Ex 253 Sketch $y = \frac{1}{x^2-1}$

(1) x-int $\rightarrow$ $0 = y = \frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$ $\rightarrow$ no sol'n

y-int $\rightarrow$ $x=0$ $\quad y = \frac{1}{(0)^2-1} = \frac{1}{-1} = -1$

$(0, -1)$

(2) VA suspect $x = 1, -1$

$x=1$ / $\lim_{x \to 1^+} \frac{1}{x^2-1} = +\infty$

$\lim_{x \to 1^-} \frac{1}{x^2-1} = -\infty$

} VA @ $x=1$

$x=-1$ / $\lim_{x \to -1^+} \frac{1}{x^2-1} = -\infty$

$\lim_{x \to -1^-} \frac{1}{x^2-1} = +\infty$

} VA @ $x=-1$

(3) HA $\lim_{x \to \infty} \frac{1}{x^2-1} = 0^+$

$\lim_{x \to -\infty} \frac{1}{x^2-1} = 0^+$

(4) CP $y' = -\frac{2x}{(x^2-1)^2}$

$0 = y' = \frac{-2x}{(x^2-1)^2} \rightarrow x = 0$ $\quad y(0) = \frac{1}{0^2-1} = -1$

$(0, -1)$

⑤ PI $\quad y'' = \dfrac{6x^2+2}{(x^2-1)^3}$

$0 = y'' = \dfrac{6x^2+2}{(x^2-1)^3}$ $\qquad 0 = 6x^2+2$

$-\frac{2}{6} = x^2 \rightarrow$ no soln

NO PI

⑥ OK

⑦ MAX/MIN: test $x=0$ ← MAX

$y''(0) = \dfrac{6(0)^2+2}{((0)^2-1)^3} = \dfrac{2}{(-1)^3} = -2 < 0$

⑧ OK

⑨

	$x-1$	$x+1$	y'
$(-\infty,-1)$	−	−	+
$(-1,1)$	−	+	−
$(1,\infty)$	+	+	+

	$-2x$	$(x-1)^2$	$(x+1)^2$	y'
$(-\infty,-1)$	+	+	+	+
$(-1,0)$	+	+	+	+
$(0,1)$	−	+	+	−
$(1,\infty)$	−	+	+	−

	$6x^2+2$	$(x-1)^3$	$(x+1)^3$	y''
$(-\infty,-1)$	+	−	−	+
$(-1,1)$	+	−	+	−
$(1,\infty)$	+	+	+	+