

COMM 215

Midterm Review

Basics of Probability

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Lots of problems that come up in applications involve phenomena for which exact predictions are either impossible or very difficult. The best we can do in such cases is to determine the *probability* of the possible outcomes. We can think of probability as a number between 0 and 1 that measures how likely an event is to happen. An event with probability zero would be an impossible event and an event with probability one would be certain to happen.

An **experiment** is an activity with an observable result. A **sample space** of an experiment is a set of outcomes such that in each trial of the experiment, *one and only one* of these outcomes occurs. Each element of a sample space is called a **simple event**. An **event** is a subset of a sample space.

Given a sample space $S = \{E_1, E_2, \dots, E_n\}$ with n simple events, a **probability distribution** for S is an assignment of a number $P(E_i)$ to each simple event E_i so that

1. $0 \leq P(E_i) \leq 1$, for all $i = 1, \dots, n$
2. $P(E_1) + P(E_2) + \dots + P(E_n) = 1$.

The **probability of an event** E , denoted by $P(E)$, is the sum of the probabilities of all simple events contained in E .

In practice, probability distributions are often estimated by experiment. For example, if you have a coin and want to know what $P(\text{heads})$ is, you could flip it one million times and collect statistics. The number of *heads* obtained divided by one million could then be used as a very good estimate of $P(\text{heads})$. If an experiment is performed

N times and the event E occurs $n(E)$ times, then the ratio $n(E)/N$ is called the **relative frequency** of E . In general, if the number of trials N is high enough, the relative frequency is a good approximation of $P(E)$. In fact, we could have defined $P(E)$ by

$$P(E) = \lim_{N \rightarrow \infty} \frac{n(E)}{N}.$$

Another method commonly used to assign a probability distribution to a sample space is by simply assuming it. For instance, in many situations it is reasonable to assume that all outcomes in the sample space are *equally likely* to occur. If you have a coin and have no reason to suspect that it is not fair, it is then reasonable to assume that both outcomes *heads* and *tails* are equally likely to occur. In this case we would assign

$$P(\text{heads}) = P(\text{tails}) = 1/2.$$

Statistical methods could then be used to test if the assumption that the coin is fair is acceptable or not.

If $S = \{E_1, E_2, \dots, E_n\}$ and if we assume that all simple events are **equally likely** to occur, then

$$P(E_1) = P(E_2) = \dots = P(E_n) = \frac{1}{n}.$$

In this case we can calculate the probability of any event E as follows

$$P(E) = \frac{\text{number of simple events in } E}{\text{number of simple events in } S}.$$

► The above formula works *only* when the simple events in the sample space are equally likely. For example, if a die is weighted and the outcomes are not equally likely, then the above formula would not apply.

Basic Counting Principles

Multiplication Principle

Consider a multistep process in which

Step 1 has n_1 possible outcomes,

Step 2 has n_2 possible outcomes,

...

Step r has n_r possible outcomes.

Then, the entire process has $n_1 \times n_2 \times \cdots \times n_r$ possible outcomes.

Inclusion-Exclusion Principle

If A and B are two finite sets, then

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

A **permutation of n distinct items taken r at a time** is an *ordered* list of r distinct items chosen from a set of n distinct items. The number of such permutations is given by

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}.$$

Observe that $P(n, n) = n!$.

A **combination of n distinct items taken r at a time** is an *unordered* set of r distinct items chosen from a set of n distinct items. The number of such combinations is given by

$$C(n, r) = \frac{n!}{r!(n-r)!}.$$

Another notation for $C(n, r)$ is $\binom{n}{r}$.

Problems

- (a) How many bit strings of length 8 are possible?
(b) How many of these start with a 1 or end with 00?
Answers: (a) 256 (b) 160
- How many functions are there from a set with 6 elements to a set with 4 elements.
Answer: 4096
- How many one-to-one functions are there from a set with 6 elements to a set with
(a) 4 elements (b) 6 elements (c) 10 elements.
Answers: (a) 0 (b) 720 (c) 151 200
- Each user on a computer system has a password, which is six to eight characters long, where each character is a letter (case sensitive) or a digit. If each password must contain at least one digit and at least one letter, how many possible passwords are there?
Answer: $167\,410\,838\,583\,040 \approx 1.67 \times 10^{14}$
- (a) How many different 7-place license plates are possible if the first 3 places are for capital letters and the other 4 for digits?
(b) What if no letter and digit can be repeated in a single license plate?
(c) What if adjacent letters and digits have to be different?
Answers: (a) 175 760 000 (b) 78 624 000 (c) 118 462 500
- The 10 letters ABCDEFGHIJ are used to form strings of length 7 (order matters).
(a) How many possible strings are there if we do not allow repetition?

