SOLUTIONS

$$(\cos x)y' + (\sin x)y = \sin x \cos^3 x$$

$$y' + (\tan x)y = \sin x \cos^2 x \quad 1$$

IF: $\mu = \exp\left(\int \tan x \, dx\right) = \exp(\ln|\sec x|)$

$$= |\sec x| = \frac{1}{|\cos x|} = \frac{1}{\cos x} \quad 1$$

$$\therefore \mu y = \frac{y}{\cos x} = \int \sin x \cos x \, dx = \frac{\sin^2 x}{2} + \frac{C}{2}$$

1

$$\frac{y}{\cos x} = \frac{C + \sin^2 x}{2} \quad 1$$

$$y = \frac{C\cos x + \sin^2 x \cos x}{2} \quad 1$$

OR

$$y = \frac{C + \sin^2 x}{2\sec x}$$

$$y = \frac{(D - 1 + \sin^2 x)\cos x}{2} = \frac{[D - (1 - \sin^2 x)]\cos x}{2}$$

$$= \frac{D\cos x - \cos^3 x}{2}$$

SOLUTIONS

$$x^2+y^2+xyy'=0$$

(a) $P=x^2+y^2$ $\quad P_y=2y$

$Q=xy$ $\quad Q_x=y$

NOT EXACT

(2)

(b) $$\mu^*(x)=\frac{P_y-Q_x}{Q}=\frac{2y-y}{xy}=\frac{y}{xy}=\frac{1}{x}$$

$$\therefore\ \mu=\exp\int\frac{1}{x}dx=e^{\ln x}=x$$

(3)

(c) $$x^3+xy^2+x^2yy'=0$$

$P=x^3+xy^2$ $\quad P_y=2xy$

$Q=x^2y$ $\quad Q_x=2xy$

EXACT!

$$\int P\,dx=\int x^3+xy^2\,dx=\frac{x^4}{4}+\frac{x^2y^2}{2}+c(y)$$

$$\int Q\,dy=\int x^2y\,dy=\frac{x^2y^2}{2}+\delta(x)$$

(3)

$$\therefore\ 0=f(x,y)=\frac{x^4}{4}+\frac{x^2y^2}{2}+C$$ (2)

SOLUTIONS

$$\frac{y^2}{x^2} - y' = 0$$

(a)

$$\frac{y^2}{x^2} = \frac{dy}{dx}$$

$$\int \frac{dy}{y^2} = \int \frac{dx}{x^2} \quad (1)$$

$$-\frac{1}{y} = C - \frac{1}{x} \quad (1)$$

$$= \frac{Cx}{x} - \frac{1}{x}$$

$$= \frac{Cx - 1}{x}$$

$$y = \frac{x}{1 - Cx} \quad (1)$$

(b)

$$y' = \frac{1}{x^2} y^2 \qquad \left\{\begin{array}{l} \alpha = 2 \\ 1 - \alpha = -1 \\ u = \frac{1}{y} \end{array}\right\} (1)$$

$$\frac{y'}{(-1)} = \frac{1}{x^2} y^2$$

$$y' = -\frac{1}{x^2} y^2$$

$$\frac{du}{dx} = -\frac{1}{x^2} \cancel{y^2}$$

$$\int du = -\int \frac{1}{x^2} dx \quad (2)$$

$$u + C = \frac{1}{x}$$

$$u = \frac{1}{x} - C \quad (1)$$

$$\frac{1}{y} = \frac{1}{x} - C = \frac{1}{x} - \frac{Cx}{x} = \frac{1 - Cx}{x}$$

$$y = \frac{x}{1 - Cx} \quad (1)$$

SOLUTIONS

(c) $u = \frac{y}{x} \rightarrow y' = u'x + u$

$u^2 - u - u'x = 0$

$u(u-1) = x\frac{du}{dx}$ 1

$\underbrace{\int \frac{dx}{x} = \int \frac{du}{u(u-1)}}_{1} = \int \frac{A}{u} + \frac{B}{u-1}\,du$ 1 to realize integration by partial fractions

NUMERATOR:

$1 = A(u-1) + Bu$

at $u=1$ $\quad 1 = A(0) + B \rightarrow B = 1$

at $u=0$ $\quad 1 = A(-1) + B(0) \rightarrow A = -1$ 1

$\therefore \ln x + \ln C = \ln|u-1| - \ln|u| = \ln\left|\frac{u-1}{u}\right| = \ln\left|1 - \frac{1}{u}\right|$

$Cx = 1 - \frac{1}{u}$

$\frac{1}{u} = 1 - cx$

$u = \frac{1}{1-cx}$

2 for work

$\frac{y}{x} = \frac{1}{1-cx}$

$y = \frac{x}{1-cx}$ 1 for answer in explicit form