

## MATH 1004 - A COMPLETE PROCEDURE FOR CURVE SKETCHING

Assume you are being asked to sketch the curve  $f(x)$ . The following is a series of steps you should follow to achieve this. **NOTE** that this procedure is in a slightly different order than what was done in class. This procedure is better to apply universally, and is complete.

### 1. INTERCEPTS

$x$ -intercept: Set  $y = 0$ , then solve for  $x$ .

$y$ -intercept: Set  $x = 0$ , then solve for  $y$ .

### 2. VERTICAL ASYMPTOTES

Look at  $f(x)$ . For all values (call them  $a_i$ ) that make the function undefined, take the limit:

$$\lim_{x \rightarrow a_i} f(x)$$

Do this for **ALL**  $a_i$  (it is possible to have more than one  $a$  that makes  $f$  undefined!). Every  $a_i$  whose limit goes to  $+\infty$  **OR**  $-\infty$  corresponds to a **vertical asymptote**.

### 3. HORIZONTAL ASYMPTOTES

Now take the limits:

$$\lim_{x \rightarrow \pm\infty} f(x)$$

If this limit **converges to a finite value**, then you have a **horizontal asymptote** at that value.

### 4. CRITICAL POINTS

Determine  $f'(x)$ , set it equal to 0, and find all values of  $x$  which satisfy that equation:

$$f'(x) = 0$$

The value which satisfy this equation, call them  $c_i$ , are the  $x$ -components of the critical points. Now plug each of the critical values,  $c_i$ , into the **original** function to determine the  $y$ -component of the critical points (*i.e.* **evaluate**  $f(c_i)$  for all  $c_i$ ). Careful **not** to plug them into  $f'(x)$ !

### 5. POINTS OF INFLECTION

A point of inflection is a point at which the shape of the curve changes from concave-up to concave-down, or vice versa. Concave-up can be thought of as the shape of a parabola opening upwards, and concave-down is like a parabola opening downwards. Much like finding the critical points, if you apply the **SAME** procedure to  $f''(x)$ , you'll get the points of inflection:

$$f''(x) = 0$$

In this case, the value which satisfy this equation, call these  $p_i$ , are the  $x$ -components of the points of inflection. Now plug each of the  $p_i$  values into the **original** function to determine the  $y$ -component of the points of inflection (*i.e.* **evaluate**  $f(p_i)$  for all  $p_i$ ). Again, careful not to plug the  $p_i$ 's into  $f''(x)$  or  $f'(x)$ . We need to evaluate  $f(p_i)$ !

### 6. START YOUR SKETCH

Give yourself lots of space on your page. Draw in the intercepts on the axes. Then draw in the asymptotes with a dotted line. Finally draw in the **critical points** and **points of inflection** we've just determined.

## 7. LOCAL MAXIMA AND LOCAL MINIMA

Using the expression for  $f''(x)$  and the **critical points from the previous step**, evaluate  $f''(c_i)$  **for each** critical point  $x$ -component,  $c_i$ . Any resulting value which is positive tells us that particular critical point is a **local minimum**. Conversely, any resulting value which is negative tells us that particular critical point is a **local maximum**. If the result is 0 then this is inconclusive, and we need more information, **but** we can say that if the critical point is **also** a point of inflection, then this is a **saddle point**.

## 8. PLOT SOME MORE

Now, we've just determined what's a max, a min, or a saddle at **EVERY** critical point. Draw in a **SMALL** part of the curve of the shape of a max/min/saddle, however applicable.

## 9. SIGN DECOMPOSITION TABLES (SDT)

Generate a SDT for  $f(x)$ ,  $f'(x)$ , and  $f''(x)$ . Recall that you put all the factors of  $f(x)$ ,  $f'(x)$ , or  $f''(x)$  (depending which SDT you're in the process of doing) along the top, and the subintervals go down the side (which correspond to the **zeros** of the factors you just wrote along the top for the given SDT). Complete the SDT by filling in + and - signs in the body of the table.

## 10. COMPLETE THE SKETCH

Use the SDTs to draw in the contours for each subinterval of the graph.

- (a) SDT of  $f(x)$  will tell you for each subinterval if you are **above** or **below** the  $x$ -axis. If the sign is positive, you're above the  $x$ -axis, and if the sign is negative, you're below the  $x$ -axis. The subintervals break at each  $x$ -intercept, and any undefined value of  $f(x)$ .
- (b) SDT of  $f'(x)$  will tell you for each subinterval if the function is **increasing** or **decreasing**. Recall that an interpretation of the derivative is the slope of the tangent line: So positive is increasing, and negative is decreasing. Here the subintervals break at each critical point,  $c_i$ , and again at any undefined value of  $f(x)$ .
- (c) SDT of  $f''(x)$  will tell you for each subinterval if the shape of the curve is **concave-up** (positive sign) or **concave-down** (negative sign). **NOTE** that the changes in concavity occur **at the points of inflection**, so these subintervals break at each  $p_i$  (and also as in the other two SDT tables - at all undefined value of  $f(x)$ ).

## 11. FINALIZE

Make sure your graph has everything it needs on it (like labels, points, etc.), it looks clean and tidy, and you're finished!