Assignment 1 with solution

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Q1

A grower estimates that if he picks his apple crop now, he will obtain 1,500 boxes of apples, which he can sell at \$2 a box. However, he thinks his crop will increase by 120 boxes of apples for each week he delays picking, but that the price will drop at a rate of 10¢ a box per week; in addition, he estimates approximately 25 boxes a week will spoil for each week he delays picking. When should he pick his crop to obtain the largest total cash return? How much will he receive for his crop at that time?

Solution

Let t = time from the present (in weeks)

Volume of apples at any time = (1,500 + 120t – 25t) = 1,500+95t

Price at any time = \$2.00 – \$0.10t

Total Cash Return (TCR) = (1,500 + 95t) (\$2.00 – \$0.10t)

$= \$3,000 + \$40t - \$9.5t^2$

Set the first derivative equal to zero and solve for t.

dTCR/dt = \$40 – \$19t = 0

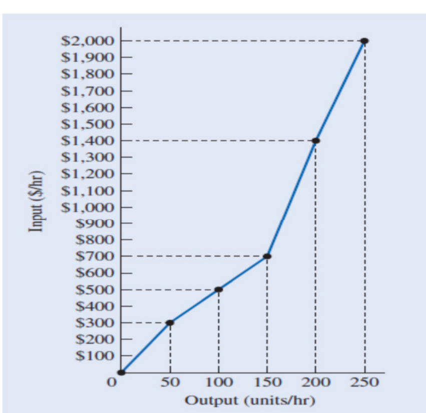
t = \$40/\$19 = 2.1 weeks

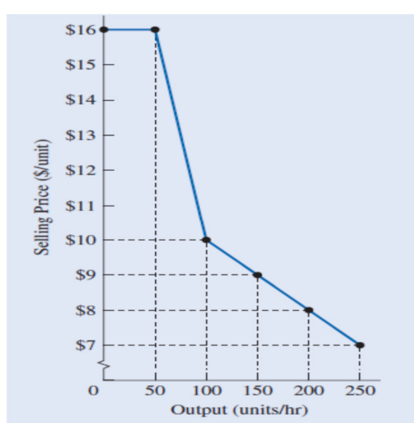
At t = 2.1 weeks:

Total Cash Return (TCR) $= \$3,000 + \$40\ (2.1) - \$9.5\ (2.1)^2 = \$3,042$

Q2

On her first engineering job, Joy Hayes was given the responsibility of determining the production rate for a new product. She has assembled data as indicated on two graphs:





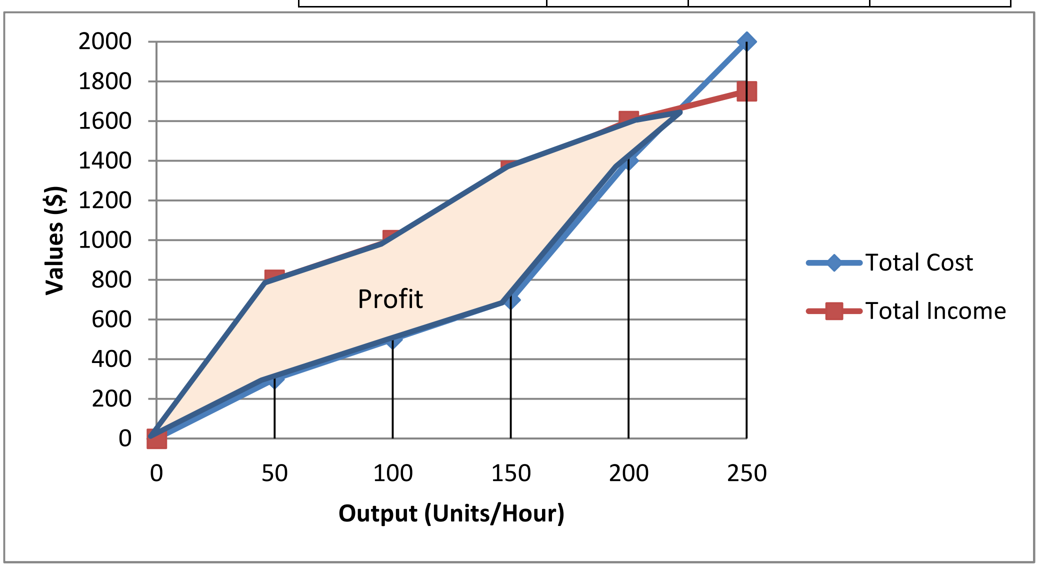
(a) Select a suitable economic criterion and estimate the production rate based upon it.

