Concordia University
Department of Economics

Econ 221/2 Section B
Fall 2014-Midterm Exam 1
Friday, October 3, 8:45 am - 10:00 am

Name:	I.D.:

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear and to the point.
- This test has 9 pages, 7 problems and is worth 40 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*8 points*) A company bids for two contracts. The probability of getting contract 1 is 0.4, the probability of getting contract 2 is 0.5, and the probability of getting both contracts is 0.2.

(a) What is the probability that this company gets exactly one of these two contracts? (1 point)

Event A: getting contract 1

Event B: getting contract 2

$P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B) = 0.4 + 0.5 - 0.2 = 0.7$

(b) What is the probability that this company gets neither of these two contracts? (1 point)

$P(\bar{A} \bigcap \bar{B}) = 1 - P(A \bigcup B) = 1 - 0.7 = 0.3$

(c) What is the probability that this company gets contract 2 given it has got contract 1? (1 point)

$P(B|A) = \frac{P(A \bigcap B)}{P(A)} = \frac{0.2}{0.4} = 0.5$

(d) What is the probability that this company gets contract 2 given it has failed to get contract 1? (1 point)

$P(B|\bar{A}) = \frac{P(\bar{A} \bigcap B)}{P(A)} = \frac{P(B) - P(A \bigcap B)}{1 - P(A)} = \frac{0.5 - 0.2}{1 - 0.4} = 0.5$

(e) Prove whether or not getting contract 1 and contract 2 are independent events. (2 point)

They are independent event because $P(B|A) = P(B) = 0.5$.

(f) Prove whether or not getting contract 1 and getting contract 2 are mutually exclusive events. (2 point)

They are not mutually exclusive because $P(A \bigcap B) = 0.2$.

2. (*6 points*) Alice is facing two options as the winner of Best Costume in a Halloween party. Option 1: She gets \$500. Option 2: She can spin a Wheel of Fortune. The Wheel of Fortune is divided into three segments. Segment 1 is a circular section with the central angle of 18 degrees. If the pointer stops in segment 1, she gets \$2000. Segment 2 is a circular section with the central angle of 72 degees. If the pointer stops in segment 2, she gets \$500. Segment 3 is circular section with the central angle of 270 degrees. If the pointer stops in segment 3, she does not get any money.

(a) If Alice chooses to spin the Wheel of Fortune, what is the probability that the pointer stops in segment 1?(2 points)

Event A: the pointer stops in segment 1

Event B: the pointer stops in segment 2

Event C: the pointer stops in segment 3

Note that the pointer can stop at anywhere. We use uniform distribution to model this experiment.

$P(A) = \frac{18}{360} = 0.05$

(b) Do you expect Alice to win more money by spinning the Wheel of Fortune than taking the \$500 cash? Explain. (2 points)

$P(B) = \frac{72}{360} = 0.20$

$P(C) = \frac{270}{360} = 0.75$

$\mu = 2000.05 + 500 \times 0.20 + 0 \times 0.75 = 200$

spinning the Wheel of Fortune, Alice expects to get \$200. She will be better off by taking the \$500 cash.

(c) What is the variance of the amount of money won by spinning the Wheel of Fortune? (2 points)

$\sigma^2 = 2000^2 \times 0.05 + 500^2 \times 0.20 + 0^2 \times 0.75 = 250000$

3. (*4 points*) Only 4% of people have Type AB blood. A bloodmobile has 12 vials of blood on a rack. If the distribution of blood types at this location is consistent with the general population, what is the probability that Canadian Blood Services finds AB blood in...(keep 4 decimals)

(a) Two of the 12 samples?(2 points)

$P(2) = \frac{12!}{2!(12-2)!} \times 0.04^2 \times (1-0.04)^{(12-2)} = 0.0702$

(b) No more than two of the 12 samples?(2 points)

$P(0) = \frac{12!}{0!(12-0)!} \times 0.04^0 \times (1-0.04)^{12} = 0.6127$

$P(1) = \frac{12!}{1!(12-1)!} \times 0.04^1 \times (1-0.04)^{(12-1)} = 0.3064$

$F(2) = P(0) + P(1) + P(2) = 0.9893$

4. (*6 points*) Statistics Canada provides data on the number of reportable accidents involving dangerous goods. In a recent year, the average rate of occurrence of such accidents was 8.15 per week.

