

University of Ottawa
MAT 1332 Practice Midterm Exam Answers

Answers:

1. (a) $\int_{-5}^5 \frac{14}{y^2 + y - 12} dy = \int_{-5}^5 \left(\frac{-2}{y+4} + \frac{2}{y-3} \right) dy = \int_{-5}^{-4} \frac{-2}{y+4} dy + \int_{-4}^5 \frac{-2}{y+4} dy + \int_{-5}^3 \frac{2}{y-3} dy + \int_3^5 \frac{2}{y-3} dy = -\infty + \infty + \infty - \infty$. This is an indeterminate form and hence is unresolvable.
(We made an error and the midterm won't be that hard!)

1. (b) $\int_1^e \frac{1}{x(1 + (\ln(x))^2)} dx = \arctan(\ln(e)) - \arctan(\ln(1)) = \arctan(\ln(1)) = \frac{\pi}{4}$

2. $y(t) = \sqrt{14t \sin(t) + 14 \cos(t) + 11}$

3. $\int \frac{x^3 + 8x^2 + 5}{x^2 + 5x - 14} dx = \frac{1}{2}x^2 + 3x - 6 \ln|x+7| + 5 \ln|x-2| + c$

4.(a) $\int_1^\infty \frac{1}{t \ln(t)} dt = \infty$, divergent. Note that $\lim_{t \rightarrow 1^+} \ln(\ln(t)) = \ln(0^+) = -\infty$

4.(b) $\int_0^\infty \frac{e^{3t}}{1 + e^{6t}} dt = \frac{1}{3}(\arctan(e^{3x}))|_0^\infty = \frac{\pi}{12}$, convergent

4.(c) $\int_1^\infty \frac{\ln(x)}{x^7} dx = \frac{1}{36}$, convergent

5. (a) Area = $\int_{-2}^{1/5} (-9x + 2 - 5x^2) dx \approx 8.87$

5. (b) Volume = $\pi \int_{-2}^{1/5} [(-9x + 2)^2 - (5x^2)^2] dx \approx 428.18$

6. $z(t) = \int 8te^{-0.07t} dt = \left(\frac{-8t}{0.07} - 1632.65 \right) e^{-0.07t} + C$. We have $z(0) = 5$, so $C = 1637.65$.

Hence $\lim_{t \rightarrow \infty} z(t) = 1637$ zombies.

7. $f_{\text{avg}} = \frac{2}{\pi} \int_0^{\pi/2} \cos(x)(1 - \sin^2(x)) dx = \frac{4}{3\pi}$