(b) Joy's boss told her: "I want you to maximize output with minimum input." Joy wonders if it is possible to meet her boss's criterion. She asks your advice. What would you tell her?

Solution

The suitable criterion is to maximize the difference between output and input. Or simply, maximize net profit. The data from the graphs may be tabulated as follows:

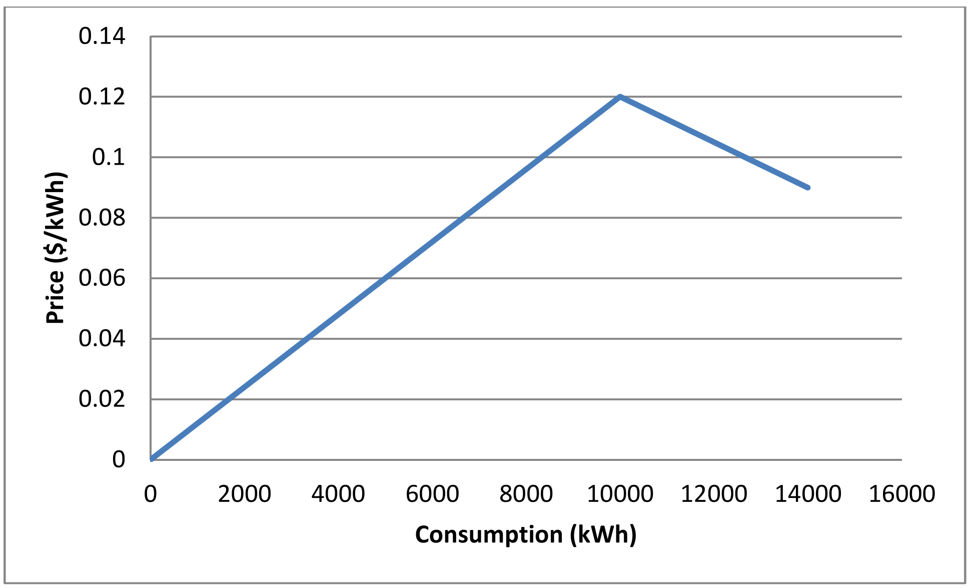
Output Units/Hour	Total Cost	Total Income	Net Profit
0	0	0	0
50	300	800	500
100	500	1000	500
150	700	1350	650
200	1400	1600	200
250	2000	1750	-250



(b) Minimum input is, of course, zero, and maximum output is 250 units/hour (based on the graph). Since one cannot achieve maximum output with minimum input, the statement makes no sense.

Q3
Electricity is sold for $0.10 per kilowatt-hour (kWh) for the first 12,000 units each month and $0.08/kWh for all remaining units. If a firm uses 15,000kWh/month what is its average and marginal cost?

Solution
If firm uses 15,000 kWh/month, then cost is:
$0.10/kWh for first 12,000 kWh
$0.08/kWh for the remaining 3,000 kWh
Marginal Cost (cost for the next kWh) = **$0.08** since we are in the 2nd bracket of the cost structure.
Total Cost = $0.10/kWh * 12,000 kWh + $0.08/kWh * 3,000 kWh = $1,440
Average Cost = $1,440 / 15,000kWh = $0.096/kWh



Q4
Two new rides are being compared by a local amusement park in terms of their annual operating costs. The two rides are assumed to be able to generate the same revenue (hence the focus on costs). The Tummy Tugger has fixed costs of $15,000 a year and variable costs of $2.75 per visitor. The Head Buzzer has fixed costs of $5,000 per year and variable costs of $4.5 per visitor. Answer the following questions so the amusement park can make the needed comparison.

