

an eCONCORDIA EXAMINATION
Department of Mathematics & Statistics

Course	Number	Section
MATH	209	EC

Examination	Date	Pages
Final	August 2015	3

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Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.

Evaluation

- ▷ This examination counts for 50% towards your final grade
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MARKS

- [12] 1. (a) Find $\lim_{x \rightarrow \infty} \frac{7 - x^2}{4x^3 - 6x^2 + 8}$.
- (b) Find the value of each of the following:
- i. $\lim_{x \rightarrow 5} \frac{x^2 - 25}{(x - 5)}$
 - ii. $\lim_{x \rightarrow 4} \frac{x^2 - 3x + a}{(x - 1)}$ where a is a real number.
- (c) Given $\lim_{x \rightarrow 6} f(x) = -5$ and $\lim_{x \rightarrow 6} g(x) = 4$, find
- i. $\lim_{x \rightarrow 6} [-3g(x)]$
 - ii. $\lim_{x \rightarrow 6} [f(x)]^2$
 - iii. $\lim_{x \rightarrow 6} [g(x) \cdot f(x)]$

- [4] 2. Using the definition of the derivative $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$, find the derivative if $f(x) = 3 + 4x^3$.

- [17] 3. (a) If $f(x) = e - 4x^{17} - 5x^3$, find $f'(x)$.
- (b) If $f(x) = (x^3 + 2x^2 - 1)(x^2 - 9)$, find $f'(x)$.
- (c) If $f(x) = (7e^x + 3)/(x^2 - 4)$, find $f'(x)$.
- (d) If $y = e^{(x^2-7x)}$, then $\frac{dy}{dx} = ?$
- (e) If $y = \ln(3x^3 - 5)^4$, then $\frac{dy}{dx} = ?$
- (f) If $y = \frac{1}{\sqrt[5]{x-2}}$, then $\frac{dy}{dx} = ?$
- (g) Find y' given $xy = e^{2y} + \pi$.
- [8] 4. Does the line tangent to the graph of $f(x) = e^x$ at $x = 1$ pass through the origin? Are there any other lines tangent to the graph of f that pass through the origin? Explain.
- [5] 5. A cube with a side of 12 centimeters is coated with ice 0.1 centimeters thick. Use differentials to estimate the volume of the ice.
- [6] 6. The total profit (in dollars) from the sale of x lawn mowers is
- $$P(x) = 30x - 0.03x^2 - 750, \quad 0 \leq x \leq 1000$$
- (a) Find the average profit per mower if 50 mowers are produced.
- (b) Find the marginal average profit at a production level of 50 mowers, and interpret the results.
- (c) Use the results from parts (a) and (b) to estimate the average profit per mower if 51 mowers are produced.
- [5] 7. Suppose a point is moving along the graph of $x^2 + y^2 = 34$. When the point is at $(-3, 5)$, its x coordinate is increasing at a rate of 0.6 units per second. How fast is the y coordinate changing at that moment?

[18] **8.** Compute the following:

(a) $\int x(3x^2 + 5)dx$

(b) $\int x^2(2x^3 + 7)^{18}dx$

(c) $\int \frac{1}{x^2 + 7}x dx$

(d) $\int e^{3-5x} dx$

(e) $\int \frac{x}{\sqrt{x+7}} dx.$

(f) $\int (x+3)^{-4} dx.$

[8] **9.** Evaluate the following integrals [give the exact answer].

(a) $\int_2^3 e^{-x^2} x dx$

(b) $\int_0^3 (t^3 + 5) dt$

[5] **10.** A note will pay \$40,000 at maturity 12 years from now. How much should you be willing to pay for the note now if money is worth 2.5%, compounded continuously?

[5] **11.** Find the area bounded by the graphs of $y = x^2 - 3x$; $y = 0$, $-2 \leq x \leq 2$.

[7] **12.** Find the interval(s) on which the graph of $f(x) = x^4 - 2x^3$ is concave upward, the interval(s) on which the graph of f is concave downward, and the inflection point(s).