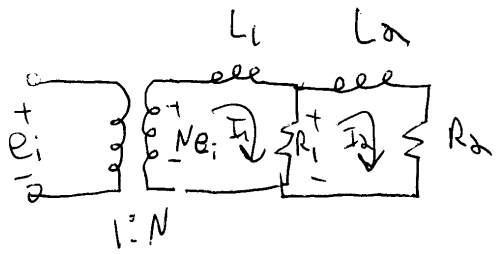
Assignment 2, Q1



Transformer multiplies e_i by N

Loop 1: $Ne_i = V_{L_1} + V_{R_1}$

$$Ne_i = L_1 I_1' + R_1(I_1 - I_2)$$

Loop 2: $V_{R_1} = V_{L_2} + V_{R_2}$

$$R_1(I_1 - I_2) = L_2 I_2' + I_2 R_2$$

$$\therefore I_1 = \left(\frac{R_2}{R_1} + 1\right) I_2 + \frac{L_2}{R_1} I_2'$$

differentiate $$I_1' = \left(\frac{R_2}{R_1} + 1\right) I_2' + \frac{L_2}{R_1} I_2''$$

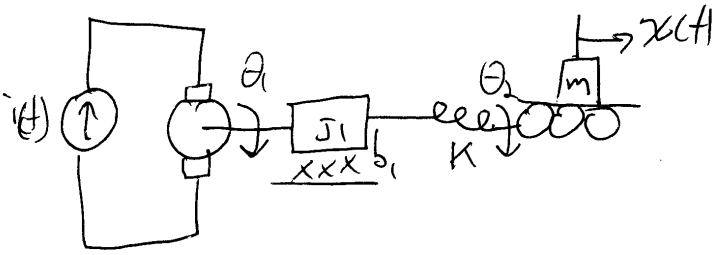
Sub into loop 1:

$$Ne_i = L_1\left(\left(\frac{R_2}{R_1} + 1\right) I_2' + \frac{L_2}{R_1} I_2''\right) + R_1\left(\left(\frac{R_2}{R_1} + 1\right) I_2 + \frac{L_2}{R_1} I_2' - I_2\right)$$

$$Ne_i = R_2 I_2 + \left[L_1\left(\frac{R_2}{R_1} + 1\right) + L_2\right] I_2' + \frac{L_1 L_2}{R_1} I_2''$$

$$\boxed{e_i = \frac{R_2}{N} I_2 + \frac{L_1(R_2 + R_1) + L_2 R_1}{N R_1} I_2' + \frac{L_1 L_2}{N R_1} I_2''} \leftarrow$$

Q2



$$x(t) = r\theta_2$$

Torque ~~force~~ in spring is $K(\theta_1 - \theta_2)$

Force on m is $\frac{\text{torque}}{\text{radius}} = \frac{K(\theta_1 - \theta_2)}{r}$

$$\therefore \quad m\ddot{x} = \frac{K(\theta_1 - \theta_2)}{r}$$

$$rm\ddot{\theta}_2 = \frac{K(\theta_1 - \theta_2)}{r}$$

$$\theta_1 = \frac{r^2 m}{K}\ddot{\theta}_2 + \theta_2$$

$$\dot{\theta}_1 = \frac{r^2 m}{K}\dddot{\theta}_2 + \dot{\theta}_2$$

$$\ddot{\theta}_1 = \frac{r^2 m}{K}\theta_2^{(4)} + \ddot{\theta}_2$$

Q continued.

Σ Torques on $J_1 = J_1\ddot{\theta}_1$

Torques are: motor, $K_m i(t)$

damper, $-b\dot{\theta}_1$

spring, $-K(\theta_1 - \theta_2)$

$\therefore \quad K_m i(t) - b\dot{\theta}_1 - K(\theta_1 - \theta_2) = J_1\ddot{\theta}_1$

Sub in expressions for $\theta_1, \dot{\theta}_1, \ddot{\theta}_1$

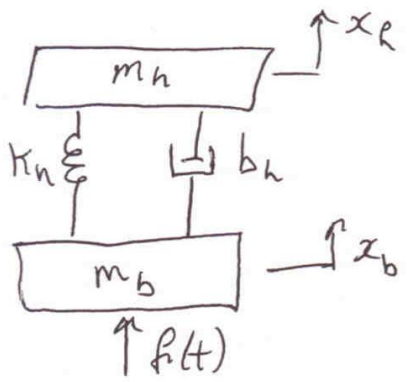
$$K_m i(t) = J_1\left(\frac{r^2 m}{K}\theta_2^{(4)} + \dddot{\theta}_2\right) + b\left(\frac{r^2 m}{K}\ddot{\theta}_2 + \dot{\theta}_2\right) + K\left(\frac{r^2 m}{K}\ddot{\theta}_2 + \theta_2 - \theta_2\right)$$

group terms $\quad K_m i(t) = J_1\frac{r^2 m}{K}\theta_2^{(4)} + b\frac{r^2 m}{K}\ddot{\theta}_2 + (J_1 + r^2 m)\dddot{\theta}_2 + b\dot{\theta}_2$

recall $\theta_2 = \frac{x(t)}{r}$

$$\therefore \quad \boxed{i(t) = \frac{J_1 r m}{K K_m} x^{(4)}(t) + \frac{b r m}{K K_m} x^{(3)}(t) + \left(\frac{J_1 + r^2 m}{r K_m}\right) x^{(2)}(t) + \frac{b}{r K_m} x^{(1)}(t)}$$

Question #3



$$m_h \ddot{x}_h = -k_h(x_h - x_b) - b_h(\dot{x}_h - \dot{x}_b) \qquad (1)$$

$$m_b \ddot{x}_b = f(t) + k_h(x_h - x_b) + b_h(\dot{x}_h - \dot{x}_b) \qquad (2)$$

Set $y(t) = x_h(t) - x_b(t)$

Divide (1) by m_h, then divide (2) by m_b and subtract results, then simplify to get

$$\ddot{y}(t) + b_k\left[\frac{1}{m_k} + \frac{1}{m_b}\right]\dot{y}(t) + k_k\left[\frac{1}{m_k} + \frac{1}{m_b}\right]y(t) = -\frac{f(t)}{m_b}$$