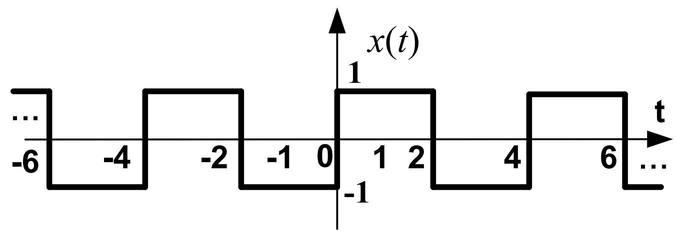
Question1: (8 % = 1% for each part)

Consider the signal $x(t) = \sum_{k=-\infty}^{\infty} x_1(t-4k)$, with

$$x_1(t) = \begin{cases} -1, & -2 < t < 0 \\ 1, & 0 < t < 2 \\ 0, & otherwise \end{cases}$$



(a) Evaluate Fourier series coefficients a_k of the signal $x(t)$ (Hint: use the symmetry of $x(t)$ when expressing a_k)

(b) Sketch the frequency domain representation of the Fourier series a_k for k=-3, -2, -1, 0, 1, 2 and 3,

(c) Show that: $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau = \sum_{k=-\infty}^{\infty} y_1(t-4k)$, with

$$y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau = \begin{cases} |t| - 2, & |t| < 2 \\ 0, & \text{otherwise} \end{cases},$$

(d) Evaluate Fourier series coefficients b_k of the signal $y(t)$ (Hint: use the integral property of CT Fourier series $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \overset{F.S}{\leftrightarrow} b_k = \frac{1}{j\omega_0 k} a_k = \frac{1}{j\frac{2\pi}{T}k} a_k$)

(e) Sketch the frequency domain representation of the Fourier series b_k for k=-3, -2, -1, 0, 1, 2 and 3,

(f) Evaluate the quantity $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$ (Hint: evaluate $y(0)$ using CT Fourier series)

(g) Evaluate the quantity $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^4} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \cdots$ (Hint: and apply CT Fourier series Parseval's relation to $y(t)$)

(h) Evaluate and sketch the CT Fourier Transform of the signal $y(t)$

Question 2: (8% = 4 % for each part)

Consider a continuous-time system which has input of signal $x(t) = e^{-2t}u(t) - e^{-3t}u(t)$ and output $y(t) = e^{-|t|}$

- **(a)** Find the frequency response $H(j\omega)$ and impulse response $h(t)$ of the above system
- **(b)** Find the impulse response $h_{inv}(t)$ of the inverse of the above system found in (a) (Hint: the frequency response of the inverse is $H_{inv}(j\omega) = 1/H(j\omega)$)

Question 3: (8% = 4% for each part)

Consider a discrete-time system which has input of signal $x[n] = (1/2)^n u[n]$ and output $y[n] = (1/4)^n u[n]$

- **(a)** Find the frequency response $H(e^{j\omega})$ and impulse response $h[n]$ of the above system
- **(b)** Find the impulse response $h_{inv}[n]$ of the inverse of the above system found in (a) (Hint: the frequency response of the inverse is $H_{inv}(e^{j\omega}) = 1/H(e^{j\omega})$)

Question 4: (6 % = 2% for each part)

Use the properties of Fourier Transforms and Basic Fourier transform pairs to find Fourier Transform and inverse Fourier transform of the following signals:

- **(a)** $x(t) = t\dfrac{d}{dt}\left[e^{-|t|}\cos(t)\right]$
- **(b)** $X(j\omega) = 4j\dfrac{d}{d\omega}\left[\dfrac{\sin(2\omega-1)}{(2\omega-1)}\right]$
- **(c)** $x(t) = \sum_{n=0}^{\infty}(n+1)\left(\dfrac{1}{2}\right)^n \delta(t-n)$ (Hint: use $\sum_{n=0}^{\infty}(n+1)a^n = \dfrac{d}{da}\left(\sum_{n=0}^{\infty}a^n\right) = \dfrac{d}{da}\left(\dfrac{1}{1-a}\right) = \dfrac{1}{(1-a)^2}$, for $|a| < 1$).

Question 5: (8% = 2% for each part)

Consider a system described by the differential equation

$$\frac{1}{2}\frac{d^2y(t)}{dt^2} + \frac{3}{2}\frac{dy(t)}{dt} + y(t) = x(t)$$

- **(a)** Sketch the block diagram of the system
- **(b)** Evaluate the Frequency response of the system

- **(c)** Evaluate the impulse response of the system using properties of CT Fourier Transforms and Basic CT Fourier transform pairs
- **(d)** Is the system is stable? Justify your answer

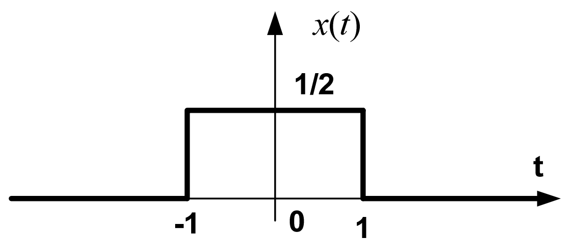
Question 6: (8% = 2% for each part)

Consider a system described by the difference equation

$$\frac{1}{4}y[n-2]-y[n-1]+y[n]=x[n]$$

- **(a)** Sketch the block diagram of the system
- **(b)** Evaluate the Frequency response of the system
- **(c)** Evaluate the impulse response of the system using properties of DT Fourier Transforms and Basic DT Fourier transform pairs
- **(d)** Is the system is stable? Justify your answer (Hint: use $\sum_{n=0}^{\infty}(n+1)a^n=\frac{d}{da}\left(\sum_{n=0}^{\infty}a^n\right)=\frac{d}{da}\left(\frac{1}{1-a}\right)=\frac{1}{(1-a)^2}$, for $|a|<1$).

Question 7: (4 % = 1% for each part)



- **(a)** Evaluate $y(t)=x(t)\otimes x(t)$ using the convolution integral
- **(b)** Evaluate the Fourier Transform of $y(t)=x(t)\otimes x(t)$ (Hint: use convolution property of CT Fourier Transform)
- **(c)** Evaluate the quantity $\int_{-\infty}^{\infty}\frac{\sin^2(\omega)}{\omega^2}d\omega$ (Hint: evaluate $y(0)$ using CT Fourier transform)
- **(d)** Evaluate the quantity $\int_{-\infty}^{\infty}\frac{\sin^4(\omega)}{\omega^4}dt$ (Hint: apply CT Fourier Transform Parseval's relation to $x(t)\otimes x(t)$)