(a) During that year, what was the probability of getting two such accidents in a given week? (2 points)

$P(2) = \frac{e^{-8.15} 8.15^2}{2!} = 0.0096$

(b) During that year, what was the probability of getting more than two such accidents in a given week? (2 points)

$P(0) = \frac{e^{-8.15} 8.15^0}{0!} = 0.0003$

$P(1) = \frac{e^{-8.15} 8.15^1}{1!} = 0.0024$

$P(X > 2) = 1 - F(2) = 1 - P(0) - P(1) - P(2) = 0.9877$

(c) Suppose the probability of the occurrences of such accidents on a particular day is the same over the year. During that year, what was the probability of getting more than two such accidents in a given day? (2 points)

$\lambda = \frac{8.15}{7} = 1.1623$

$P(0) = \frac{e^{-1.1623} 1.1623^0}{0!} = 0.3128$

$P(1) = \frac{e^{-1.1623} 1.1623^1}{1!} = 0.3625$

$P(2) = \frac{e^{-1.1623} 1.1623^2}{2!} = 0.2113$

$P(X > 2) = 1 - F(2) = 1 - P(0) - P(1) - P(2) = 0.1134$

5. (*6 points*) For the 900 trading days from January 2003 through July 2006, the daily closing price of IBM stock (in \$) is well modelled by a normal distribution with mean \$85.60 and standard deviation \$6.20. According to this model, what is the probability that on a randomly selected day in this period the stock price closed...(keep 4 decimals)

(a) Above \$91.80? (2 points)

Define $Z = \frac{X-\mu_X}{\sigma_X}$

$P(X > 91.80) = P(Z > \frac{91.80-85.60}{6.20}) = P(Z > 1) = 1 - P(Z < 0) - P(0 < Z < 1) = 1 - 0.5 - 0.3413 = 0.1587$

(b) Below \$98.00? (2 points)

$P(X < 98.00) = P(Z < \frac{98.00-85.60}{6.20}) = P(Z < 2) = P(Z < 0) + P(0 < Z < 2) = 0.5 + 0.4772 = 0.9772$

(c) Between \$73.20 and \$98.00? (2 points)

$P(73.20 < X < 98.00) = P(\frac{73.2-85.60}{6.20} < Z < 2) = P(-2 < Z < 2) = 2P(0 < Z < 2) = 0.9544$

6. (*4 points*) A study in the *International Conference on Social Robotics* (Vol. 6414, 2010) investigated the trend in the design of social robots. The study found that of 106 social robots, 63 were built with legs only, 20 with wheels only, 8 with both legs and wheels, and 15 with neither legs or wheels. Suppose you randomly select 10 of the 106 social robots and count the number with neither legs or wheels. (keep 4 decimals)

(a) What is the probability that there is one with neither legs nor wheels? (2 points)

Suppose X robots with neither legs or wheels are selected

$P(1) = \frac{C_1^{15} C_{10-1}^{106-15}}{C_{10}^{106}} = 0.3691$

(b) What is the probability that there are more than two with neither legs nor wheels? (2 points)

$P(0) = \frac{C_0^{15} C_{10-0}^{106-15}}{C_{10}^{106}} = 0.2018$

$P(2) = \frac{C_2^{15} C_{10-2}^{106-15}}{C_{10}^{106}} = 0.2801$

$P(X > 2) = 1 - P(0) - P(1) - P(2) = 0.1490$

7. (*6 points*) There is a sign in the university library elevator indicating a 16-person limit as well as a weight limit of 2500 pounds. Suppose that the weights of students, faculty, and staff are approximately normally distributed with a mean weight of 150 pounds and a standard deviation of 25 pounds.

(a) If there are 16 people in the elevator, what is their expected total weight? (2 points)

Let X_i denote the weight of person i. The total weight is $Y = \sum_{i=1}^{16} X_i$

$\mu_Y = 16E[X_i] = 16 \times 150 = 2400$

(b) What is the variance of the total weight of these 16 people? (2 points)

$\sigma_Y^2 = \sum_{i=1}^{16} Var[X_i] + 2\sum_{i=1}^{15}\sum_{j=i+1}^{16} Cov[X_i, X_j] = \sum_{i=1}^{16} Var[X_i] = 16 \times 25^2 = 10000$

The second equality holds because of the independence of individual i's and individual j's weight.

(c) What is the probability that these 16 people in the elevator will exceed the weight limit? (2 points)

Let $Z = \frac{Y-\mu_Y}{\sigma_Y} = \frac{Y-2400}{20}$. Z follows the standard normal. distribution

$P(Y > 2500) = P(Z > \frac{2500-2400}{100}) = P(Z > 1) = 0.1587$