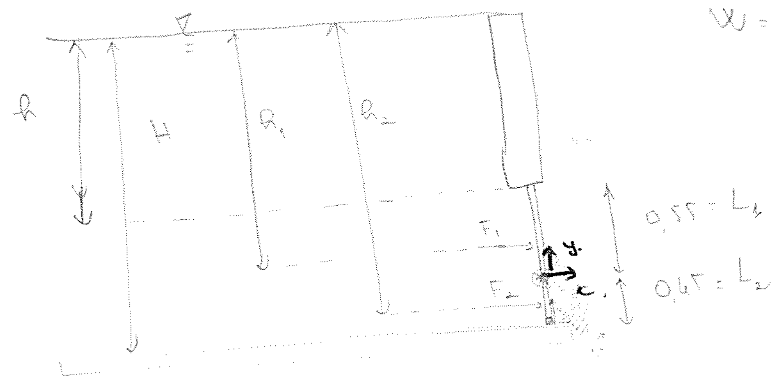
Assignment 1

Problem 2



$w = 2\,m$

$h_1 = H - L_2 - y$

$h_2 = H + y - L_2$

Trouver H pour que la porte s'ouvre ?

$M_{ouvrir} = M_{fermer}$

$\Rightarrow \int_A y\,P_1\,dA = \int_A y\,P_2\,dA$

$\Rightarrow \int_0^w \int_0^{L_1} y\,\rho g h_1\,dy\,dz = \int_0^w \int_0^{L_2} y\,\rho g h_2\,dy\,dz$

$\Rightarrow \int_0^w \int_0^{L_1} y\left(\rho g(H - L_2 - y)\right)dy\,dz = \int_0^w \int_0^{L_2} y\,\rho g(H - L_2 + y)\,dy\,dz$

$\Rightarrow \left[\left(\rho g(H - L_2)\frac{y^2}{2}\right) - \frac{1}{3}y^3\rho g\right]_0^{L_1} w = w\left(\left[\rho g(H - L_2)\frac{y^2}{2} + \frac{1}{3}y^3\rho g\right]\right)_0^{L_2}$

$\rho g(H - L_2)\frac{L_1^2}{2} - \frac{1}{3}L_1^3\rho g = \rho g(H - L_2)\frac{L_2^2}{2} + \frac{1}{3}L_2^3\rho g$

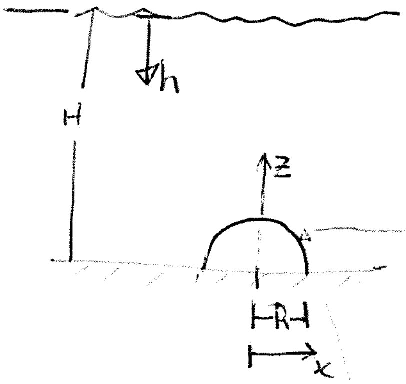
$\Rightarrow \rho g H\frac{L_1^2}{2} - \rho g L_2\frac{L_1^2}{2} - \frac{1}{3}L_1^3\rho g - \rho g H\frac{L_2^2}{2} + \rho g\frac{L_2^3}{2} - \frac{1}{3}\rho g L_2^3 = 0$

$\Rightarrow H\left(\rho g\frac{L_1^2}{2} - \rho g\frac{L_2^2}{2}\right) = \rho g L_2\frac{L_1^2}{2} + \frac{1}{3}L_1^3\rho g - \rho g\frac{L_2^3}{2} + \frac{1}{3}\rho g L_2^3$

$\Rightarrow H = \frac{-\rho g\frac{L_2^3}{2} + \rho g L_2\frac{L_1^2}{2} + \frac{1}{3}L_1^3\rho g + \frac{1}{3}\rho g L_2^3}{\rho g\left(\frac{L_1^2}{2} - \frac{L_2^2}{2}\right)} = \frac{0{,}45 \times \frac{0{,}55^2}{2} + \frac{1}{3}0{,}55^3 - \frac{0{,}45^3}{2} + \frac{1}{3}0{,}45^3}{\frac{0{,}55^2}{2} - \frac{0{,}45^2}{2}}$

$\Rightarrow \boxed{H = 2{,}16\,m}$

Q1. Assignment / Devoir 2



$H = 10\,m$

$R = 1\,m$

$F_{net} = ?$

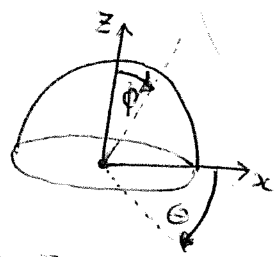
$$\underline{F = \int -p\, dS} \quad , \quad p = \rho g h$$

$$h = (H - z)$$

R = const , θ and φ = variable $\quad dS = \hat{n}\, dA$

$$\vec{r} = [\underbrace{\varrho \sin(\phi)\cos(\theta)}_{x_s}, \underbrace{\varrho \sin(\phi)\sin(\theta)}_{y_s}, \underbrace{\varrho \cos\phi}_{z_s}] \qquad \hat{n} = \frac{\partial \vec{r}}{\partial \theta} \times \frac{\partial \vec{r}}{\partial \phi}$$

