

Stat 2507**Assignment 2 Solution****Summer 2016****Due: Monday, Jun 13, 2016 in the class @ 6:05pm-7:30pm**

You should pick up your marked assignment from the TA during your lab time
 Assignment 2 Solution has 11 questions, for a total of 100 marks

The marking scheme is as follows:

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Marks	5	12	8	5	10	10	10	10	10	10	10	100
Score:												

Activate "Enable command" from Editor on the toolbars menu when the session window is active.

Question 1) 5 marks

Use the following minitab commands to generate 8 observations:

```
MTB > set C11
DATA> 7 8 4 10 10 3 4 9
DATA> end
MTB > Base 2.
MTB > random 8 C12;
SUBC> normal 5 3.7.
MTB > formula C1 = C11+C12
MTB > name C1 "Q1Data"
```

Using Minitab for the following:

- (1 mark) Compute the Std of the data in column "Q1Data". **Std = 4.31846**
- (2 marks) Add 2.3 to each number in C1 and store them in C2. Compute the new Std: **Std = 4.31846**. How does your answer compare with that in part (a)? **Same**.
- (2 marks) Multiply the data in (a) by 4 and compute the Std: **Std = 4.31846**. How does your answer compare with that in part (a)? **4 times larger**.

Question 2) 12 marks

Suppose that X has a binomial distribution with $n = 31$ and $p = 0.7$ i.e $X \sim Bin(31, 0.7)$. Using the following Minitab command, answer the questions that follow:

```
MTB > cdf;
SUBC> binomial 31 0.7.
MTB > invcdf 0.8;
SUBC> binomial 31 0.7.
```

- (2 marks) $P(X = 15) = P(X \leq 15) - P(X \leq 14) = 0.00954 - 0.00340 = 0.0061420$
- (2 marks) $P(X < 20) = P(X \leq 19) = 0.19241$ and $P(X \leq 20) = 0.3121$
- (1 mark) $P(X > 20) = 1 - P(X \leq 20) = 1 - 0.3121 = 0.6879$
- (1 mark) $P(X \geq 20) = 1 - P(X \leq 19) = 1 - 0.1924 = 0.8076$
- (2 marks) $P(18 < X < 23) = P(X \leq 22) - P(X \leq 18) = 0.6135 - 0.1069 = 0.5066$

- (f) (2 marks) If you simulate 10000 values of X , what would be the expected number of values that are less than 15? $(10000)(0.00340) = 34$.
- (g) (2 marks) $P(X \leq k) \geq 0.8$ k between 23 and 24 but take $k = 24$ as X is discrete

Question 3) 8 marks

Suppose that X has a hypergeometric distribution with parameters $N = 32, M = 12$ and $n = 7$.

- (a) (2 marks) Calculate the probabilities below, using the following Minitab command:

```
MTB > cdf 4;
SUBC> hypergeometric 32 12 7.
```

$$P(X \leq 4) = \underline{0.9496} \text{ and } P(X > 4) = \underline{1 - P(X \leq 4) = 1 - 0.9496 = 0.0504}$$

- (b) (1 mark) Calculate the value of "a" such that $P(X \leq a) > 0.8$ for the above hypergeometric distribution, using the following Minitab command:

```
MTB > invcdf 0.8;
SUBC> hypergeometric 32 12 7.
```

$$\underline{a = 4}$$

- (c) Suppose that $X \sim N(\mu = 12, \sigma^2 = 9)$. Using the following Minitab command:

```
MTB > cdf x;
SUBC> normal 12 3.
MTB > invcdf 0.44;
SUBC> normal 12 3.
```

- i. (3 marks) Calculate the given probabilities

$$P(X > 14) = \underline{1 - P(X \leq 14) = 1 - 0.7475 = 0.2525} \text{ and}$$

$$P(13 \leq X \leq 18) = \underline{P(X \leq 18) - P(X \leq 13) = 0.9773 - 0.6306 = 0.3467}$$

- ii. (2 marks) Calculate "a" where $P(X \leq \underline{11.55}) = 0.44$.

Question 4) 5 marks

Calculate the mean μ and variance σ^2 of random variables using Minitab. Use file data "Stat 2507 A2Data-Su2016"

```
MTB > Formula C3 = SUM(C1*C2)
MTB > Formula C4 = SUM((C1^2)*C2) - C3^2
MTB > Formula C5 = SQRT(C4)
MTB > Formula C6 = C3 - 2*C5
MTB > Formula C7 = C3 + 2*C5
```

- (a) (1 mark) Obtain the mean of x , i.e., $\mu = E(X)$?

$$\underline{\mu = E(X) = \sum_x xP(X = x) = 2.22805}$$

- (b) (2 marks) Obtain the variance of x , i.e., $\sigma^2 = \text{Var}(X)$ and Std σ ?

