Concordia University
Department of Economics

Econ 221, Winter 2015
Midterm Exam 1
Sunday, February 22, 1:00 pm - 3:00 pm

Name:	I.D.:	Section:

Read all of the following information before starting the exam:

- This is a two-hour exam.
- This paper is graded out of 20 marks.
- The exam comprises two sections. Section A contains 4 multiple-choice questions and Section B contains 5 short-answer questions. Candidates should attempt all questions.
- Section A: Circle the best answer to each question in Section A. The space next to the choices can be used for calculations but they will not be considered as part of your answer.
- Section B: Answers to questions in Section B should be written in the spaces provided. Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, explain all relevant mathematics.
- Standard normal table is attached in the last page.
- Good luck!

Section A: 4 Multiple Choice Questions

1. (1 point) The standard deviation of the sample data 13, 14, 17, and 20 equals:(B)
   - A. 2.74
   - B. 3.16
   - C. 7.98
   - D. 9.16

2. (1 point) The amount of time you have to wait at a particular stoplight is uniformly distributed between zero and two minutes. is the probability that you have to wait more than 30 seconds for the stoplight? (C)
   - A. 0.25
   - B. 0.50
   - C. 0.75
   - D. 1.00

3. (1 point) Suppose X and Y are two random variables with $E(X) = 1.50$, $E(Y) = 0.55$, $Cov(X, Y) = -0.025$, $Var(X) = 0.25$, and $Var(Y) = 0.2475$. Let $W = 2X - 3Y$. What is the value of $E(W)$ and $Var(W)$? (A)
   - A. $E(W) = 1.35$ and $Var(W) = 3.5275$
   - B. $E(W) = 1.35$ and $Var(W) = 3.8275$
   - C. $E(W) = 1.50$ and $Var(W) = 3.5275$
   - D. $E(W) = 1.50$ and $Var(W) = 3.8275$

4. (1 point) Suppose you roll a pair of dice. Let A be the event that you observe an even number. Let B be the event that you observe a number greater than seven. What is $A \cap \bar{B}$? (D)
   - A. [9, 11]
   - B. [8, 10, 12]
   - C. [3, 5, 7]
   - D. [2, 4, 6]

Section B: 5 Questions

1. (6 points) In a recent survey, 300 teenagers were asked whether they had been to movie or dinner within the past month. Define the random variables as follows:

   $X = 1$ if teenager had been to movie within the past month; $X = 0$ otherwise

   $Y = 1$ if teenager had been out to dinner within the past month; $Y = 0$ otherwise

   The joint probability distribution of variables X and Y is shown in the table below.

	Y=0	Y=1
X=0	0.1368	0.2432
X=1	0.1550	0.4650

   (a) (2 points) Calculate the marginal probability distributions of X and Y.
   $P(X = 0) = 0.3800$, $P(X = 1) = 0.6200$
   $P(Y = 0) = 0.2918$, $P(Y = 1) = 0.7082$

   (b) (2 points) Calculate $P(Y|X = 1)$.
   $P(Y = 0|X = 1) = \frac{P(X=1,Y=0)}{P(X=1)} = \frac{0.1550}{0.62} = 0.25$
   $P(Y = 1|X = 1) = 1 - P(Y = 0|X = 1) = 0.75$

   (c) (2 points) Explain, using probabilities, whether going to movie and going out to dinner are statistically independent.
   They are not statistically independent, because $P(Y = 0|X = 1) \neq P(Y = 0)$.

2. (2 points) A publisher wishes to evaluate the effectiveness of a marketing campaign. 75% of all potential professors were reached in a focused advertising program. 28% of those contacted adopted the book while 8% of the adoptions came from professors who did not receive the promotional material. Define the following events of interest:

   A1 = Professor received advertising material

   A2 = Professor did not receive advertising material

   B1 = Professor adopts the book

   B2 = Professor does not adopt the book

   What is the probability that a professor who adopts the book received the advertising material?

$$
\begin{aligned}
P(A_1|B_1) &= \frac{P(B_1|A_1)P(A_1)}{P(B_1|A_1)P(A_1) + P(B_1|A_2)P(A_2)} \\
&= \frac{0.28 \times 0.75}{0.28 \times 0.75 + 0.08 \times 0.25} \\
&= 0.9130
\end{aligned}
$$

3. (2 points) A very large logging operation has serious problems keeping their skidders operating properly. The equipment fails at the rate of 3 breakdowns every 48 hours.

   What is the probability of a single breakdown within 24 hours is? (keep 4 decimals)
   $P(t < 0.5) = 1 - e^{-3\times 0.5} = 0.7769$

4. (2 points) The probability that a person catches a cold during the cold and flu season is 0.4. Assume that 10 people are chosen at random.

   What is the probability that exactly four of them will catch a cold? (keep 4 decimals)
   $P(X = 4) = C_4^{10} P^4 (1 - P)^6 = \frac{10!}{4!(10-4)!} 0.4^4 (1 - 0.4)^6 = 0.2508$

5. (4 points) The length of time it takes to be seated at a local restaurant on Friday night is normally distributed with a mean of 15 minutes and a variance of 25 minutes (keep 4 decimals)

   (a) (2 points) What is the probability that you have to wait more than 20 minutes to be seated?

$$\begin{aligned} P(X > 20) &= P(Z > \frac{20-15}{5}) \\ &= P(Z > 1) \\ &= 1 - P(Z < 1) \\ &= 0.1587 \end{aligned}$$

   (b) (2 points) What is the probability that you have to wait between 10 and 20 minutes to be seated?

$$\begin{aligned} P(10 < X < 20) &= P(\frac{10-15}{5} < Z < \frac{20-15}{5}) \\ &= P(-1 < Z < 1) \\ &= P(Z < 1) - P(Z < -1) \\ &= 2P(Z < 1) - 1 \\ &= 0.6826 \end{aligned}$$

Standard Normal Probabilities

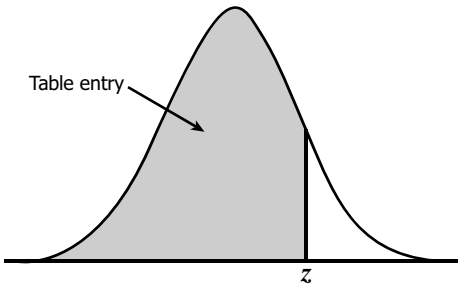

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998