$\varrho = R$



$\theta = [0, 2\pi]$

$\phi = [0, \frac{\pi}{2}]$

$$\therefore \quad \frac{\partial \vec{r}}{\partial \theta} = R[\sin\phi(-\sin\theta), \sin\phi\cos\theta, 0]$$

$$\frac{\partial \vec{r}}{\partial \phi} = R[\cos(\phi)\cos\theta, \cos\phi\sin\theta, R(-\sin\phi)]$$

$$\hat{n} = R^2 [-\sin^2\phi\cos\theta, \sin^2\phi\sin\theta, \underbrace{-\sin\phi\cos\phi\sin^2\theta - \cos\phi\sin\phi\cos^2\theta}_{-\sin\phi\cos\phi}]$$

$\therefore (\sin^2\theta + \cos^2\theta = 1)$

Test $\hat{n}$ @ $\phi = 0, \theta = 0 \rightarrow \hat{n} = [0, 0, 0]$

∴ test again/encore

@ $\phi = \frac{\pi}{2}, \theta = 0 \rightarrow \hat{n} = [-1, 0, 0]$ which points inwards

∴ multiply $\hat{n}$ by -1

$$\rightarrow \hat{n} = -R^2[-\sin^2\phi\cos\theta, \sin^2\phi\sin\theta, -\sin\phi\cos\phi]$$

$$F = \int_{\phi=0}^{\frac{\pi}{2}} \int_{\theta=0}^{2\pi} -\rho g(H - \overbrace{R\cos\phi}^{z_s})(-R^2)[-\sin^2\phi\cos\theta, \sin^2\phi\sin\theta, -\sin\phi\cos\phi]$$

<u>x and y component</u>: will cancel out due to symmetry

(you can see this if you try to integrate)

$F_x = 0, F_y = 0$

Z component:

$$F_z = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} -\rho g (R^2)(H - R\cos\phi)(-\sin\phi\cos\phi)\, d\theta\, d\phi$$

$$= +2\pi \rho g R^2 \int_0^{\frac{\pi}{2}} \left(-H \sin\phi\cos\phi + R\sin\phi\cos^2\phi\right) d\phi$$

integration tables

$$= \rho g\, 2\pi R^2 \left[-H\left(-\frac{1}{2}\cos^2\phi\right) + R\left(-\frac{1}{3}\cos^3\phi\right)\right]_0^{\frac{\pi}{2}}$$

$$= \rho g\, 2\pi R^2 \left(-\frac{H}{2} + \frac{R}{3}\right)$$

$$= -\rho g \underbrace{H\pi R^2}_{\forall_{cyl}} + \rho g \underbrace{\frac{2}{3}\pi R^3}_{\forall_{Hemisphere}} = -\rho g \pi \left(HR^2 - \frac{2}{3}R^3\right)$$

notice: same as weight of water above

$$\rho \simeq 1000 \frac{kg}{m^3}$$

$$g = 9.81 \frac{m}{s^2}$$

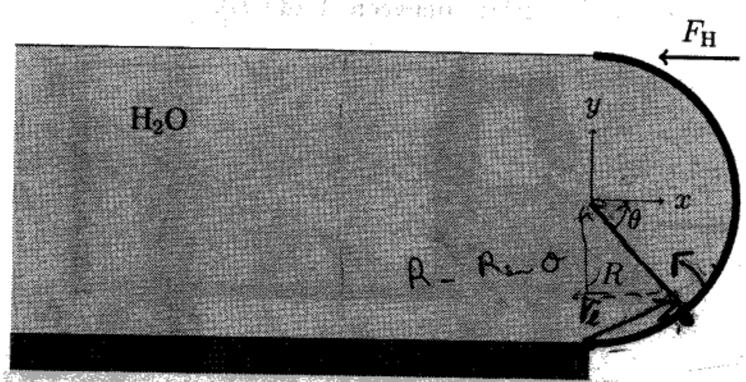
$$\underline{= -287644\, N}$$

Net Force $= F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$\therefore \boxed{F_{net} \approx 288\, kN \downarrow}$$

Name:

2) A gate is formed from a half cylinder. It has a width, W; a height, $2R$; and is hinged at the bottom. What horizontal force, F_H is needed to hold the gate in equilibrium?



