Exponents and Radicals Module Review **Name: SOLUTIONS**

1) Evaluate the following exponents/radicals **without** the help of your calculator (using exponent rules): (2 mark each)

a) $49^{-\frac{1}{2}}$

$= \frac{1}{49^{\frac{1}{2}}}$

$= \frac{1}{\sqrt{49}}$

$= \frac{1}{7}$

b) $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

$= \left(\frac{125}{8}\right)^{\frac{2}{3}}$

$= \left(\sqrt[3]{\frac{125}{8}}\right)^{2}$

$= \left(\frac{5}{2}\right)^{2}$

$= \frac{25}{4}$

c) $\left(\frac{32}{18}\right)^{\frac{3}{2}}$

$= \left(\frac{16}{9}\right)^{\frac{3}{2}}$

$= \left(\sqrt{\frac{16}{9}}\right)^{3}$

$= \left(\frac{4}{3}\right)^{3}$

$= \frac{64}{27}$

d) $(-8)^{\frac{2}{3}}$

$= \left(\sqrt[3]{-8}\right)^{2}$

$= (-2)^{2}$

$= 4$

2) Evaluate the following exponents/radicals **with** the help of your calculator: (1 mark each)

a) $25^{-2} = 0.0016$

b) $\sqrt[5]{36} = 2.047672511$

c) $7^{\frac{2}{5}} = 2.177906424$

d) $(-5)^{\frac{2}{3}} = 2.924017735$

e) $-5^{\frac{2}{3}} = -2.924017735$

d) $\left(\sqrt[3]{4}\right)^{4} = 6.349604208$

3) Simplify the following and write as a single radical if possible. (2 marks each)

a) $7\sqrt{2} - 3\sqrt{2} + \sqrt{3}$

$= 4\sqrt{2} + \sqrt{3}$

b) $3\sqrt{x}\sqrt{x-1}$

$= 3\sqrt{x(x-1)}$

$= 3\sqrt{x^2 - x}$

c) $2\sqrt{3x} + 4\sqrt{27x} - 2\sqrt{48x}$

$= 2\sqrt{3x} + 4\sqrt{9 * 3x} - 2\sqrt{16 * 3x}$

$= 2\sqrt{3x} + 4 * 3\sqrt{3x} - 2 * 4\sqrt{3x}$

$= 2\sqrt{3x} + 12\sqrt{3x} - 8\sqrt{3x}$

$= 6\sqrt{3x}$

d) $7\sqrt{2}\sqrt{6}$

$= 7\sqrt{12}$

$= 7\sqrt{4 * 3}$

$= 7 * 2\sqrt{3}$

$= 14\sqrt{3}$

e) $-\sqrt[3]{16y} + 4\sqrt[3]{2y} - 3\sqrt[3]{y}$

$= -\sqrt[3]{8 * 2y} + 4\sqrt[3]{2y} - 3\sqrt[3]{y}$

$= -2\sqrt[3]{2y} + 4\sqrt[3]{2y} - 3\sqrt[3]{y}$

$= 2\sqrt[3]{2y} - 3\sqrt[3]{y}$

f) $5\sqrt{2x^3} - 3x\sqrt{2x}$

$= 5\sqrt{x^2 * 2x} - 3x\sqrt{2x}$

$= 5x\sqrt{2x} - 3x\sqrt{2x}$

$= 2x\sqrt{2x}$

g) $-\sqrt[3]{27x^4} + 4x\sqrt[3]{x} - 3\sqrt[3]{x^5}$

$= -\sqrt[3]{3 * 3 * 3x^3 * x} + 4x\sqrt[3]{x} - 3\sqrt[3]{x^3 * x^2}$

$= -3x\sqrt[3]{x} + 4x\sqrt[3]{x} - 3x\sqrt[3]{x^2}$

$= x\sqrt[3]{x} + -3x\sqrt[3]{x^2}$

$= x\sqrt[3]{x} + -3x\sqrt[3]{x^2}$

h) $-2\sqrt{75x^3} + 4x\sqrt{12x} + 5\sqrt{x}\sqrt{27}$

$= -2\sqrt{25x^2 * 3x} + 4x\sqrt{4 * 3x} + 5\sqrt{x}\sqrt{9 * 3}$

$= -2(5x)\sqrt{3x} + 4x(2)\sqrt{3x} + 5\sqrt{x}(3)\sqrt{3}$

$= -10x\sqrt{3x} + 8x\sqrt{3x} + 15\sqrt{x}\sqrt{3}$

$= -2x\sqrt{3x} + 15\sqrt{3x}$

4) Simplify the following (if possible): (2 marks each)

a) $2x^{\frac{1}{2}} \times \left(-5x^{\frac{2}{3}}\right)$

$= -10x^{\frac{1}{2}+\frac{2}{3}}$

$= -10x^{\frac{7}{6}}$

b) $(8x + 7)^{\frac{3}{2}}$

Cannot further simplify (2 terms under radical sign)

c) $3x^{-\frac{2}{5}} \times (2 - 4x + 7x^3)$

$= 6x^{-\frac{2}{5}} - 12x^{1-\frac{2}{5}} + 21x^{3-\frac{2}{5}}$

$= 6x^{-\frac{2}{5}} - 12x^{\frac{5}{5}-\frac{2}{5}} + 21x^{\frac{15}{5}-\frac{2}{5}}$

