Sandwich/Squeeze Theorem

Without loss of generality, but in general, examining tendencies as $x \to \infty$, considering $f(x), l(x), h(x)$, where l is considered to be the "low" function, and h is considered to be the "high" function, then provided:

$$l(x) \leq f(x) \leq h(x)$$

We can solve the limit $\lim_{x\to\infty} f(x)$ if we bind below with $l(x)$, and we bind above with $h(x)$.

THEN

$$\lim_{x\to\infty} l(x) \leq \lim_{x\to\infty} f(x) \leq \lim_{x\to\infty} h(x)$$

If we know $\lim_{x\to\infty} l(x)$ and $\lim_{x\to\infty} h(x)$, then the limit of $f(x)$ might be solvable. So, provided we are clever, and choose l and h such that $\lim_{x\to\infty} l(x) = \lim_{x\to\infty} h(x)$, then given f is bound by l and h, then $\lim_{x\to\infty} f(x)$ can be solved.

i.e.

1. Provided the functions satisfy:
   $l(x) \leq f(x) \leq h(x)$
2. We evaluate the limits:
   $\lim_{n\to\infty} l(x) \leq \lim_{n\to\infty} f(x) \leq \lim_{n\to\infty} h(x)$
3. IF $L = \lim_{x\to\infty} l(x) = \lim_{x\to\infty} h(x)$
4. THEN $\lim_{x\to\infty} f(x) = L$ as well

Example: Solve the limit $\lim_{x \to \infty} \frac{\sin 2x}{x^2}$

We realize that $\sin 2x$ oscillates between 1 and -1. We express this as:

$$f(x) = \sin 2x \ \in \ [-1,1]$$

$$-1 \leq \sin 2x \leq 1 \ \ \rightarrow \underset{\substack{\uparrow \\ l(x)}}{\frac{-1}{x^2}} \leq \frac{\sin 2x}{x^2} \leq \underset{\substack{\uparrow \\ h(x)}}{\frac{1}{x^2}}$$

$$\left.\begin{aligned} \lim_{x \to \infty} l(x) &= \lim_{x \to \infty} \frac{-1}{x^2} = 0 \\ \lim_{x \to \infty} h(x) &= \lim_{x \to \infty} \frac{1}{x^2} = 0 \end{aligned}\right\} = L$$

Therefore, by the sandwich theorem, $\lim_{x \to \infty} \frac{\sin 2x}{x^2} = 0$ (given it's bound below by $\frac{-1}{x^2}$ and above by $\frac{1}{x^2}$).