MAT 2377
Practice Exam

1. Suppose that for a very large delivery of integrated circuits, each circuit has a probability of failure of 0.09. Find the probability that no more than 2 pieces fail in a random sample of size 20.

   (A) 0.13 (B) 0.90 (C) 0.73 (D) 0.20 (E) none of the preceding

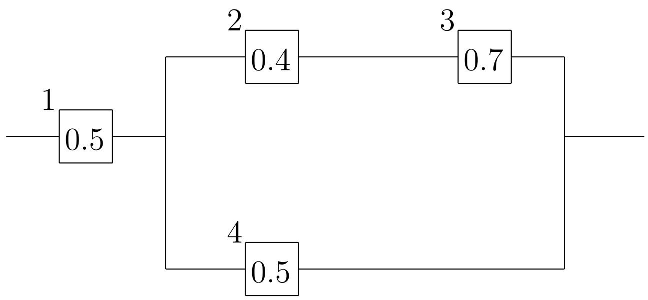
2. A scientist inoculated several mice one by one, with a germ of an illness until he found a mouse that has contracted the disease. If the probability of contracting the disease is 1/6, what is the probability that 8 mice are required ?

   (A) 0.0465 (B) 0.935 (C) 0.074 (D) 0.260 (E) none of the preceding

3. A medical research team wanted to evaluate a screening method proposed for Alzheimer's disease. The method was evaluated on 450 patients randomly selected patients with the Alzheimer's disease (in 436 cases the screening was positive). Moreover, the method was also assessed on 500 patients without disease. The screening was positive for 5 of these cases. We know Canada only 7.7% of the population aged 65 and older have the Alzheimer's disease. Find the probability that a Canadian 65 years and older has Alzheimer's disease given that the screening was positive.

   (A) 0.08 (B) 0.89 (C) 0.97 (D) 0.11 (E) none of the preceding

4. Consider the following circuit with four components. We say that it is functional if there is a path of functional components from left to right. The probability that the component is functional is indicated. Suppose that the components are independent. What is the probability that the circuit is **not** working ?



(A) 0.32 (B) 0.16 (C) 0.035 (D) 0.68 (E) none of the preceding

5. In a NiCd battery, a fully charged cell is composed of Nickelic hydroxide. Nickel is an element that has multiple oxidation states. Let X be the charge of the nickel, which has the probability mass function as follows :

x	$f_X(x)$
0	0.18
1	0.34
2	0.33
4	0.15

Determine the mean charge of the nickel.

(A) 2.0 (B) 1.5 (C) 1.6 (D) 1.45 (E) none of the preceding

6. Let X be a random variable with mean $\mu_X = .75$ and probability density function

$$f_X(x) = 3\,x^2, \quad 0 < x < 1.$$

Determine the variance of the random variable X.

(A) 0.0375 (B) 0.1936 (C) 0.7746 (D) 0.3873 (E) none of the preceding

7. Suppose that $X_1, X_2, \ldots, X_{81}$ are independent random variable with the same probability density function

$$f_X(x) = \frac{1}{2}\exp(-x/2), \quad x > 0.$$

Approximate the following probability

$$P\left(\sum_{i=1}^{81} X_i > 170\right).$$

Hint : Central Limit Theorem

(A) 0.67 (B) 0.16 (C) 0.33 (D) 0.95 (E) 0.001

8. A random sample of size $n_1 = 16$ is selected from a normal population of mean 175 and variance 288. A second random sample of size $n_2 = 9$ is selected from a normal population with mean 80 and variance 162. Let $\overline{X}_1$ and $\overline{X}_2$ represent of the respective sample means. Find the probability that $\overline{X}_1 + \overline{X}_2$ is larger than 156.5.

