MAT 134IC, W11 Diagnostic

1. An equation for the plane passing through the points $(1, 2, -1)$ and $(2, 3, 1)$, which is parallel to the x-axis is: 4 54

A. $7x - 3y + -2z = 3$
B. $2y - z = 5$
C. $x + y - z = 4$
D. $x - y = -1$
E. $2y - z = 0$
F. $x + y + z = 2$

Since the plane is $\parallel$ to the x-axis, its normal will be $\perp$ to the x-axis, and so any cartesian equation describing it will have 0 coefficient on "x."

Hence B and E are the only candidats. Since $(1, 2, -1)$ does not satisfy $2y - z = 0$, B must be correct.

2. Find an equation of the plane which passes through the point $(1,\ 8,\ -7)$ and which is perpendicular to the line whose parametric equations are:

$$x = 2 + 2t,\ y = 1 + t,\ z = 3 - 4t;\ t \in \mathbf{R}.$$

A. $2x + y - 4z = 6$
B. $2x + y + 3z = -11$
C. $2x - 3y + 7z = -71$
D. $2x - 4y + z = -28$
E. $2x + y - 4z = 38$
F. $-4x + y + 2z = 10$

A normal to the plane is $n = (2, 1, -4)$, so A and E are the only possibilities. Since $2(1) + 1(8) - 4(-7) = 38$, E is correct

3. Find parametric equations for the line L passing through $(-1,\ 1,\ 1)$ and which is perpendicular to the plane $3x - y + 2z = 1$.

A. $x = -1 + 3t,\ \ y = 1 + t,\ \ z = 1 - 2t,\ \ t \in \mathbf{R}$
B. $x = -1 + 3t,\ \ y = 1 - t,\ \ z = 1 + 2t,\ \ t \in \mathbf{R}$
C. $x = -1 - 6t,\ \ y = 1 + t,\ \ z = 1 - t,\ \ t \in \mathbf{R}$
D. $x = -1 - 3t,\ \ y = 1 - t,\ \ z = 1 - 2t,\ \ t \in \mathbf{R}$
E. $x = -1 - t,\ \ y = 1 + t,\ \ z = 1 + t,\ \ t \in \mathbf{R}$
F. $x = 1 - 4t,\ \ y = 1 - t,\ \ z = -1 - 3t,\ \ t \in \mathbf{R}$

A direction vector for L is $d = (3,-1,2)$. The only possibility is therefore B (which does indeed contain $(-1,1,1)$.)

4. If $u = (3,\ -3,\ -1)$, $v = (-5,\ 4,\ -4)$ and $w = (0,\ 3,\ 4)$, find $\|(2u + v) \times w\|$.

A. $2\sqrt{5}$
B. $2\sqrt{10}$
C. $5\sqrt{2}$
D. $10\sqrt{5}$
E. 25
F. $5\sqrt{5}$

$$2u + v = (6,-6,-2) + (-5,4,-4) = (1,-2,-6)$$

Hence $(2u+v)\times w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -6 \\ 0 & 3 & 4 \end{vmatrix} = (10, -4, 3)$

Hence $\|(2u+v)\times w\|^2 = 10^2 + (-4)^2 + 3^2 = 125$,

So $\|(2u+v)\times w\| = \sqrt{125} = \sqrt{5.25} = 5\sqrt{5}$

5. Find a scalar equation for the plane with vector parametric equation

$$v = (2,4,3) + s(1,2,-1) + t(3,-4,0);\ s,t \in \mathbf{R}.$$

A. $4x - 3y + 10z = -50$

B. $4x + 3y - 10z = 50$

C. $4x - 3y + 10z = 50$

D. $-4x + 3y + 10z = 50$

E. $4x + 3y + 10z = -50$

F. $4x + 3y + 10z = 50$

A normal to the plane is

$$n = (1,2,-1) \times (3,-4,0)$$

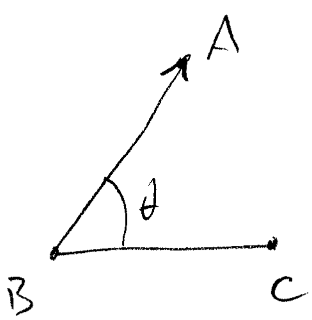
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -4 & 0 \end{vmatrix}$$

$= (-4,-3,-10)$. Hence the only possibilities are E and F.

