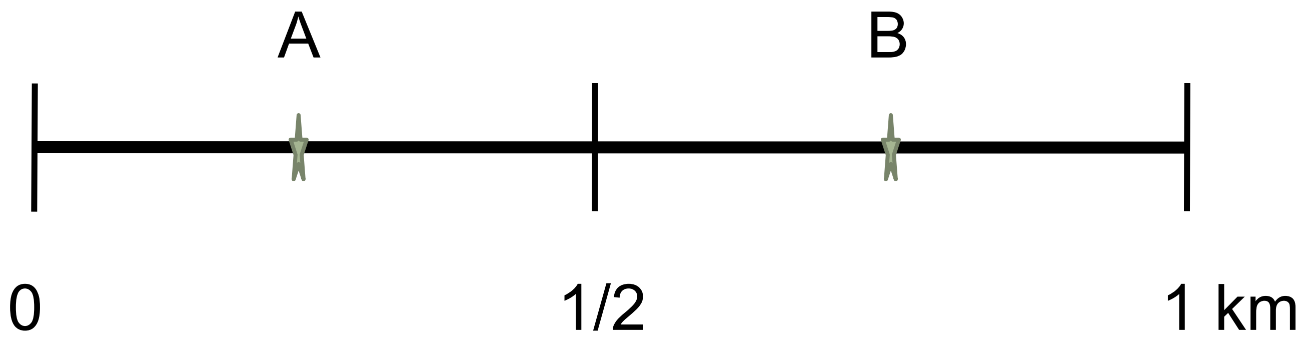
Assumptions

- There are 2 firms selling an identical good – ice cream – on a beach that is one km long.
- The ice cream sells at a fixed price, p.
- Each firm's cost is c and p > c.
- Buyers are distributed uniformly along the beach.
- The farther buyers have to travel for ice cream, the higher their cost to reach the seller. This means that customers will patronize the ice cream stand that is the closest to them.

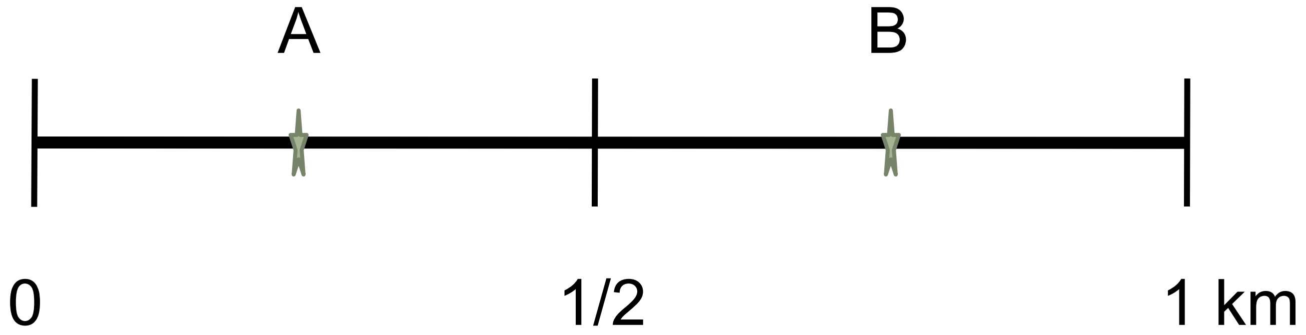
- The firms can relocate their ice cream stands without cost.
- Where does each firm locate along the beach?
- Let’s look at it visually.



- The 2 ice cream vendors can locate anywhere along the beach.

