Solutions MATH 208 Final Exam – April 2013

Question 1

1. **[5]**Find the linear cost function for Joanne's T-shirt production
   Cost $= 3.5x + 90$
2. **[5]**How many T-shirts must she produce and sell in order to break even?
   Revenue $= 9x = 3.5x + 90 =$ Cost, when $x = 16.36$
   Joanne must sell **17** T-shirts to exceed her costs.

Question 2

A. **[2]** $\left(\frac{5}{6}\right)^x = \frac{36}{25} = \left(\frac{5}{6}\right)^{-2} \Rightarrow x = -2$

B. **[2]** $(4)^{x+1} = (8)^{2-x}$
$(2)^{2x+2} = (2)^{6-3x} \Rightarrow 2x + 2 = 6 - 3x \Rightarrow 5x = 4 \Rightarrow x = \frac{4}{5}$

C. **[2]** $\log_5(x+6) + \log_5(x+2) = 1$
$\log_5[(x+6)(x+2)] = 1 \Rightarrow (x+6)(x+2) = 5$
$x^2 + 8x + 7 = 0 \Rightarrow (x+7)(x+1) = 0 \Rightarrow x = -7$ *or* $x = -1$ (but -7 is not possible)

D. **[2]** $\log_2(2x) = 4 - \log_2(x+2)$
$\log_2(2x)(x+2) = 4 \Rightarrow 2x^2 + 4x = 2^4 = 16 \Rightarrow x^2 + 2x - 8 = 0$
$(x+4)(x-2) = 0 \Rightarrow x = -4$ *or* $x = 2$,

E. **[2]** $\log_3\left[\frac{5x}{x-2}\right] = 2$
$\frac{5x}{x-2} = 3^2 = 9 \Rightarrow 5x = 9(x-2) = 9x - 18$
$18 = 4x \Rightarrow x = 9/2$

Question 3

A. **[5]** $\sum_{k=0}^{41} f(k) = -28\sum_{k=0}^{41} k + \sum_{k=0}^{41} 6$
$-28\left[\frac{(41)(42)}{2}\right] + 6 \times 42 = -23856$

B. **[5]** $\sum_{h=0}^{30} g(h) = 5\sum_{h=0}^{30}(1.8)^x = 5\left[\frac{1-1.8^{31}}{1-1.8}\right] = 512,068,039.3$

Question 4

1. **[2]**Find the amount of each semi-annual interest payment.
   $4000 * 0.06 = \$240.00$ **OR** 4000*0.06/2 = **$120.00**
2. **[4]**Sinking Fund: Find the amount of each payment.
   $FV = 4000 = P \times \left(\frac{1.08^5 - 1}{0.08}\right) \Rightarrow P = \$681.83$
3. **[4]**Table showing amount in the sinking fund after each deposit.

Year	Deposit	Interest	Balance
1	681.83	0	681.83
2	681.83	54.5464	1418.206
3	681.83	113.4565	2213.493
4	681.83	177.0794	3072.402
5	681.83	245.7922	4000.025

Question 5

1. **[3]**How large are the monthly payments?
   Amount to finance $= 212000 - 0.2 * 212000 = 169600$
   Monthly payments: $n = 12 * 30 = 360$; $i = \frac{0.072}{12} = 0.006$
   PV$= 169600 = P\left[\frac{1-1.006^{-360}}{0.006}\right] \Rightarrow P = \$\mathbf{1151.22}$
2. **[3]**Loan balance after 96th payment
   Remaining payments: $360 - 96 = 264$
   Outstanding balance (after 96 pmts) $= 1151.22\left[\frac{1-1.006^{-264}}{0.006}\right] = \$\mathbf{152,320.58}$
3. **[4]**How much interest will they pay during the 7th year of the loan?
   Outstanding balance (after 72 payments) $= 1151.22\left[\frac{1-1.006^{-288}}{0.006}\right] = 157{,}609.90$
   Outstanding balance (after 84 payments) $= 1151.22\left[\frac{1-1.006^{-276}}{0.006}\right] = 15{,}5060.12$
   Amount repaid in 7th year $= 157609.90 - 155060.12 = 2549.78$
   Total payments in 7th year $= 12 * 1151.22 = 13{,}814.64$
   Total interest paid in 7th year $= 13814.64 - 2549.78 = \$\mathbf{11,264.86}$

Question 6

1. **[4]**Convert the system to an augmented matrix.
   $x - y + 5z = -6; 3x + 3y - z = 10; x + 3y + 2z = 5$

$$M := \begin{bmatrix} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{bmatrix}$$

2. **[6]**Calculate the reduced row echelon form of the matrix and use it to find the solution.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$ Solution: $\mathbf{x = 1; y = 2; z = -1}$

Question 7

1. **[3]**Write the technological matrix M for the island economy.

$$M := \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0 \end{bmatrix}$$

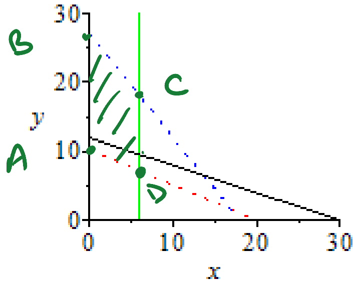
Transpose: $$A1 := \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0 \end{bmatrix}$$

2. **[3]**Set up the equation that must be satisfied by the inputs:
   $0.8x_1 - 0.2x_2 - 0.1x_3 = 30$
   $-0.2x_1 + 0.9x_2 - 0.1x_3 = 15$
   $-0.1x_1 - 0.2x_2 + x_3 = 20$

3. **[4]**Find the respective inputs satisfying these demands.
   $\mathbf{x_1 = 49.15;\ x_2 = 31.05;\ x_3 = 31.13}$

Question 8

[10]Maximize or minimize $P(x, y) = 10x + 25y$



Point	$P(x, y) = 10x + 25y$
A: (0, 10)	250
B: (0.27)	675 – Max
C: (6, 18)	510
D: (6,7)	235 – Min

Question 9

1. **[3]**How many different samples are there?
   ${}_{100}C_5$= **75,287,520**
2. **[3]**How many of the samples contain 2 defective fuses?
   ${}_{10}C_2 * {}_{90}C_3$= 45*117480 **= 5,286,600**
3. **[4]**How many of the samples contain at least 1 defective fuse?
   ${}_{100}C_5 - {}_{90}C_5$ = 75,287,520 – 43,959,268 = **31,338,252**

Question 10

1. **[5]**If John chooses his guests at random, what is the probability that Mary and Laura are invited?
   ${}_{26}C_8/{}_{28}C_{10} = \frac{1562275}{13123110} = \mathbf{0.1190\ or\ 11.90\%}$
2. **[5]**If John decides to invite 5 men and 5 women, what is the probability that Mary and Laura are invited?
   ${}_{10}C_3/{}_{12}C_5 = \frac{120}{792} = \mathbf{0.1515\ or\ 15.15\%}$