$= 6x^{-\frac{2}{5}} - 12x^{\frac{3}{5}} + 21x^{\frac{13}{5}}$

d) $\left(\frac{8}{27}x^6\right)^{\frac{2}{3}}$

$= \frac{8^{\frac{2}{3}}}{27^{\frac{2}{3}}}x^{6*\left(\frac{2}{3}\right)}$

$= \frac{2^2}{3^2}x^4$

$= \frac{4}{9}x^4$

e) $\left(2x^{-\frac{3}{4}}+4\sqrt[3]{x}\right)\times\left(-\sqrt{x^3}+5x\right)$

$=\left(2x^{-\frac{3}{4}}+4x^{\frac{1}{3}}\right)\times\left(-x^{\frac{3}{2}}+5x\right)$

$=-2x^{-\frac{3}{4}+\frac{3}{2}}+10x^{-\frac{3}{4}+1}-4x^{\frac{1}{3}+\frac{3}{2}}+20x^{\frac{1}{3}+1}$

$=-2x^{\frac{3}{4}}+10x^{\frac{1}{4}}-4x^{\frac{11}{6}}+20x^{\frac{4}{3}}$

f) $\left(\frac{9}{4}x^3\right)^{-\frac{5}{2}}$

$=\left(\frac{4}{9x^3}\right)^{\frac{5}{2}}$

$=\left(\sqrt{\frac{4}{9x^3}}\right)^5$

$=\left(\frac{2}{3x^{\frac{3}{2}}}\right)^5$

$=\frac{32}{729x^{\frac{15}{2}}}$

5) Simplify and Rationalize the following radical expressions:

a) $\sqrt{3}(7\sqrt{2}-3\sqrt{3})$

$=7\sqrt{6}-3\sqrt{9}$

$=7\sqrt{6}-9$

b) $(3\sqrt{5}-1)(2\sqrt{5}-3\sqrt{2})$

$=6\sqrt{25}-9\sqrt{10}-2\sqrt{5}+3\sqrt{2}$

$=30-9\sqrt{10}-2\sqrt{5}+3\sqrt{2}$

c) $2\sqrt{10}(3\sqrt{2}-4\sqrt{5})+\sqrt{2}(9+\sqrt{10})$

$=6\sqrt{20}-8\sqrt{50}+9\sqrt{2}+\sqrt{20}$

$=7\sqrt{4*5}-8\sqrt{25*2}+9\sqrt{2}$

$=14\sqrt{5}-40\sqrt{2}+9\sqrt{2}$

$=14\sqrt{5}-31\sqrt{2}$

d) $\frac{3\sqrt{3}-5\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$=\frac{3\sqrt{3}-5\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$=\frac{3\sqrt{9}-3\sqrt{6}-5\sqrt{6}+5\sqrt{4}}{\sqrt{9}+\sqrt{6}-\sqrt{6}-\sqrt{4}}$

$=\frac{9-8\sqrt{6}+10}{3-2}$

$=19-8\sqrt{6}$

e) $\frac{5\sqrt{5}+\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

$=\frac{5\sqrt{5}+\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}\times\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$

$=\frac{10\sqrt{25}+2\sqrt{10}+15\sqrt{10}+3\sqrt{4}}{4\sqrt{25}-6\sqrt{10}+6\sqrt{10}-9\sqrt{4}}$

$=\frac{50+17\sqrt{10}+6}{20-18}$

$=28+\frac{17\sqrt{10}}{2}$

f) $\frac{(\sqrt{7}+\sqrt{5})(2\sqrt{5}-3\sqrt{7})}{\sqrt{5}-2\sqrt{7}}$

$=\frac{2\sqrt{35}-3\sqrt{49}+2\sqrt{25}-3\sqrt{35}}{\sqrt{5}-2\sqrt{7}}$

$=\frac{-\sqrt{35}-11}{\sqrt{5}-2\sqrt{7}}\times\frac{\sqrt{5}+2\sqrt{7}}{\sqrt{5}+2\sqrt{7}}$

$=\frac{-\sqrt{25*7}-2\sqrt{49*5}-11\sqrt{5}-22\sqrt{7}}{\sqrt{25}+2\sqrt{35}-2\sqrt{35}-4\sqrt{49}}$

$=\frac{-5\sqrt{7}-14\sqrt{5}-11\sqrt{5}-22\sqrt{7}}{5-28}$

$=\frac{-27\sqrt{7}-25\sqrt{5}}{-23}$

$=\frac{25\sqrt{5}+27\sqrt{7}}{23}$