September 21st

MAT 1330, Fall 2016 Assignment 1
Due Thursday September 22 by 8:00pm.
Late assignments will not be accepted; nor will unstapled assignments. Professors in the math department will not lend you a stapler; do not ask for one.

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QUESTION 1. For any function f, we denote by D_f the maximal domain. Consider the two functions

$$f(x) = \frac{x}{x+1}, \qquad g(x) = \cot(x) = \frac{\cos(x)}{\sin(x)}.$$

Find the domains of the functions f, g, $f \circ g$, and $g \circ f$.

f of g $\to f(g(x))$

$$= \frac{\frac{\cos(x)}{\sin(x)}}{\frac{\cos(x)}{\sin(x)} + 1} = \frac{\frac{\cos(x)}{\sin(x)}}{\frac{\cos(x)+\sin(x)}{\sin(x)}}$$

$$= \frac{\cos(x)}{\sin(x)} \times \frac{\cancel{\sin(x)}}{\cos(x)+\sin(x)}$$

$$\boxed{f(g(x)) = \frac{\cos(x)}{\cos(x)+\sin(x)}}$$

g of f $\quad g(f(x)) = \dfrac{\cos\left(\frac{x}{x+1}\right)}{\sin\left(\frac{x}{x+1}\right)}$

Answer:

$D_f = \boxed{x \in \mathbb{R} \; (-\infty,-1),(-1,\infty)}$

$D_g = \boxed{x \in \mathbb{R} \quad x \neq n\pi \text{ where } n \in \mathbb{Z}}$

$D_{f\circ g} = \boxed{x \in \mathbb{R},\ x \neq 3\pi/4 + n\pi \text{ where } n \in \mathbb{Z}}$

$D_{g\circ f} = \boxed{x \in \mathbb{R},\ x \neq \frac{\pi}{2} + n\pi,\ \pi + n\pi} - 1$, where $n \in \mathbb{Z}$

$(\pi/2 + n\pi) - 1,\ (\pi + n\pi) - 1$

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Goal: Study stability, more examples

September 21^{st}

$= f(x)$? tines of graph $x^* = f(x^*)$

Question 2. (a) Solve the following inequality. (Continue on back)

$$\left|\frac{2}{5x^2+6x+7}\right| < \frac{1}{3}$$

Answer: $\boxed{(-\infty, -1) \text{ \& } (-\tfrac{1}{5}, +\infty)}$

Case 1

$$\frac{2}{5x^2+6x+7} - \frac{1}{3} < 0$$

$$\frac{6 - (5x^2+6x+7)}{(3)(5x^2+6x+7)} < 0$$

$$\frac{-5x^2-6x-1}{(3)(5x^2+6x+7)} < 0$$

Q.F (numerator)

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{+6 \pm \sqrt{36-20}}{-10}$$

$$x = \frac{+6+\sqrt{16}}{-10} \qquad x = \frac{+6-\sqrt{16}}{-10}$$

$$x = -1 \qquad x = -1/5$$

Q.F (denominator)

$$x = \frac{-6 \pm \sqrt{36-140}}{10}$$

No sol$^{\underline{n}}$ since there is no square root of negative

$-\infty$		-1		$-\frac{1}{5}$		$+\infty$
	$-$		$+$		$-$	

$\therefore \frac{-5x^2-6x-1}{(3)(5x^2+6x+7)} < 0$ @

$(-\infty, -1)$ & $(-\frac{1}{5}, +\infty)$

(b) Solve the following inequality.

$$\frac{1}{x+1} < \frac{1}{x^2+4x+3}.$$

Answer: $\boxed{(-\infty, -3) \text{ \& } (-2, -1)}$

$$\frac{1}{(x+1)} < \frac{1}{(x+1)(x+3)}$$

$x \neq -1, -3$

$$\frac{1}{(x+1)} - \frac{1}{(x+1)(x+3)} < 0$$

$$\frac{x+3-1}{(x+1)(x+3)} < 0$$

$$\frac{x+2}{(x+1)(x+3)} < 0$$

	$-\infty$ to -3	-3 to -2	-2 to -1	-1 to $+\infty$
$x+2$	$-$	$-$	$+$	$+$
$x+1$	$-$	$-$	$-$	$+$
$x+3$	$-$	$+$	$+$	$+$
$\therefore$	$-$	$+$	$-$	$+$

