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Basic Math Review
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In this course, we assume that you not only can hire a full-time worker, but also a worker for any fraction of full-time. You cannot only sell scoops of ice cream, but also a third of a scoop or any other fraction. While this assumption (like most assumptions) makes the model less realistic, the concepts remain the same as if we used discrete numbers. When I refer, for instance, to an extra marginal unit of ice-cream sold, I don't mean a full scoop of ice cream; I refer to a very, very tiny fraction of an extra scoop sold. This makes life much easier for us: It allows us to use derivatives. We can compute the additional revenue of selling one more unit (which is very small) of ice cream by taking the derivative of revenues with respect to the quantity of ice-cream sold.

In class I will sometimes use examples with discrete numbers to illustrate things, but in all problem sets and exams, we will use derivatives to determine marginal products etc.

Partial Derivatives

If $y = f(x)$ is a function of variable x, then we call the derivative $f'(x)$, dy/dx or f_x. If $f(x, y)$ is a function of variables x and y, then we call the derivative of that function $f(x, y)$ with respect to x f_x or $\partial f(x, y)/\partial x$ and the derivative of that function $f(x, y)$ with respect to y $\partial f(x, y)/\partial y$ or f_y. A derivative is the slope of a function or the increase of the function value for an increase in its argument. For example, if $y = f(x)$, then $f'(x)$ is the amount by which y increases when you increase x by one (inifinitesimal) unit.

Here are just some simple rules for derivatives. It is informal and inexhaustive, so consult a math book for details if you are interested.

(a) The derivative of a constant is zero.

(b) The derivative of x^n with respect to x is nx^{n-1}.

(c) The derivative of a sum is equal to the sum of the derivatives:

$$d[f(x) + g(x)]/dx = df(x)/dx + dg(x)/dx.$$

(d) Product rule: if two functions are multiplied, $f(x)g(x)$, then the derivative is

$$d[f(x)g(x)]/dx = (df(x)/dx)g(x) + (dg(x)/dx)f(x).$$

(e) Quotient rule: if one function is divided by another, $f(x)/g(x)$, then the derivative is

$$d[f(x)/g(x)]/dx = \frac{(df(x)/dx)g(x) - (dg(x)/dx)f(x)}{[g(x)]^2}.$$

(f) Chain rule: if one function is a function of another function, $f(g(x))$, then the derivative is

$$d[f(g(x))]/dx = [df(g(x))/dg(x)][dg(x)/dx].$$

Total Derivatives

If we have a function of several variables, then the total change in function value can be decomposed into the change emanating from each variable. So for example, if my revenues come from selling ice cream and coffee, then the total change in revenue is equal to the change in revenue coming from selling more (or less) coffee and the change in revenue coming from selling more (or less) ice cream.

If $z = f(x_1, x_2, \ldots, x_n)$, then the total change in z is

$$dz = f_{x1}dx_1 + f_{x2}dx_2 + \cdots + f_{xn}dx_n,$$

where dx_1 is the change in x_1 (as usual in very small units so we can use derivatives) and similarly for x_2 etc.

Examples

Some examples. See the solutions on the next page. Remember that $x^{-n} = 1/x^n$.

1. If $f(x) = 5x^2 + 17$, what is f_x?
2. If $f(x) = 3x^{1/3} + 12x + 4000$, what is f_x?
3. If $f(x) = 0.8x^{-1/8} - 1/x^3$, what is f_x?
4. If $f(x) = (2x + x^3)(4 - x^{-3})$, what is f_x?
5. If $f(x) = (7x + x^9)/(5 + x^2/2)$, what is f_x?
6. If $f(x) = (3x^2 + 12x^{1/2})^{1/3}$, what is f_x?
7. If $f(x, y) = 4y + 3x$, what is f_x and what is f_y?
8. If $f(x, y) = 2yx^{2.5}$, what is f_x and what is f_y?
9. If $f(x, y) = (4x + 2y^{1/2})(x^4 - y^3/2)$, what is f_x and what is f_y?
10. If $f(x, y) = (x^{-5}y^2)^3 + x^2$, what is f_x and what is f_y?
11. If $z = f(x, y) = 0.25x^3 + y^{1.1}$, what is the total derivative? If x increases by two and y decreases by five small units, what is the change in z?
12. If $z = f(x, y) = 4(xy - 2x)$, what is the total derivative? If initially $x = 4$ and $y = 3$, and then z increases by one and y decreases by two small units, then what is the change in x?
13. If $z = f(x, y) = (2x + y^{1/2})^2$, what is the total derivative? If initially $x = 1/2$ and $y = 1$, and then x increases by four and z increases by two small units, then what is the change in y?

Solutions

1. If $f(x) = 5x^2 + 17$, $f_x = 10x$
2. If $f(x) = 3x^{1/3} + 12x + 4000$, $f_x = x^{-2/3} + 12$
3. If $f(x) = 0.8x^{-1/8} - 1/x^3$, $f_x = -0.1x^{-9/8} + 3x^{-4}$
4. If $f(x) = (2x + x^3)(4 - x^{-3})$, $f_x = (2 + 3x^2)(4 - x^{-3}) + (2x + x^3)(3x^{-4})$
5. If $f(x) = (7x + x^9)/(5 + x^2/2)$, $f_x = [(7 + 9x^8)(5 + x^2/2) - (7x + x^9)(x)]/(5 + x^2/2)^2$
6. If $f(x) = (3x^2 + 12x^{1/2})^{1/3}$, $f_x = (6x + 6x^{-1/2})(1/3)(3x^2 + 12x^{1/2})^{-2/3}$
7. If $f(x, y) = 4y + 3x$, $f_x = 3$ and $f_y = 4$
8. If $f(x, y) = 2yx^{2.5}$, $f_x = 5yx^{1.5}$ and $f_y = 2x^{2.5}$
9. If $f(x, y) = (4x+2y^{1/2})(x^4 - y^3/2)$, $f_x = 4(x^4 - y^3/2) + (4x+2y^{1/2})(4x^3)$ and $f_y = (y^{-1/2})(x^4 - y^3/2) + (4x + 2y^{1/2})(-1.5y^2)$
10. If $f(x, y) = (x^{-5}y^2)^3 + x^2$, $f_x = (-5x^{-6}y^2)3(x^{-5}y^2)^2 + 2x$ and $f_y = (x^{-5}2y)3(x^{-5}y^2)^2$
11. If $z = f(x, y) = 0.25x^3 + y^{1.1}$, $dz = 0.75x^2dx + 1.1y^{0.1}dy$. If $dx = 2$ and $dy = -5$, $dz = 1.5x^2 - 5.5y^{0.1}$
12. If $z = f(x, y) = 4(xy - 2x)$, $dz = 4(y - 2)dx + 4xdy$. If initially $x = 4$ and $y = 3$, and $dz = 1$ and $dy = -2$, then $1 = 4dx - 32$ or $dx = 33/4$
13. If $z = f(x, y) = (2x + y^{1/2})^2$, $dz = (2)2(2x + y^{1/2})dx + (1/2y^{-1/2})2(2x + y^{1/2})dy$. If initially $x = 1/2$ and $y = 1$, and then $dx = 4$ and $dz = 2$, then $2 = 32 + 2dy$ or $dy = -15$.