$x = R\cos\theta$
$y = -R\sin\theta$
$z = z$ 1

$d\vec{F}_p = -p\hat{n}\,dS$ 1 $p = \rho g(R-y) = \rho g(R + R\sin\theta) = \rho g R(1+\sin\theta)$

$\vec{r} = R\cos\theta\,\hat{i} - R\sin\theta\,\hat{j} + z\hat{k}$

2 $\hat{n}\,dS = \left(\frac{\partial r}{\partial\theta} \times \frac{\partial r}{\partial z}\right) d\theta\,dz = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -R\sin\theta & -R\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} d\theta\,dz = \left(-R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}\right) d\theta\,dz$

1 $d\vec{F}_p = -\rho g R(1+\sin\theta)\left(-R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}\right) d\theta\,dz$

1 "lever arm": $\vec{r}_\ell = R\cos\theta\,\hat{i} + R(1-\sin\theta)\hat{j} + z\hat{k}$

$d\vec{M} = \vec{r}_\ell \times d\vec{F}_p$ 1 $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ R\cos\theta & R(1-\sin\theta) & z \\ -\cos\theta & \sin\theta & 0 \end{vmatrix} \left(-\rho g R^2(1+\sin\theta)\,d\theta\,dz\right) =$

$= \left[-z\sin\theta\,\hat{i} - z\cos\theta\,\hat{j} + \left(\cancel{R\cos\theta\sin\theta} + R\cos\theta - \cancel{R\cos\theta\sin\theta}\right)\hat{k}\right]\left(-\rho g R^2(1+\sin\theta)\,d\theta\,dz\right)$

only look at moment around "z"

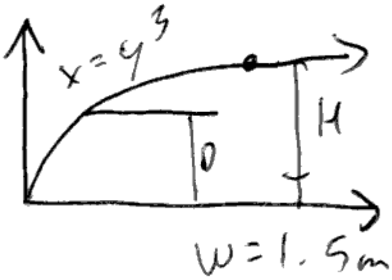
$M_z = \int_0^W \int_{-\pi/2}^{\pi/2} -\rho g R^3 (1+\sin\theta)\cos\theta\,d\theta\,dz = -\rho g R^3 W \int_{-\pi/2}^{\pi/2} (1+\sin\theta)\cos\theta\,d\theta$ 1

$= -\rho g R^3 W \left[+\sin\theta + \frac{\sin^2\theta}{2}\right]_{-\pi/2}^{\pi/2} = -2\rho g R^3 W$ 1

1 $F = \frac{M_z}{2R} = \rho g R^2 W$

3

4a Q2



$$\vec{r} = u^3\hat{i} + u\hat{j} + v\hat{k}$$

$$\frac{\partial \vec{r}}{\partial u} = 3u^2\hat{i} + \hat{j}$$

$$\frac{\partial \vec{r}}{\partial v} = \hat{k}$$

$$\left(\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3u^2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{i} - 3u^2\hat{j}$$

Points in correct direction (check $u=0$)
points in $\hat{i}$ dir.

$$p_{rel} = \rho g h = \rho g (D-y) = \rho g (D-u)$$

$$d\vec{F}_p = -p\hat{n}\,dS = -\rho g (D-u)\left[\hat{i} - 3u^2\hat{j}\right] du\,dv$$

$$d\vec{M} = \vec{r} \times dF_p = -\rho g (D-u) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u^3 & u & v \\ 1 & -3u^2 & 0 \end{vmatrix} du\,dv$$

lever arm is same as parametrization of surface in this case

$$= -\rho g (D-u) \begin{bmatrix} 3u^2 v\,\hat{i} \\ v\,\hat{j} \\ -3u^5 - u\,\hat{k} \end{bmatrix} du\,dv$$

only consider rotation around hinge

$\therefore$ only worry about M_z

$$dM_z = \rho g (D-u)(u + 3u^5)\,du\,dv$$

$$M_z = \int_0^W \int_0^D \rho g (D-u)(u+3u^5)\,du\,dv$$

$$= \rho g W \int_0^D (Du - u^2 + 3Du^5 - 3u^6)\,du$$

$$= \rho g W \left[\frac{Du^2}{2} - \frac{u^3}{3} + \frac{3Du^6}{6} - \frac{3}{7}u^7 \right]_0^D$$

$$= \rho g W \left[\frac{D^3}{2} - \frac{D^3}{3} + \frac{D^7}{2} - \frac{3D^7}{7} \right]$$

$$= (1000\,kg/m^3)(9.81\,m/s^2)(1.5\,m)\left[\frac{(1.2m)^3}{2} - \frac{(1.2m)^3}{3} + \frac{(1.2m)^7}{2} - \frac{3}{7}(1.2m \right.$$

$$= 8004\,N$$

$$\sum M = 0 \qquad M - FH = 0$$

$$F = \frac{M}{H} = \frac{8004\,N}{1.4\,m} = \boxed{5.7\,kN}$$