$\therefore \frac{x+2}{(x+1)(x+3)} < 0$ @

$(-\infty, -3)$ & $(-2, -1)$

Goal: study stability, more examples September 21^{st}

Recall: $x_{t+1} = f(x_t)$. 2 types of graph $x^* = f(x^*)$

QUESTION 3. Suppose that a population of bacteria is monitored daily. Its volume is multiplied by the same number every day. Hence, the volume V_t on day t satisfies the DTDS

$$V_{t+1} = rV_t.$$

Suppose that the initial volume is $V_0 = 8$ml and the volume at day 17 is $V_{17} = 16$ml.
(a) Find the value of r.

$V_{t+1} = rV_t$

$V_0 = 8mL$

$V_1 = rV_0$

$V_2 = rV_1 = r^2V_0$

$V_{17} = rV_{17} = r^{17}V_0$

Now solve for r

$V_{17} = r^{17}V_0$

$\frac{(16)}{8} = \frac{r^{17}(8)}{8}$

$\sqrt[17]{2} \quad \sqrt[17]{r^{17}}$

$\boxed{r = \sqrt[17]{2}}$

∴ The value of r is $\sqrt[17]{2}$

Answer: $r = \sqrt[17]{2}$ ≈ 1.04

(b) On which day is the volume equal to 64ml?

Find V_{day}

Given: $r = \sqrt[17]{2}$

V.

$V_{t+1} = rV_t$

$\frac{(64)}{\sqrt[17]{2}} = \frac{(\sqrt[17]{2})(V)}{\sqrt[17]{2}}$

≈ 61.4

$V_{t+1} \approx 62.4$

$V_{t+1} \approx 62$

∴ On the 62^{nd} day the volume is equal to 64mL

Answer: $V_{t+1} = 62.1$

QUESTION 4. Suppose that when you finish your studies and it is time to pay back your loans, you have a debt of \$ 100,000. From now on, at the beginning of each month, the bank adds 0.5% of the current value in interest. At the end of each month, you pay \$1,000. You receive a monthly statement of the remaining value of your loan, denoted L_t, immediately after your t-th payment and before any interest is applied. (In particular, $L_0 = 100,000$.)
(a) Write down the DTDS for L_t and the updating function.

$L_0 = 100,000$
$r = 1.005$
$c = -1000$

The DTDS is: $L_{t+1} = \boxed{1.005L_t - 1000}$

The updating function is: $f(L) = \boxed{1.005L - 1000}$

(b) When you receive the statement for L_{16}, you realize that you have lost the statement for L_{15}. Find the formula that calculates L_{15} from L_{16}.

$L_{15+1} = 1.005L_{15} - 1000$

$L_{16} = 1.005L_{15} - 1000$

$L_{16} + 1000 = 1.005L_{15}$

$L_{15} = \frac{L_{16}+1000}{1.005}$

$L_{15} = \boxed{\frac{L_{16}+1000}{1.005}}$

(c) Write down the general solution formula for the DTDS.

Answer: $\boxed{L_t = r^t(L_0 - L^*) + L^*}$ where L^* is your equilibria point found by $L^* = \frac{c}{(1-r)}$

(d) How many months does it take to pay down the entire loan? Calculate the smallest t for which $L_t \leq 0$.

$L_t = r^t(L_0 - L^*) + L^*$

① Find L^*

$L^* = \frac{(-1000)}{(1-1.005)} = 200\,000$

② Plug into L_t

$L_t = 0$ because that is the value when the whole loan is paid off.

③ $0 = (1.005)^t(100000 - 200000) + 200000$

$= (1.005)^t(-100000) + 200000$

$-200000 = (1.005)^t(-100000)$

$2 = (1.005)^t$

$\log 2 = t(\log 1.005)$

$\boxed{t = 138.9 \text{ months}}$

Answer: $\boxed{t = 138.9 \text{ months}}$

(e) Your bank changes its mailing policies: To save money, they only send out a statement every other month. Your payment schedule does not change. Find the corresponding updating function.

$L_{t+2} = f(L_{t+1}) = f(f(L_t)) = f \circ f(L_t)$

Answer: $L_{t+2} = \boxed{f(L_t + 1)}$