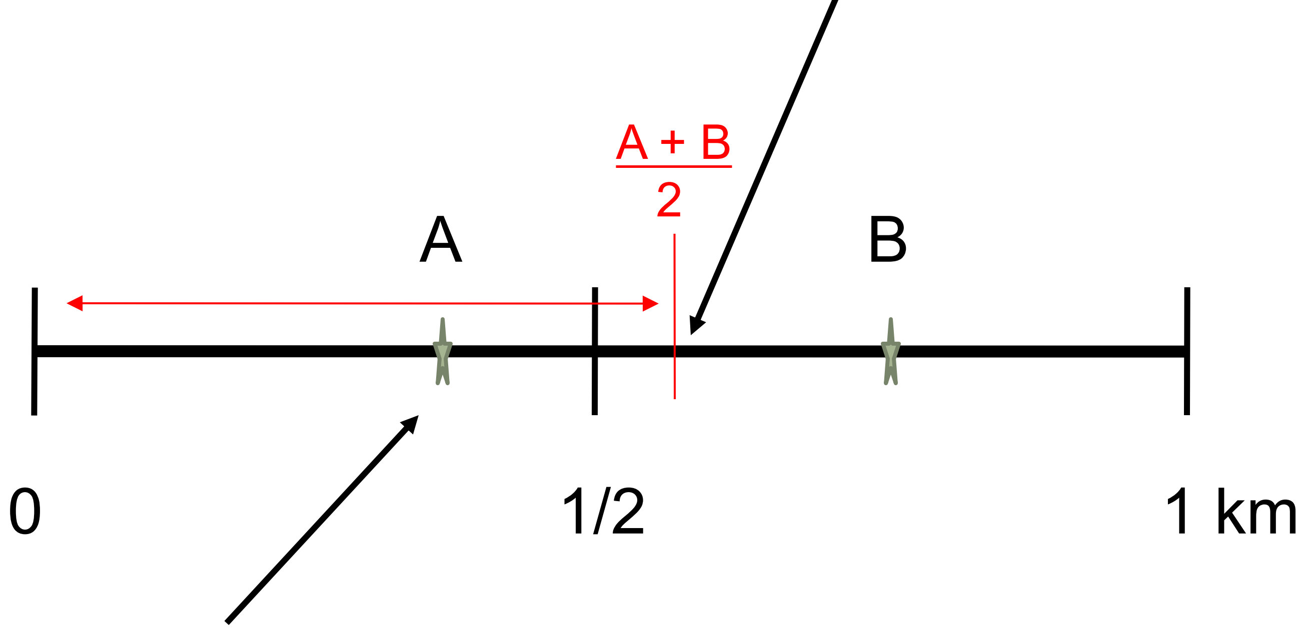


- Suppose each vendor, A and B, initially locates as above.



- Buyers along the first half-km of the beach will buy from A and those along the second half-km will buy from B.

Buyers at the point (A+B)/2 are indifferent as to from whom they buy. They are equally distant from either vendor.



- Vendor A now decides to move her stand to the right. Buyers close to the halfway mark who used to buy from B will now buy from A who is closer.

- Vendor A’s market segment extends from 0 to $(A + B)/2$.
- Vendor B’s market segment extends from $(A + B)/2$ to 1.

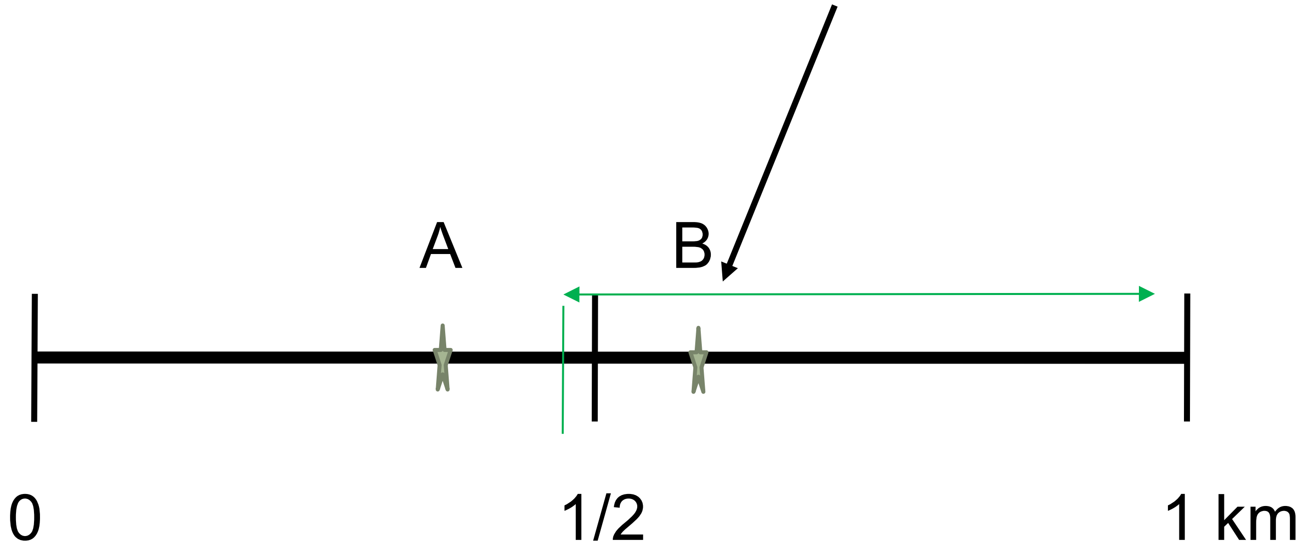
- Because the n customers are uniformly distributed, the number of A’s customers is the length of his market segment times n:
- $n(A + B)/2$ = number of A’s customers
- whenever A is to the left of B $(A < B)$