Since $4(2) + 3(4) + 10(3) = 50$, F is correct.

6. If $A = (3,\ 0,\ 9)$, $B = (2,\ 4,\ 1)$ and $C = (1,\ 4,\ 0)$, find the angle $\angle ABC$.

A. $\pi/2$
B. $\pi/3$
C. $\pi/4$
D. $\pi/6$
E. $3\pi/4$
F. $4\pi/3$



$$\cos\theta = \frac{(A-B)\cdot(C-B)}{\|(A-B)\|\ \|C-B\|}.$$

$A - B = (1,-4,8)$; $C - B = (-1,0,-1)$, so

$$\cos\theta = \frac{-9}{\sqrt{1+16+64}\ \sqrt{2}} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}.$$ Hence

$\theta = 3\pi/4$.

7. If $u = (3,\ 3,\ 3)$ and $v = (2,\ 1,\ 3)$ find the orthogonal projection of u on v, i.e. $\mathrm{proj}_v u$.

A. $\frac{9}{7}(2,\ 1,\ 3)$

B. $\frac{12}{7}(2,\ 1,\ 3)$

C. $\frac{9\sqrt{14}}{7}(2,\ 1,\ 3)$

D. $\frac{2}{2}(3,\ 3,\ 3)$

E. $\frac{9}{7}(3,\ 3,\ 3)$

F. $6\sqrt{3}\ (1,\ 1,\ 1)$

$$\mathrm{Proj}_v u = \frac{u \cdot v}{\|v\|^2} v$$

$$= \frac{(3,3,3)\cdot(2,1,3)}{4+1+9} \cdot (2,1,3)$$

$$= \frac{18}{14} \cdot (2,1,3) = \frac{9}{7}(2,1,3)$$

8. Find the volume of the parallelepiped determined by the vectors $u = (1,\ 1,\ -1)$, $v = (2,\ 0,\ 1)$ and $w = (1,\ -1,\ 3)$.

A. -2
B. 4
C. 6
D. 8
E. 16
F. 2

$$\mathrm{vol} = | u \cdot v \times w |$$

$$v \times w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 3 \end{vmatrix} = (1, -5, -2)$$

Hence $u \cdot v \times w = (1,1,-1)\cdot(1,-5,-2)$

$= -2.$

Hence the volume is 2.

9. What is the area of the triangle with vertices (3, 0, - 2), (5, 2, -1) and (5, 9, 0)?

A. 11

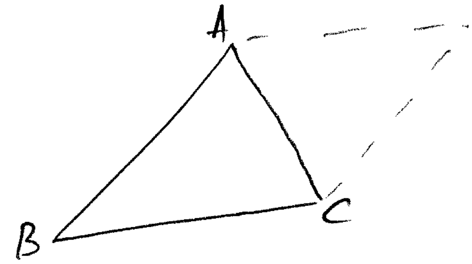
B. 13

C. 15

D. $\frac{13}{2}$

E. $\frac{15}{2}$

F. $\frac{17}{2}$



$$\text{area} = \frac{1}{2}\|(A-B)\times(C-B)\|$$

$$(A-B)\times(C-B) = (-2,-2,-1)\times(0,7,1)$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -2 & -1 \\ 0 & 7 & 1 \end{vmatrix}$$

$$= (5, +2, -14)$$

Thus $\text{area} = \frac{1}{2}\sqrt{25+4+196} = \frac{1}{2}\sqrt{225} = \frac{15}{2}$

10. The intersection of the planes with equations $x + 11y - 4z = 40$ and $x - y = -8$ is the line with (vector) parametric equation:

A. $(-4,\ 4,\ 0) + t(1,\ 3,\ 1),\ t \in \mathbf{R}$

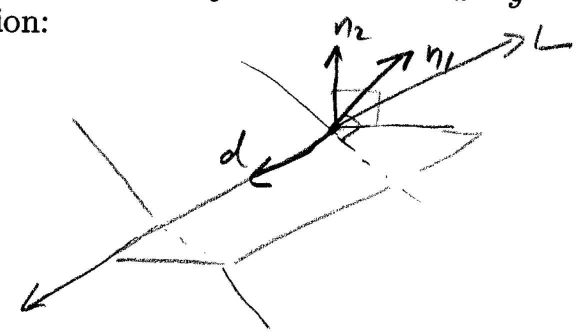
B. $(4,\ 4,\ 0) + t(1,\ 1,\ 8),\ t \in \mathbf{R}$

