SOLUTION

(a) $\displaystyle\int_3^\infty \frac{(\ln n)^3}{n}\,dn$ $\begin{cases} u = \ln n \\ du = \frac{dn}{n} \end{cases}$ $\displaystyle = \int_{\ln 3}^{\infty} u^3\,du = \frac{1}{4}u^4 \Big|_{\ln 3}^{\infty}$

setup 1 work 2

$= \infty$ $\therefore$ $\displaystyle\sum_{n=3}^{\infty} \frac{(\ln n)^3}{n}$ diverges. conclusion 1

(b) for $n > 3$, $\ln n > 1$ $\therefore$ $\dfrac{(\ln n)^3}{n} > \dfrac{1}{n}$ 1

Since $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n}$ diverges, then by comparison $\displaystyle\sum_{n=3}^{\infty} \frac{(\ln n)^3}{n}$

also diverges. 1

SOLUTION

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{2n!} \qquad a_n = \frac{(-1)^n 3^n}{2n!} \Rightarrow a_{n+1} = \frac{(-1)^{n+1} 3^{n+1}}{2(n+1)!}$$

$$= \frac{(-1)^{n+1} 3 \cdot 3^n}{2(n+1)\, n!}$$

So
$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\dfrac{(-1)^{n+1} 3\cdot 3^n}{2(n+1)n!}}{\dfrac{(-1)^n 3^n}{2n!}}\right| = \frac{3\cdot 3^n}{2(n+1)n!} \cdot \frac{2n!}{3^n}$$

① a_{n+1}

① work

$$= \frac{3}{n+1}$$

Since $\underbrace{L = \lim_{n\to\infty} \frac{3}{n+1} = 0 < 1, \text{ then}}_{\text{① proper outcome}}$ $\sum a_n$ conv. abs.

① conclusi

SOLUTION

$$\sum_{n=0}^{\infty} \frac{(-1)^n n^{2n}}{2^n (n^2+1)^n}$$

$$|a_n|^{\frac{1}{n}} = \left| \frac{(-1)^n n^{2n}}{2^n (n^2+1)^n} \right|^{\frac{1}{n}} = \left(\frac{n^{2n}}{2^n (n^2+1)^n} \right)^{\frac{1}{n}} = \frac{n^2}{2(n^2+1)}$$ 1 work

So $L = \lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{2(n^2+1)} = \frac{1}{2} < 1$ 1 outcome

Since $L<1$, then $\sum a_n$ conv. abs. 1 conclusion

SOLUTION

$$\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{n^2+1}}{n+1} \qquad \underbrace{a_n = \frac{(-1)^n \sqrt{n^2+1}}{n+1} \Rightarrow b_n = \frac{\sqrt{n^2+1}}{n+1}}_{1}$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{\sqrt{n^2+1}}{n+1} = 1 \neq 0 \quad 1$$

So the series diverges. 1

NOTE: The Alternating Series test is not applicable since b_n is NOT decreasing, but showing $\lim_{n\to\infty} b_n \neq 0$ is acceptable

→ Valid as either Alternating <u>or</u> n^{th} term.

SOLUTIONS

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ If $\sum |a_n|$ conv, then $\sum a_n$ conv. abs.

So $\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ 1 for solving $\sum |a_n|$

By the $\underbrace{\text{p-series } p = 3/2 > 1, \text{ so } \sum |a_n| \text{ conv,}}_{1}$

therefore $\underbrace{\sum a_n \text{ conv. abs.}}_{1}$

SOLUTION

$$\sum_{n=0}^{\infty} \frac{(-2)\,3^n}{4(-5)^{n-1}} = \sum_{n=0}^{\infty} \frac{(-2)(-5)\;3^n}{4\;(-5)(-5)^{n-1}} = \sum_{n=0}^{\infty} \frac{10 \cdot 3^n}{4 \cdot (-5)^n}$$

① **factor**

$$= \frac{10}{4} \underbrace{\sum_{n=0}^{\infty} \left(-\frac{3}{5}\right)^n}_{\text{geometric series}} = \frac{10}{4}\left(\frac{1}{1+\left(\frac{3}{5}\right)}\right) = \frac{10}{4} \cdot \frac{1}{8/5} = \frac{\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}} \cdot \frac{5}{8}$$

① **formula**

$$= \frac{25}{16}$$ ① **answer**