- (b) How many strings begin with the letter G if repetitions are allowed?
- (c) How many strings contain the substring GAB if repetitions are not allowed?
- (d) How many strings begin or end with the substring GAB if repetitions is allowed?
- (e) How many strings contain the letters A and B, with A somewhere to the left of B, if repetitions are not allowed?

Answers: (a) 604800 (b) 1 000 000 (c) 4200 (d) 19990 (e) 141120

7. How many different subsets of 5 letters are possible from the the 26 letters of the alphabet?

Answer: 65780

8. In how many ways can we select a chairperson, secretary, and treasurer form a group of 25 persons?

Answer: 13800

9. How many strings of five decimal digits

- (a) do not contain the same digit five times?
- (b) begin with an odd digit?
- (c) have exactly three digits that are 5s?

Answers: (a) 99 990 (b) 50 000 (c) 810

10. A biologist is attempting to classify 20,000 species of insects by assigning 3 letter initials (not necessarily distinct) to each species. Is it possible to classify all the species in this way? *Answer:* No

11. We have 25 lightbulbs distributed as follows and we select 5 at random (order does not matter).

40-W	60-W	75-W
9	6	10

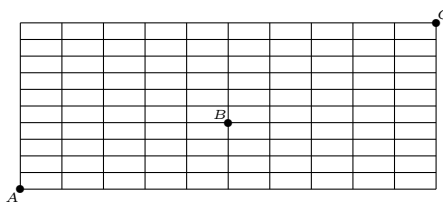
- (a) In how many ways can this be done.
- (b) How many selections will contain exactly two 75-W bulbs.
- (c) How many selections will contain at least one 75-W bulbs.
- (d) How many selections will contain two 40-W and three 60-W bulbs

Answers: (a) 53130 (b) 20475 (c) 50127 (d) 720

12. How many 5-card hands are possible from a standard deck of 52 cards? How many of these hands will have exactly three hearts and two spades? How many hands will have 3 kings and a pair?

Answers: 2598960, 22308, 288

13. Consider the following 10×10 grid.



- (a) How many paths joining the two points A and C are possible if we start at point A and are only allowed to travel right and up on the grid?
- (b) Of all the paths from A to C found in (a), how many pass through point B?

Answers: (a) 184756 (b) 58212

14. How many 10-bit strings contain five consecutive zeros or five consecutive ones?

Answer: 222

15. How many different ways are there to choose a dozen donuts from the five varieties at a donut shop?

Answer: 1820

16. The number 5 can be expressed as a sum of 3 positive integers, taking order into account, in 6 ways, namely

$$1 + 1 + 3 = 1 + 3 + 1 = 3 + 1 + 1 = 1 + 2 + 2 = 2 + 1 + 2 = 2 + 2 + 1.$$

Let m and n be positive integers such that $m \leq n$. In how many ways can n be expressed as a sum of m positive integers, taking order into account? *Answer:* $\binom{n-1}{m-1}$

Binomial Theorem

For any set S with n elements, the number of subsets of S with r elements is given by

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}.$$

A useful combinatorial formula is Pascal's identity

$$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}, \quad 1 \leq r \leq n.$$

It can be proved by the following combinatorial argument. Consider a set S with $n+1$ elements and fix attention on a particular element in the set, call it element a . There are $\binom{n}{r-1}$ subsets of S with r elements that contain a , and there are $\binom{n}{r}$ subsets of S with r elements that do not contain a . Since there are a total of $\binom{n+1}{r}$ subsets of S with r elements, Pascal's identity holds.

