

MAT1330 D - Assignment 1 - Winter 2016 (due Jan 25 at 16 :00)

Calculus for the Life Sciences I

Prof : Abdulrahman Karouma

This assignment must be printed out, stapled and filled in.¹

Last name : _____ First name : _____

Student number : _____ DGD number : _____

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature :

1. Find the domain of $f(x) = \frac{\sqrt{x^2 + 1}}{x^2 + 4x + 1}$. Solution :

2. Solve the inequality : $2 + \left| \frac{1}{x} - 6 \right| \geq 14$. Solution :

1. Remark : The correct answer requires justification written logically : you must convince me that you know why your solution is correct. You must answer these questions in the space provided. Use the backs of pages if necessary.

3. Suppose someone has three drinks of alcohol that bring the alcohol content in their body to 42 grams. Then the person stops drinking. Each hour, %45 of the alcohol are eliminated from the body.

(a) Write the DTDS for the amount of alcohol in the body on an hourly basis.

(b) Identify the initial condition and give the general solution.

(c) If the amount of alcohol in the body has to be below 8 grams before one can drive, how long does the person have to wait before they can drive?

The person has to wait hours

4. Consider the DTDS modelling an organism with a %25 growth per hour

$$P_{t+1} = f(P_t) = P_t + 0.25P_t.$$

Find and interpret the inverse of the updating function $f(P_t)$. Specifically, is it "%25 shrinkage per hour" ?

5. The size of a bird population on an island located close to the coast of Newfoundland depends on the local growth rate (birth minus death) and the migration between the island and Newfoundland. A discrete dynamical system modelling this population is

$$x_{n+1} = 0.70x_n + 60, \quad n = 0, 1, 2, 3, \dots$$

where x_n is the size of the bird population on the island after n years. 0.70 is the local growth rate and 60 is the yearly increase due to migration.

- (a) if $x_0 = 100$, then

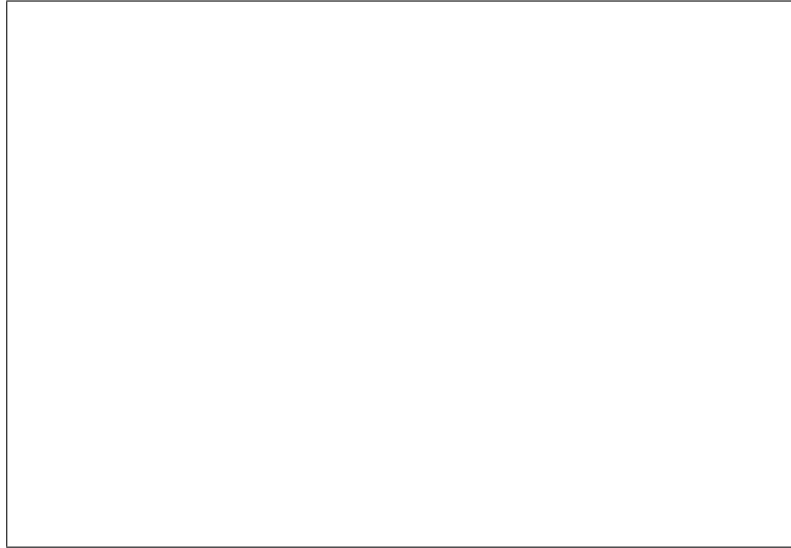
$$x_1 = \boxed{}, \quad x_2 = \boxed{}, \quad x_3 = \boxed{}$$

- (b) Give the updating function f of the dynamical system. $f(x) =$

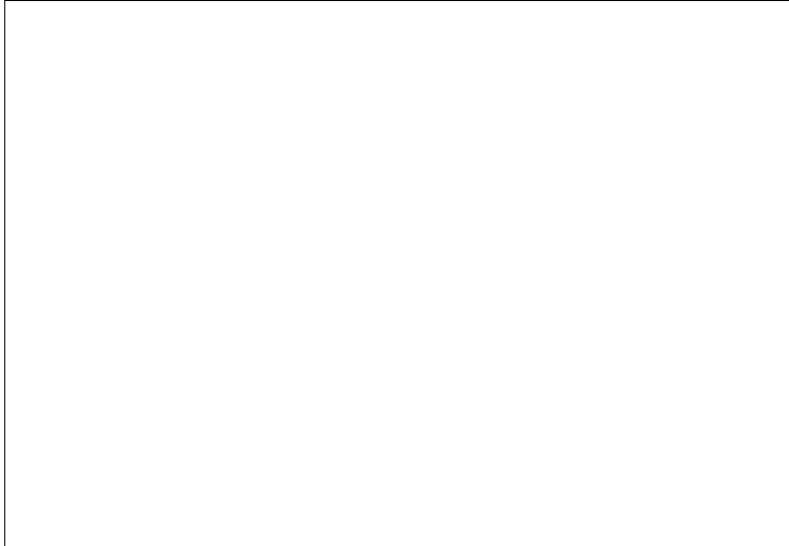
- (c) Find the equilibrium point x^* of the dynamical system. $x^* =$

- (d) Give the solution of the dynamical system with $x_0 = 100$. $x_n =$

- (e) Draw the solution of the dynamical system with $x_0 = 100$ (four points are enough).



- (f) Draw the cobweb diagram of the dynamical system with $x_0 = 100$ (four iterations are enough).



- (g) Determine the stability of the equilibrium point using the cobweb diagram.

The equilibrium point is

6. The dynamical system

$$x_{n+1} = \frac{\alpha x_n}{1 + \beta x_n}, \quad n = 1, 2, 3, 4, \dots$$

plays a role in the analysis of nonlinear models of gene and neural networks. α and β are positive parameters. Suppose that $\alpha = 4$ and $\beta = 2$,

- (a) If $x_0 = 2.5$, then $x_1 =$, $x_2 =$, $x_3 =$

- (b) Find the equilibrium points x^* of the dynamical system. $x^* =$