(A) 0.5987 (B) 0.4013 (C) 0.6231 (D) 0.4235 (E) none of the preceding

9. Let X_1, X_2, X_3, X_4, X_5 be a random sample from a population with a mean μ and a variance σ^2. Consider the following estimators for μ :

$$\hat{\Theta}_1 = (X_1 + X_2)/2, \quad \hat{\Theta}_2 = 2X_5 - X_1.$$

Which of the two estimators has the smallest variance ?

(A) $\widehat{\Theta}_1$ has the smallest variance.
(B) $\widehat{\Theta}_2$ has the smallest variance.
(C) The variances are equal.
(D) Insufficient information is given.

10. A machine produces cylindrical shaped metallic parts. A sample of pieces is selected and the diameters are

1.01	0.97	1.03	1.04	0.99	0.98
0.99	1.04	1.03	1.01		

Determiner a 99% confidence interval for the mean diameter. You can assume that the population is normal.

(A) [0.989,1.022]
(B) [0.983,1.035]
(C) [0.991,1.034]
(D) [.987,1.024]
(E) none of the preceding

11. The air pressure in tires chosen at random from a certain model of car is normally distributed with mean 31 $\mathsf{lb/in}^2$ and a standard deviation of 0.2 $\mathsf{lb/in}^2$. What is the probability that the pressure in a tire chosen at random is between 30,5 and 31,5 $\mathsf{lb/in}^2$.

(A) 0.6827 (B) 0.3173 (C) 0.5000 (D) 0.4245
(E) none of the preceding

12. Past experience has indicated that the strength of the wire used in the manufacture of equipment for drapery is normally distributed and that $\sigma^2 = 2$. A random sample of 25 specimens was examined and the mean strength is $\bar{x} = 98$ pounds per square inch. Find a 95% confidence interval for the mean strength.

(A) [97.216,98.784] (B) [97.216,98.554] (C) [97.456,98.784]
(D) [97.446,98.554] (E) none of the preceding

13. We want to test the hypothesis that the contents of containers using a particular lubricant is more than 10 litres. The contents of 10 containers randomly selected are measured :

10.2	9.7	10.1	10.3	10.1
9.8	9.9	10.4	10.3	9.5

We summarize the data with the following sums :

$$\sum_{i=1}^{10} x_i = 100.3 \quad \text{and} \quad \sum_{i=1}^{10} x_i^2 = 1006.79.$$

Compute the p-value for the one-sided test assuming that the population is normal.

(A) $.05 < P < .10$ (B) $.10 < P < .20$ (C) $.25 < P < .40$
(D) $.50 < P < .80$ (E) none of the preceding

14. The boiling point for each of 16 specimens of a certain brand of vegetable oil has been measured. The sample mean is $\bar{x} = 94.32$. Suppose that the boiling point follows a normal distribution with $\sigma = 1.2$. Test $H_0 : \mu = 95$ against $H_1 : \mu \neq 95$ with $\alpha = 0.01$. Determine the p-value of the test and give the conclusion.

(A) Reject H_0, $p = 0.012$ (B) Do not reject H_0, $p = 0.023$
(C) Reject H_0, $p = 0.023$ (D) Do not reject H_0, $p = 0.012$.

15. A manufacturer is interested in the output voltage of power of energy used in a PC. Assume that the voltage performance is normally distributed with $\sigma^2 = 0.25$. The manufacturer wants to test $H_0: \ mu = 5$ against $H_0 : \mu \neq 5$ with $n = 8$. For $\alpha = 0.05$ determine the critical region in terms of $\overline{x}$.

(A) $\overline{x} < 4.83$ or $\overline{x} > 5.17$ (B) $\overline{x} < 4.65$ or $\overline{x} > 5.35$
(C) $\overline{x} < 4.12$ or $\overline{x} > 4.72$ (D) $\overline{x} < 5.12$ or $\overline{x} > 5.72$
(E) none of the preceding

16. Refer to Question 15, determine the probability of committing an error of type II, if the true mean voltage is 5.1.

(A) 0.915 (B) 0.084 (C) 0.05 (D) 0.723 (E) none of the preceding

17. An expert wants to determine the average time (in seconds) to drill holes in some metal flange. Determine the sample size required to be 99% certain that the sample average is within 15 seconds of the true mean. Suppose that $\sigma = 30$ seconds.