Binomial Theorem. For any $n \in \mathbf{N}$ and any $x, y \in \mathbf{R}$,

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.$$

Proof. We prove it by induction. For $n = 1$ we have

$$(x + y)^1 = \binom{1}{0} x^0 y^1 + \binom{1}{1} x^1 y^0 = x + y.$$

Assume the formula holds for some fixed arbitrary $n \in \mathbf{N}$. Then,

$$\begin{aligned} (x + y)^{n+1} &= (x + y)(x + y)^n \\ &= (x + y) \sum_{k=0}^n \binom{n}{k} x^k y^{n-k} \\ &= \underbrace{\sum_{k=0}^n \binom{n}{k} x^{k+1} y^{n-k}}_{\text{set } \ell=k+1} + \underbrace{\sum_{k=0}^n \binom{n}{k} x^k y^{n+1-k}}_{\text{set } \ell=k} \\ &= \sum_{\ell=1}^{n+1} \binom{n}{\ell-1} x^\ell y^{n+1-\ell} + \sum_{\ell=0}^n \binom{n}{\ell} x^\ell y^{n+1-\ell} \\ &= \sum_{\ell=1}^n \binom{n}{\ell-1} x^\ell y^{n+1-\ell} + x^{n+1} + y^{n+1} + \sum_{\ell=1}^n \binom{n}{\ell} x^\ell y^{n+1-\ell} \\ &= x^{n+1} + \sum_{\ell=1}^n \left[\binom{n}{\ell-1} + \binom{n}{\ell} \right] x^\ell y^{n+1-\ell} + y^{n+1} \\ &= x^{n+1} + \sum_{\ell=1}^n \binom{n+1}{\ell} x^\ell y^{n+1-\ell} + y^{n+1} \\ &= \sum_{\ell=0}^{n+1} \binom{n+1}{\ell} x^\ell y^{n+1-\ell} \end{aligned}$$

which completes the proof by induction.

Permutations vs. Combinations

It is **very** important to make the distinction between permutations and combinations. In permutations, order matters and in combinations order does **not** matter. The important information can be summarized by:

	Order	Number
Permutation	matters	$P(n, k) = \frac{n!}{(n-k)!}$
Combination	does not matter	$C(n, k) = \frac{n!}{(n-k)!k!}$

Examples:

A company has to select 3 officers from a pool of 6 candidates. How many different ways can this be done if:

- (a) The officers are distinct?
- (b) The officers are not distinct?

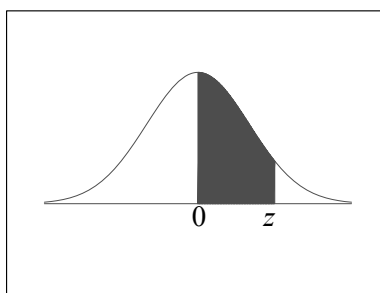
It is **very** important whether or not these officers are distinct.

(a) If the officers are distinct, we are picking a triple (s_1, s_2, s_3) with each s_i being a candidate, and order matters. This means we are finding a 3-permutation from a set of 6 elements. So there are: $P(6, 3) = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120$ distinct ways to pick these officers.

(b) If the officers are not distinct, the triples (s_1, s_2, s_3) , (s_1, s_3, s_2) , (s_3, s_2, s_1) , etc. are the same since the positions are the same. So, we are finding a 3-combinations from a set of 6 elements. So there are:

$$C(6, 3) = \frac{6!}{(6-3)!3!} = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 5 \cdot 4 = 20.$$