(a) Determine mathematically the breakeven number of visitors per year for the two rides to have equal annual costs.
(b) Develop a graph that illustrates the following (note: put visitors per year on the horizontal axis and costs on the vertical axis):

- accurate total cost lines for the two alternatives (show line, slopes, and equations)
- the break-even point for the two rides in terms of number of visitors
- the ranges of visitors per year where each alternative is preferred

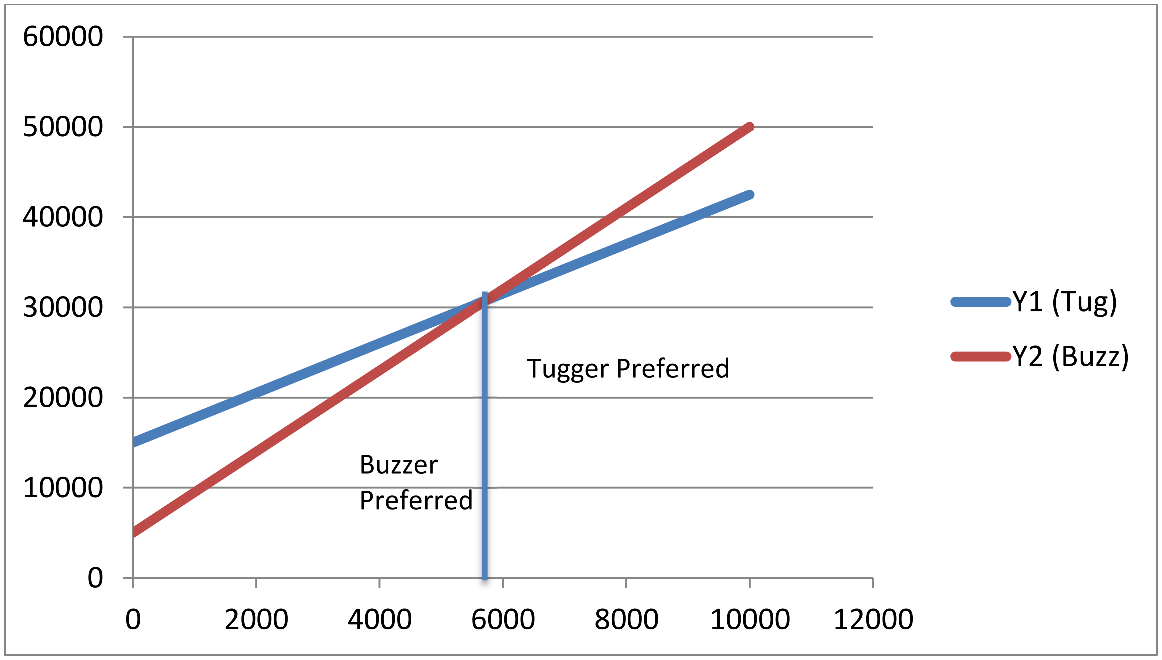
Solution

(a)
Let x = number of visitors per year
Break even when: Total Costs (Tugger) = Total Costs (Buzzer)
$\$15,000 + \$2.75x = \$5,000 + \$4.50x$
$x = 5,714$ visitors is the breakeven quantity

X	Y1 (Tug)	Y2 (Buzz)
0	15,000	5,000
4,000	26,000	23,000
8,000	37,000	41,000

(c) See the figure below

Y1 = 15000 + 2.75x
Y2 = 5000 + 4.50x



Q5
Solution

a) F = ($0.61/month) * (12 mon/yr) * (55 yrs) = $402.60

b)

$$F = A[((1+i)^N - 1)/i]$$

Tug Preferred

$$F = 0.61[((1+0.03)^{660} - 1)/0.03] = \$6,036,384,346.72$$

Q6
Solution

Buzz Preferred

a) Lump Sum Payment = $395 (F/P, 1.7
= $453.81

Y1 (Tug) = 10,000 + 2.5x
Y2 (Buzz) = 4,000 + 4x

b) Alternate Payment = $395 (F/P, 11%, 1)
= $ 438.45

Choose the alternate payment plan.

