MAT 1330, Fall 2016 Assignment 1
Due Thursday September 22 by 8:00pm.
Late assignments will not be accepted; nor will unstapled assignments. Professors in the math department will not lend you a stapler; do not ask for one.

Instructor (circle one): F. Lutscher X. Wang Y. Samia R. Mikkelborg

DGD (circle one): 1 2 3 4

Student Name ______________________ Student Number ______________

Student Name ______________________ Student Number ______________

Student Name ______________________ Student Number ______________

By signing below, we declare that this work is our own and that we have not copied from any other individual or other source.

Signatures ____

QUESTION 1. For any function f, we denote by D_f the maximal domain. Consider the two functions

$$f(x) = \frac{x}{x+1}, \qquad g(x) = \cot(x).$$

Find the domains of the functions f, g, $f \circ g$, and $g \circ f$.

For f: Denominator $\neq 0 \Leftrightarrow x \neq -1$. For g: $\cot(x) = \frac{\cos(x)}{\sin(x)}$

$\sin(x) = 0$ if $x = n\pi, n \in \mathbb{Z}$

For $f \circ g$: Avoid all x s.t. $g(x) \notin D_f$, i.e. $g(x) \neq -1$

$g(x) \neq -1 \Leftrightarrow \cos(x) \neq -\sin(x) \Leftrightarrow x \neq \frac{3\pi}{4} + n\pi$

Answer:

$D_f = \boxed{\{x \in \mathbb{R} \mid x \neq -1\}}$

$D_g = \boxed{\{x \in \mathbb{R} \mid x \neq n\pi,\ n \in \mathbb{Z}\}}$

$D_{f \circ g} = \boxed{\{x \in \mathbb{R} \mid x \neq n\pi,\ x \neq \frac{3\pi}{4} + n\pi,\ n \in \mathbb{Z}\}}$

$D_{g \circ f} = \boxed{\{x \in \mathbb{R} \mid x \neq -1,\ x \neq \frac{n\pi}{1-n\pi},\ n \in \mathbb{Z}\}}$

For $g \circ f$: Avoid all x s.t. $f(x) \notin D_g$, i.e. $f(x) = \frac{x}{x+1} \neq n\pi$

or $x \neq \frac{n\pi}{1-n\pi}$

QUESTION 2. (a) Solve the following inequality.

$$\left|\frac{2}{5x^2+6x+7}\right| < \frac{1}{3}$$

Answer: $x < -1$ or $x > -\frac{1}{5}$

Check the denominator: $5x^2+6x+7=0$ if $x = \frac{1}{10}\left[-6 \pm \sqrt{36-140}\right]$ not a real number.

$\Rightarrow$ denominator is positive.

$\Rightarrow$ Cross-multiply to get $5x^2+6x+7>6$ or $5x^2+6x+1>0$.

Solve for $=0$: $x = \frac{1}{10}\left[-6 \pm \sqrt{36-20}\right] = \begin{cases} -1 \\ -1/5 \end{cases}$

Since it is an upward parabola: $5x^2+6x+1>0$ if $x \notin [-1, -\frac{1}{5}]$

(b) Solve the following inequality.

$$\frac{1}{x+1} < \frac{1}{x^2+4x+3}.$$

Answer: $x < -3$ or $-2 < x < -1$

Notice: $x^2+4x+3 = (x+1)(x+3)$

Case 1: If $x>-1$ then $x+1>0$. Multiply by $(x+1)$: $1 < \frac{1}{x+3}$

Since $x>-1$, $x+3>0$. Multiply: $x+3<1$ $\Rightarrow$ $x<-2$ incompatible with $x>-1$.

Case 2: $x<-1$ then $x+1<0$. Multiply by $(x+1)$: $1 > \frac{1}{x+3}$

If $x>-3$ then $x+3>1$ or $x>-2$ $\Rightarrow$ $-2<x<-1$

If $x<-3$ then $x+3<1$ or $x<-2$ $\Rightarrow$ $x<-3$

QUESTION 3. Suppose that a population of bacteria is monitored daily. Its volume is multiplied by the same number every day. Hence, the volume V_t on day t satisfies the DTDS

$$V_{t+1} = rV_t.$$

Suppose that the initial volume is $V_0 = 8\text{ml}$ and the volume at day 17 is $V_{17} = 16\text{ml}$.
(a) Find the value of r.

$V_{t+1} = rV_t \Rightarrow V_t = r^t V_0 \Rightarrow V_{17} = r^{17} \cdot V_0$

$\Rightarrow r^{17} = 2 \Rightarrow r = \sqrt[17]{2}$

Answer: $r = \sqrt[17]{2}$

(b) On which day is the volume equal to 64ml?

$V_t = 64 = r^t \cdot 8.$ solve for t where $r = \sqrt[17]{2}$

$8 = (2^{1/17})^t = 2^{t/17}$ require $t/17 = 3$

or $t = 3 \cdot 17 = 51$

Answer: $V_{51} = 64$

QUESTION 4. Suppose that when you finish your studies and it is time to pay back your loans, you have a debt of \$ 100,000. From now on, every month, the bank adds 0.5% of the current value in interest. At the end of each month, you pay \$1,000. You receive a monthly statement of the remaining value of your loan, denoted L_t, immediately after your t-th payment. (In particular, $L_0 = 100,000$.)

(a) Write down the DTDS for L_t and the updating function.

The DTDS is: $L_{t+1} = \boxed{1.005 L_t - 1000}$

The updating function is: $f(L) = \boxed{1.005 L - 1000}$

(b) When you receive the statement for L_{16}, you realize that you have lost the statement for L_{15}. Find the formula that calculates L_{15} from L_{16}.

$$L_{t+1} = 1.005 L_t - 1000 \Rightarrow L_{t+1} + 1000 = 1.005 L_t$$

$$L_t = \frac{1}{1.005}(L_{t+1} + 1000)$$

$L_{15} = \boxed{(L_{16} + 1000)/1.005}$

(c) Write down the general solution formula for the DTDS.

Answer: $\boxed{L_t = (1.005)^t (L_0 - L^*) + L^*}$ $\quad L^* = \frac{-1000}{-0.005}$

(d) How many months does it take to pay down the entire loan? Calculate the smallest t for which $L_t \leq 0$.

$$0 = (1.005)^t (L_0 - L^*) + L^* \quad \text{solve for } t$$

$$(1.005)^t = -\frac{L^*}{L_0 - L^*} \qquad t = \frac{\ln\left(-\frac{L^*}{L_0 - L^*}\right)}{\ln(1.005)} = \frac{\ln(2)}{\ln(1.005)} \simeq 138.97$$

Answer: $\boxed{t = 139}$

(e) Your bank changes its mailing policies: To save money, they only send out a statement every other month. Your payment schedule does not change. Find the corresponding updating function.

$$L_{t+2} = 1.005(1.005 L_t - 1000) - 1000$$

Answer: $L_{t+2} = \boxed{(1.005)^2 L_t - 2005}$

f(x)=(1.005^2)x-2005 est la fonction itérative