Standard Normal Distribution Table

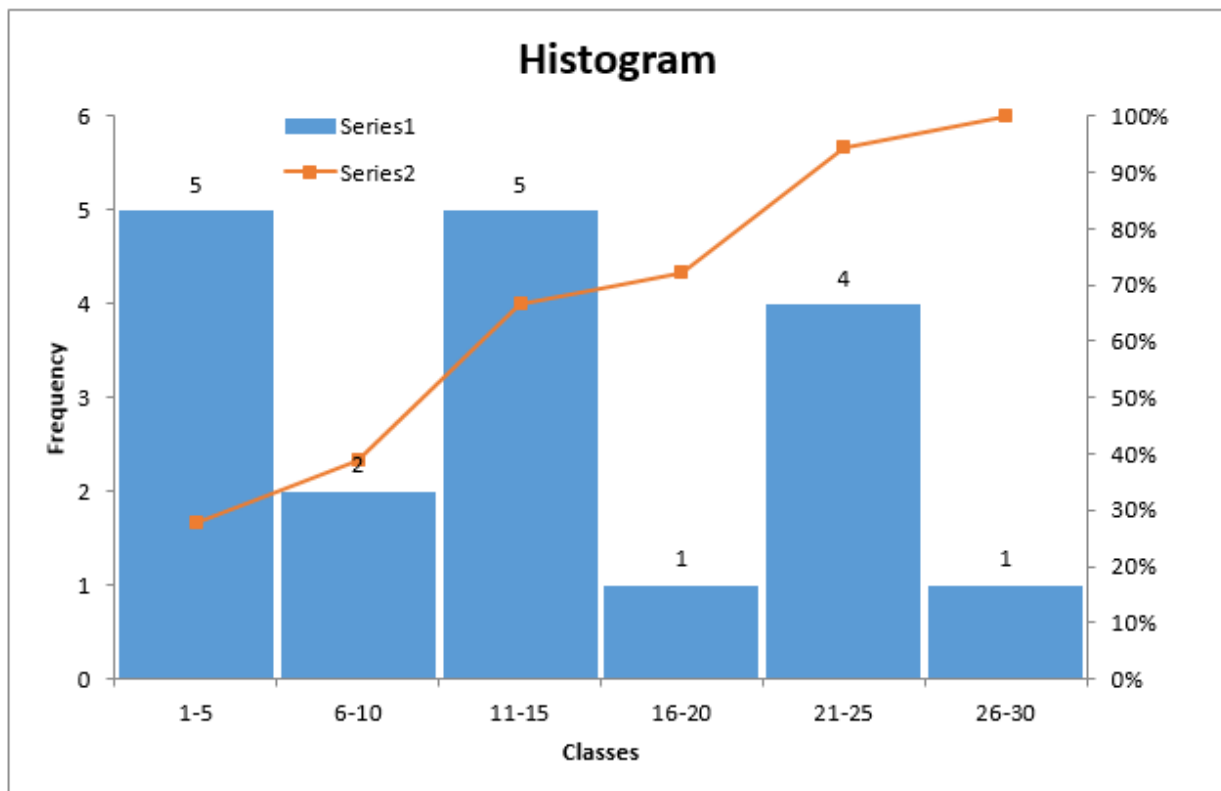


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

I. Using the data below, complete the table, create a histogram, and draw an ogive graph.

Category	Frequency			
1-5	5			
6-10	2			
11-15	5			
16-20	1			
21-25	4			
26-30	1			

Category	Frequency	Cumulative Freq.	Relative Freq.	Cumulative Rel. Freq.
1-5	5	5	27.78 %	27.78 %
6-10	2	7	11.11 %	38.89 %
11-15	5	12	27.78 %	66.67 %
16-20	1	13	5.56 %	72.23 %
21-25	4	17	22.22 %	94.45 %
26-30	1	18	5.56 %	100.00 %
	18		100.00 %	



2. Create a Stem & Leaf Diagram using the data below.

1, 10, 50, 51, 66, 69, 76, 77, 79, 82, 83, 85, 87, 88, 88, 89, 92, 92, 96, 96, 98, 108

Stem Unit = 10	Leaf Unit = 1						
0	1						
1	0						
2							
3							
4							
5	0	1					
6	6	9					
7	6	7	9				
8	2	3	5	7	8	8	9
9	2	2	6	6	8		
10	8						

3. Using the data below create a Box Plot

1, 10, 50, 51, 66, 69, 76, 77, 79, 82, 83, 85, 87, 88, 88, 89, 92, 92, 96, 96, 98, 108