Q7
Solution

The garbage company sends out bills only six times a year. Each time they collect one month's bills one month early.

100,000 customers × $6.00 × 2% per month × 6 times/yr = $72,000

Q8
Solution

a) Interest Rate per 6 months = (420,000-400,000)/$400,000= 0.0500 = 5%
Effective Interest Rate per yr. = $(1 + 0.05)^2 - 1 = 0.1025$ = 10.25%

b) For continuous compounding:

$F = Pe^{r/k}$ (k is the number of payments per year, jack deposited the money once at 6 months , therefore k for this case is 1 per six months or 2 per year)
$420,000 = $400,000 $e^{r/1}$
$r_{(\text{nominal interest per 6 months or per payment})}$ = ln ($420,000/$400,000) = 0.0488 = 4.88% per 6 months
Nominal Interest Rate (per year) = 4.88% (x2) = 9.76% per year

Q9

The I've Been Moved Corporation (IBM) receives a constant flow of funds from its worldwide operations. This money (in the form of cheques) is deposited continuously in many banks with the goal of earning as much interest as possible for IBM. One billion dollars is deposited each month, and the money earns an average of 0.5% interest a month, compounded continuously. Assume all the money remains in the accounts until the end of the month.
(a) How much interest does IBM earn each month?
(b) How much interest would IBM earn each month if it held the cheques and made deposits to its bank accounts just four times a month?

Solution

(a) Interest rate = 0.5% per month
Principal = $1,000 million
I= i_p x P/2 = $(e^{r/k}-1) \times 1\times10^9 \times 1/2 = (e^{0.005}-1) \times 1\times10^9 \times 1/2$ =**$2,506,260** every month
Note: In the above calculation, the actual principal has been halved, as the interest is paid on the average sum.
Money is being paid continuously, which is 0 at the beginning of the month, and 1 billion at the end of the month.

(b) Cheques are distributed 4 times a month.
$i_p = e^{r/k} - 1$
$i_p = e^{0.06/48} - 1$
Interest rate per payment, i_p = 0.125%
Principal 1000 million/4= $250 million every week
Thus, A = $250 million, $n = 4$
$F = A$ [F/A, 0.125%, 4] = 250 million x 4.0075
Thus, F = $1,001.88 million
Interest accumulated = $1,877,737 every month

Q10

A forklift truck costs \$31,000. A company agrees to purchase such a truck with the understanding that it will make a single payment for the balance due in three years. The vendor agrees to the deal and offers two different interest schedules. The first schedule uses an annual effective interest rate of 13%. The second schedule uses 12.75% compounded continuously.

(a) Which schedule should the company accept?

(b) What would be the size of the single payment?