C. $(-4,\ 4,\ 0) + t(1,\ 1,\ 3),\ t \in \mathbf{R}$

D. $(4,\ -4,\ 0) + t(-1,\ 1,\ -3),\ t \in \mathbf{R}$

E. $(0,\ 4,\ -4) + t(1,\ 3,\ -1),\ t \in \mathbf{R}$

F. $(4,\ 0,\ 4) + t(-1,\ 3,\ 1),\ t \in \mathbf{R}$



A direction vector for L is $n_1 \times n_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 11 & -4 \\ 1 & -1 & 0 \end{vmatrix} = (-4, -4, -12)$, or $(1, 1, 3)$. Hence C must be correct. (The point $(-4, 4, 0)$ does indeed belong to both planes.)

11. Express the following complex numbers in the form $a + bi$:

$$z_1 = \frac{1}{1+i} \quad = \quad \frac{1-i}{1+1} \; = \; \frac{1}{2} - \frac{1}{2} i ,$$

$$z_2 = (2+i)(2+2i)$$

A. $z_1 = 1 - i$; $z_2 = 4 + 4i$

B. $z_1 = -1 + i$; $z_2 = 2 + 4i$

C. $z_1 = \frac{1}{2} + \frac{1}{2}i$; $z_2 = 2 - 6i$

D. $z_1 = \frac{1}{2} - \frac{1}{2}i$; $z_2 = 2 + 6i$

E. $z_1 = 2 - \frac{1}{4}i$; $z_2 = 6 - 2i$

F. $z_1 = 1 - i$; $z_2 = 0$

Thus D must be correct!

(Indeed, $(2+i)(2+2i) = 2+6i$)

12. Find the polar form of $\dfrac{1-\sqrt{3}i}{-1+i}$

A. $\sqrt{2}\left(\cos(-\frac{5\pi}{12}) + i\sin(-\frac{5\pi}{12})\right)$

B. $\sqrt{2}\left(\cos(\frac{11\pi}{12}) + i\sin(\frac{11\pi}{12})\right)$

C. $\sqrt{2}\left(\cos(-\frac{7\pi}{12}) + i\sin(-\frac{7\pi}{12})\right)$

D. $\sqrt{2}\left(\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12})\right)$

E. $\sqrt{2}\left(\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12})\right)$

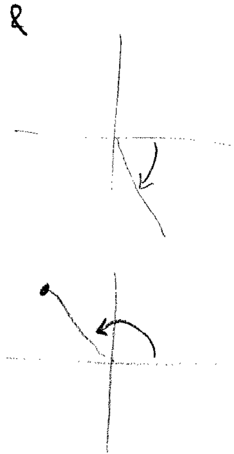
F. $\sqrt{2}\left(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12})\right)$

- $1 - \sqrt{3}i = \sqrt{1+\sqrt{3}^2} \cdot e^{i\theta} = 2e^{i\theta}$

where $\cos\theta = \frac{1}{2}$ &

$\sin\theta = -\frac{\sqrt{3}}{2}$.

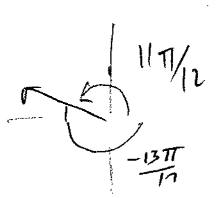
Hence $\theta = -\pi/3$



- $-1 + i = \sqrt{2}\, e^{3\pi/4}$.

Hence $\dfrac{1-\sqrt{3}i}{-1+i} = \dfrac{2e^{-i\pi/3}}{\sqrt{2}\, e^{3\pi/4}}$

Now, $-\pi/3 - 3\pi/4 = \dfrac{(-4-9)\pi}{12} = -\dfrac{13\pi}{12}$. But $e^{-i\frac{13\pi}{12}}$



$= e^{i\frac{11\pi}{12}}$. Hence B is correct

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