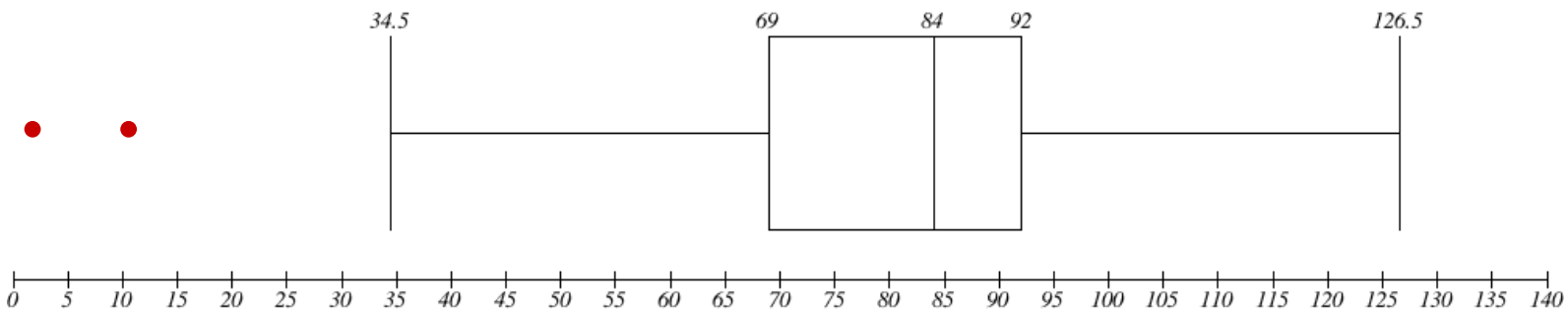
$$Q1: 0.25(22) = 5.5\text{th number} \rightarrow 6\text{th number} = 69$$

$$Q2: 0.50(22) = 11\text{th number} \rightarrow (11\text{th} + 12\text{th}) / 2 = (83+85)/2 = 84$$

$$Q3: 0.75(22) = 16.5\text{th number} \rightarrow 17\text{th number} = 92$$

$$UL: Q3 + 1.5(IQR) \rightarrow 92 + 1.5(92 - 69) = 126.5$$

$$LL: Q1 - 1.5(IQR) \rightarrow 69 - 1.5(92 - 69) = 34.5$$



4. Indicate the level of data measurement for each variable in the data set.
 - a. Airline
 - b. Day of Week
 - c. Time of Day
 - d. Male/Female
 - e. Business/Pleasure
 - f. Length of Trip
 - g. Hours to Destination
 - h. Children < 10 years (yes / no)
 - i. Pieces of Luggage
 - j. Pieces Carried On
 - k. Times Flown
 - l. Satisfaction Level (1 = bad, 10 = great)

5. A store manager tracks the number of customer complaints each week. The following data reflect a random sample of ten weeks.

11	19	4	6	8	9	6	4	0	3
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- a. Calculate the mean, median, variance, and standard deviation
- b. If one week is selected at random, what is the probability that it is the the bottom Quartile of complaints?
- c. if 6 weeks are picked at random (no replacement) what is the probability that each week picked has more complaints than the previous week? (the 2nd has more complaints than the 1st, the 3rd has more complaints than the 2nd, and so on.)

6. RLW is a Montreal based firm that specializes in high-end running shoes, and has two factories in Montreal (one in the East-End and one in Lachine) that produce its shoes. The East-End factory produces an average of 335 units per day with a standard deviation equal to 11 units. The factory in Lachine produces an average of 145 units per day with a standard deviation equal to 8 units. Based on this information, which line is relatively more consistent?

7. A Montreal real-estate company owns six office buildings in the Greater Montreal area that it leases to businesses. The lease price per square foot differs by building due to location and building amenities. Currently, all six buildings are fully leased at the prices shown here.

	\$ / Ft²	Ft²
Building 1	\$75	125,000
Building 2	\$85	37,500
Building 3	\$90	77,500
Building 4	\$45	35,000
Building 5	\$55	60,000
Building 6	\$110	130,000

- a. Compute the weighted average (mean) price per square foot for these buildings

- b. Compute the standard deviation of the price per square foot for these buildings

8. Assume that a standard deck of 52 playing cards is randomly shuffled and the first 2 cards are dealt to you. What is the probability that you have a **blackjack**? A blackjack is where one card is an ace and the other card is worth 10 points. The 10-point cards are kings, queens, jacks and 10's.

9. Employees at a large computer company earn sick leave in one-minute increments depending on how many hours per month they work. They can then use the sick leave time any time throughout the year. Any unused time goes into a sick bank account that they or other employees can use in the case of emergencies. The human resources department has determined that the amount of unused sick time for individual employees is uniformly distributed between 0 and 480 minutes. Based on this information, what is the probability that an employee will have less than 20 minutes of unused sick time?