(A) 26 (B) 27 (C) 28 (D) 29 (E) none of the preceding

18. Suppose that we have a random sample of size $n = 10$ from a normal population with $\mu = 4$ and $\sigma^2 = 9$. Let $\overline{X}$ and S^2 be the sample mean and the sample variance, respectively. Determine c such that

$$P\left(\frac{\overline{X} - 4}{S/\sqrt{10}} \geq c\right) = 0.01.$$

(A) 1.833 (B) 2.821 (C) 1.645 (D) 2.424
(E) none of the preceding

19. The joint probability density function of X and Y is

$$f(x, y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), \quad 0 < x < 1, 0 < y < 2.$$

Compute the probability that $P(Y < 1/3)$.

(A) 0.1071 (B) 0.2857 (C) 0.8571
(D) 0.3929 (E) none of the preceding

20. Consider the random variables X and Y with the following joint probability mass function :

x	y	$f_{XY}(x, y)$
1	1	.25
1	2	.25
2	1	.25
0	0	.25

Compute the coefficient of correlation between X and Y.

(A) 0.195 (B) 0 (C) 0.500 (D) 2.125 (E) none of the preceding

21. For a random sample of 12 paired observations (x_i, y_i), we compute the following sums :

$$\sum_{i=1}^{12} x_i = 25, \quad \sum_{i=1}^{12} y_i = 432$$

$$\sum_{i=1}^{12} x_i^2 = 59, \quad \sum_{i=1}^{12} y_i^2 = 15648, \quad \sum_{i=1}^{12} x_i y_i = 880.5.$$

Using the fitted regression line, a prediction of a new observation of Y when $x = 5$ is :

(A) 27.78 (B)47.77 (C) 41.87 (D) 55.97 (E) none of the preceding

22. Raw materials are studied for contamination. Suppose that the occurrences of contamination particles can be modeled as a Poisson process with a rate of 0.02 particle per kilogram of material. What is the probability that there are no particles in 10 kilograms?

(A) 0.989 (B) 0.819 (C) 0.135 (D) 0.020 (E) none of the preceding

23. Suppose that simple linear regression model

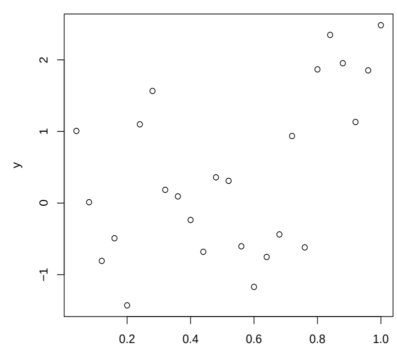
$$Y = \beta_0 + \beta_1 x + \varepsilon$$

is appropriate. From $n = 14$ observations, we compute the fitted regression line $\widehat{y} = .6649 + .83075\,x$. Given that $S_{yy} = 4.1289$ and $S_{xy} = 4.4094$, compute the estimated standard error of the slope. Note that :

$$S_{xx} = \sum_{i=1}^{n}(x_i - \overline{x})^2, S_{yy} = \sum_{i=1}^{n}(y_i - \overline{y})^2, S_{xy} = \sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y}).$$

(A) 0.3176 (B) 0.0388 (C) 0.0855 (D) 0.0073
(E) none of the preceding

24. For the following data, the sample correlation between X and Y is :



(A) 0.49 (B) 0.98 (C) -0.5 (D) -0.98

25. Consider a random sample $X_1, \ldots, X_{100}$ from a population with mean $\mu = 1$ and variance $\sigma^2 = 4$. The approximate probability that the sample mean is greater than 1.5 is :

(A) 0.99379 (B) 0.00621 (C) 0.5000 (D) 0.7742
(E) none of the preceding