Solution

P= \$31,000, $n = 3$ years, $F = ?$

(a) $i_a = 0.13$

$F = P\,(1 + i)^n = \$31{,}000\,(1.13)^3 = \$44{,}729.8$

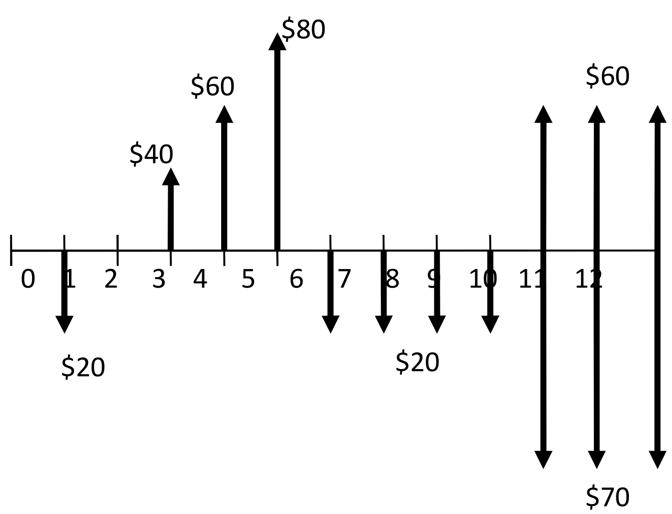
(b) $r = 0.1275$, k=1/3

$F = P\,e^{r/k} = \$31{,}000\,e^{(0.1275)(3)} = \$31{,}000\,(1.4659) = \$45{,}444.3$

We can see that although the interest rate was less with the continuous compounding, the future amount is greater because of the increased compounding periods (an infinite number of compounding periods). Thus, the correct choice for the company is to choose the 13% interest rate and discrete compounding.

Q11

Calculate the present worth of the cash flow shown in the accompanying diagram, using at most three kinds of interest factors at 3% interest compounded annually.



Solution

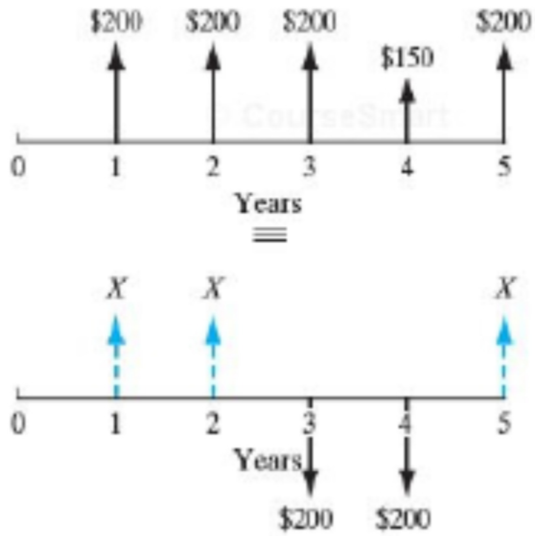
There are multiple ways to interpret the cash flows, below is one way to solve the cash flow diagram.

$$P = -20\left(P/F, 0.03, 1\right) + 40\left(P/A, 0.03, 3\right)\left(P/F, 0.03, 2\right)\left\{1 + (0.5)\left(A/G, 0.03, 3\right)\right\}$$
$$-20\left(P/A, 0.03, 4\right)\left(P/F, 0.03, 5\right) - 10\left(P/A, 0.03, 3\right)\left(P/F, 0.03, 9\right)$$
$$= \$\,54.38$$

P = -20(0.9709) + 40 (2.8286)(0.9426)[1+(0.5)(0.9803)]-20(3.7171)(0.8626) -10(2.8286) (0.7664)=53.7

Q12

Find the value of X so that the two cash flows shown in the diagram are equivalent for an interest rate of 5%.



Solution

Establishing equivalence at $n = 5$

200(F /A,5%,5)-50(F / P, 5%,1)=X (F / A, 5%,5)-(200 + X)[(F / P,5%, 2)+(F / P, 5%, 1)]
200(5.5256)-50(1.0500) =X (5.5256)-(200+X) [(1.1025) + (1.0500)]
1,052.62 = X (5.5256)-(200+X) (2.1525)
1,052.62 + 430.5= 3.3731 X
X = 439.69

Q13

For the following transactions, draw the C.F.D and find the value of G that makes the deposit series equivalent to the withdrawal series at interest rate of 10%, compounded annually.

End of period	Deposit	Withdrawal
0	$1100	
1	800	
2	600	
3	400	
4	200	
5	50	
6		G
7		2G
8		3G
9		4G
10		5G

Solution

G(P/G,10%,6) = 800(F/A,10%,4)+(1,100-200(P/G,10%,4))(F/P,10%,4)+50(P/F,10%,1)

G={ 800(4.641)+(1100-200(4.3781))(1.4641)+50(0.9091)}/9.684

G=422.01