10. Three events occur with probabilities of $P(E_1) = 0.35$, $P(E_2) = 0.25$, $P(E_3) = 0.40$. Other probabilities are: $P(B | E_1) = 0.25$, $P(B | E_2) = 0.15$, $P(B | E_3) = 0.60$.

- a. Compute $P(E_1 | B)$

$$P(E_1|B) = \frac{P(E_1 \cap B)}{P(B)}$$

$$= \frac{8.75}{36.5} = \underline{\underline{23.97\%}}$$

- b. Compute $P(E_2 | B)$

$$P(E_2|B) = \frac{P(E_2 \cap B)}{P(B)}$$

$$= \frac{3.75}{36.5} = \underline{\underline{10.27\%}}$$

- c. Compute $P(E_3 | B)$.

$$P(E_3|B) = \frac{P(E_3 \cap B)}{P(B)}$$

$$= \frac{24}{36.5} = \underline{\underline{65.75\%}}$$

Need to find $P(B)$ first!

$$P(B|E_1) = \frac{P(B \cap E_1)}{P(E_1)}$$

$$0.25 = \frac{x}{0.35} \quad x = 0.0875$$

$$= \underline{\underline{8.75\%}}$$

do this for
 $P(B|E_2)$
 $P(B|E_3)$
 the sum
 of all 3
 is = $P(B)$

	B	B'	
E1	8.75	26.25	35
E2	3.75	21.25	25
E3	24.00	16.00	40
	36.50	63.50	100

12. Until the summer of 2008, the real estate market in Fresno, California, had been booming, with prices skyrocketing. Recently, a study showed the sales patterns in Fresno for single-family homes. One chart presented in the commission's report is reproduced here. It shows the number of homes sold by price range and number of days the home was on the market.

Price Range (\$000)	Days on the Market			
	1-7	8-30	Over 30	
Under \$200	125	15	30	170
\$200-\$500	200	150	100	450
\$501-1000	400	525	175	1100
Over \$1000	125	140	35	300
	850	830	340	2020

- a. Using the relative frequency approach to probability assessment, what is the probability that a house will be on the market more than 7 days?

$$P(X > 7 \text{ days}) = P(8 \leq X \leq 30) + P(X > 30)$$

↑ "or"

$$\frac{830 + 340}{2020} = 57.92\%$$

- b. Is the event 1-7 days on the market independent of the price \$200-\$500?

$$\frac{850 \times 110}{2020} = 189 \neq 200 \text{ so the 2 events are dependent!}$$

- c. Suppose a home has just sold in Fresno and was on the market less than 8 days, what is the most likely price range for that home?

$$P(X | X < 8 \text{ days}) \rightarrow \frac{400}{850} \text{ is the highest \% so it's homes that are between \$501K and \$1000}$$

13. A company wants to know how important higher education is when selecting a new CEO. Prior studies have shown that 60% of all CEOs will be successful, 85% of successful CEOs have a university degree, while 70% of those who don't have a University degree turn out to be unsuccessful CEOs

- a. What is the probability of a CEO having a university degree

$$P(D) = 70\%$$

- b. What is the probability of a CEO not having a university degree **and** being successful

$$P(D' \cap S) = 9\%$$

- c. What is the probability of a CEO having a university degree **or** being successful

$$P(D \cup S) = P(D) + P(S) + P(D \cap S)$$

$$70 + 60 - 51 = 79\%$$

Need to complete the table first!

Recognize that since $P(S|D') = 70\%$ then $P(S|D')$ must = 30% ~ then solve for D'

$$P(S|D') = \frac{P(S \cap D')}{P(D')}$$

$$30\% = \frac{9\%}{x}$$

$$x = P(D') = 30\%$$

	S	S'	
D	51	19	70
D'	9	21	30
	60	40	100

14. Radio Shack stocks four alarm clock radios. If it has fewer than four clock radios available at the end of a week, the store restocks the item to bring the in-stock level up to four. If weekly demand is greater than the four units in stock, the store loses the sale. The radio sells for \$25 and costs the store \$15. The Radio Shack manager estimates that the probability distribution of weekly demand for the radio is as follows:

X (Weekly Demand)	P(x)
0	0.05
1	0.05
2	0.10
3	0.20
4	0.40
5	0.1
6	0.05
7	0.05

a. What is the expected weekly demand for the alarm clock radio?

$$E(x) = \text{Sum}(X_i)(P_i) = (0 \times 0.05) + (1 \times 0.05) + (2 \times 0.10) \dots = 3.6 \text{ radios}$$

b. What is the probability that weekly demand will be greater than the number of available radios?

$$P(x > 4) = P(5) + P(6) + P(7) = 40\%$$

c. What is the expected weekly profit from the sale of the alarm clock radio? (Remember: There are only four clock radios available in any week to meet demand.)

d. On average, how much profit is lost each week because the radio is not available when demanded?

$$(\$10 \times 3.6) - \$32.5 = \underline{\underline{\$3.50}}$$

Demand	Profit	Probability	E(x)
0	\$0	5%	\$0
1	\$10	5%	\$0.5
2	\$20	10%	\$2
3	\$30	20%	\$6
4	\$40	40%	\$16
5	\$40	10%	\$4
6	\$40	5%	\$2
7	\$40	5%	\$2
			\$32.50

15. Use the binomial formula to calculate the following probabilities for an experiment in which $n = 5$ and $p = 0.4$:

- a. the probability that x is at most 1

$$P(X \leq 1) = P(1) + P(0) = 33.70\%$$

- b. the probability that x is at least 4

$$P(X \geq 4) = 8.70\%$$

- c. the probability that x is less than 1

$$P(X < 1) = P(X = 0) = 7.78\%$$

16. A manufacturing firm produces a product that has a ceramic coating. The coating is baked on to the product, and the baking process is known to produce 15% defective items (for example, cracked or chipped finishes). Every hour, 20 products from the thousands that are baked hourly are sampled from the ceramic coating process and inspected. a.

- a. What is the probability that 5 defective items will be found in the next sample of 20?

$$n = 20$$

$$p = 15\%$$

$$P(X=5) = \frac{20!}{5!(20-5)!} 0.15^5 0.85^{15} = 10.28\%$$

- b. On average, how many defective items would be expected to occur in each sample of 20?

$$E(X) = np = 20 \times 15\% = \underline{\underline{3}}$$

- c. How likely is it that 15 or more non-defective (good) items would occur in a sample due to chance alone?

$$P(X \geq 15) = P(15) + P(16) + P(17) + P(18) + P(19) + P(20)$$

Where X = the number of "good" items thus use $P = 85\%$

Alternatively

$$P(X \geq 15 \text{ "good"}) = P(X < 5 \text{ "Bad"})$$

Where X = the number of "bad" items thus use $P = 15\%$

17. For a standardized normal distribution, calculate the following probabilities:

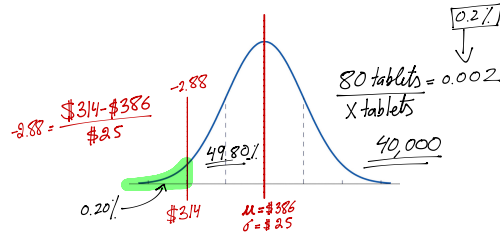
a. $P(z < 1.5)$

b. $P(z \geq 0.85)$

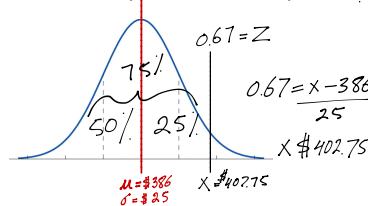
c. $P(-1.28 < z < 1.75)$

18. The average sales price of tablets was \$386 during the first quarter of this year, a 21% drop compared to the same period last year, according to a report from IMS Research. The price decline is a result of intense competition in the tablet market. Assume that the standard deviation of prices was \$25, and that tablet pricing follows a normal distribution.

- a. If a local Best Buy outlet sold 80 tablets at or below \$314, how many tablets were sold this year?



- b. Compute the third quartile of the tablet prices sold by this Best Buy outlet



- c. What is the probability of selling a tablet that's priced more than the price of the average price of a tablet + tax? (assume the tax is 15%)