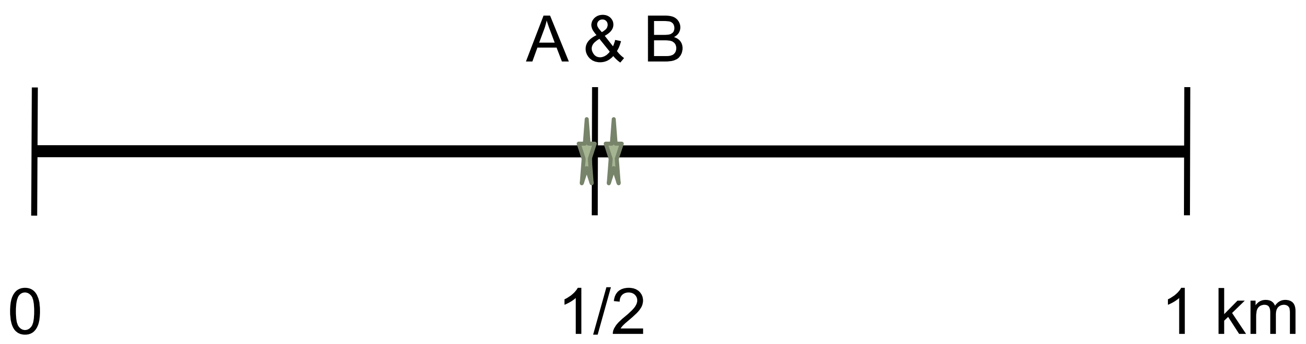
- Similarly, the number of B's customers is
- n[1 – (A + B)/2] when A < B

- A's profits are (p – c)[n(A + B)/2]
- B's profits are (p – c)[n{1 - (A + B)/2}]

- B's profits are less than A's.

- Vendor B catches on and moves his stand to the left.





- Vendor A moves right again, Vendor B moves left again and this continues until both vendors settle at the halfway mark.

- The strategy of each firm to locate at the same spot – the halfway point – results in a Nash equilibrium where each vendor splits the market of potential buyers.
- Suppose there are n buyers along the beach who will each buy one ice cream cone.
- Each vendor's profit is the profit per unit sold times the number of units sold:
- Π = (p-c)(number of customers)

- If the two vendors split the market evenly, each will earn a profit of (p-c)(n/2).
- Note that if the vendors want to increase their market share from this point, they have to differentiate their product, advertise, etc just as firms in monopolistic competition would do.

Example

- Suppose that n = 128, A = 1/2, B = 3/4 and p – c = 1. What is the point of market segmentation – that is, the point where customers would be indifferent between buying from A or B?

- (A + B)/2 = (5/4)/2 = 5/8

- What is A's profit?
- Since A < B, profit = 128(5/8) = $80
- What is B's profit?
- Profit = 128(3/8) = $48