$$\underline{\sigma^2 = \text{Var}(X) = \sum_x x^2P(X = x) - \mu^2 = 1.56089} \text{ and } \underline{\sigma = \sqrt{1.56089} = 1.24935}$$

- (c) (1 mark) Calculate the interval $[\mu \pm 2\sigma]$

$$\underline{(2.22805 \pm 2 * 1.24935) = (-0.2714, 7.27)} \text{ (Note: original text has 7.27, but calculation shows 7.27)}$$

- (d) (1 mark) What values of x are inside the interval in part (c)? **1, 2, 3, and 4**

Question 5) 10 marks

Calculate the corresponding probabilities

- (a) (5 marks) Suppose that $P(A) = 0.6$, $P(B) = 0.7$, and that events A and B are **independent**. Find $P(A \cap B)$, $P(A \cup B)$, $P(A|B)$ and $P(B|A)$.

Solution:

$$P(A \cap B) = P(A)P(B) = 0.6 \times 0.7 = 0.42 \text{ as } A \text{ and } B \text{ are independent.}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.7 - 0.42 = 0.88$$

$$P(A|B) = P(A) \text{ as } A \text{ and } B \text{ are independent.}$$

$$P(B|A) = P(B) \text{ as } A \text{ and } B \text{ are independent.}$$

- (b) (5 marks) Suppose that $P(A) = 0.4$, $P(B) = 0.5$, and that events A and B are **mutually exclusive**.

Find $P(A \cap B)$, $P(A \cup B)$, $P(A|B)$ and $P(B|A)$.

Solution:

$$P(A \cap B) = 0 \text{ as } A \text{ and } B \text{ are mutually exclusive.}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0 = 0.9$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = 0$$

Question 6) 10 marks

Suppose that a random variable X follows the distribution given by:

a	-2	-1	0	1	2
$P(X = a)$	1/8	1/4	1/4	?	1/8

Let $Y = X^2$. Find $P(Y \geq 1)$ and $E(Y)$, $\text{Var}(X)$.

Solution:

$$? = P(X = 1) = 1 - (1/8 + 1/4 + 1/4 + 1/8) = 1/4 \Rightarrow \text{since } Y = X^2 \text{ then } Y = 0, 1, 4$$

$$P(Y = 0) = P(X = 0) = 1/4, P(Y = 1) = P(X = -1) + P(X = 1) = 1/2 \text{ and}$$

$$P(Y = 4) = P(X = -2) + P(X = 2) = 1/4$$

a	0	1	4
$P(Y = a)$	1/4	1/2	1/4

$$P(Y \geq 1) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \text{ and } \mu_y = E(Y) = \sum_y yP(Y = y) = 0(1/4) + 1(1/2) + 4(1/4) = 3/2$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 = E(Y) - (E(X))^2 = 3/2 - [-1/4 - 1/4 + 1/4 + 1/4]^2 = 3/2$$

$$\sigma_y^2 = \text{Var}(Y) = \sum_y y^2P(Y = y) - \mu_y^2 = 0^2(1/4) + 1^2(1/2) + 4^2(1/4) - 9/4 = 9/4 \Rightarrow \sigma_y = 3/2$$

Question 7) 10 marks

Steve takes either a bus or the subway to go to work, with probabilities 0.25 and 0.75, respectively. When he takes the bus, he is late 40% of the time. When he takes the subway, he is late 30% of the time. If Steve is late for work on a particular day, what is the probability that he took the bus?

Solution:

Define the following events:

B : Steve takes the bus. S : Steve takes the subway. L : Steve is late for work.

It is given that $P(B) = 0.25$, $P(S) = 0.75$, $P(L|B) = 0.40$, and $P(L|S) = 0.30$.

Using Bayes' Rule,

$$P(B|L) = \frac{P(L|B)P(B)}{P(L|B)P(B)+P(L|S)P(S)} = \frac{P(L|B)P(B)}{(0.40)(0.25)+(0.30)(0.75)} = 0.3077$$

Question 8) 10 marks

Of all the Harry Potter books purchased in a recent year, about 60% were purchased for readers 14 or older. If 12 Harry Potter fans who bought books that year are surveyed, find the following probabilities.

- (a) (2 marks) Define X and its distribution.

Let X be the number of readers who are 14 or older, $X \sim Bin(n = 12, p = 0.6)$

- (b) (2½ marks) At least five of them are 14 or older.

$$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.057 = 0.943$$

- (c) (3 marks) Exactly nine of them are 14 or older.

$$P(X = 9) = C_9^{12} p^9 (1-p)^{12-9} = C_9^{12} 0.6^9 (0.4)^3 = 0.142$$

- (d) (2½ marks) Less than three of them are 14 or older.

$$P(X < 3) = P(X \leq 2) = C_0^{12} 0.6^0 (0.4)^{12} + C_1^{12} 0.6^1 (0.4)^{11} + C_2^{12} 0.6^2 (0.4)^{10} = 0.003$$

Question 9) 10 marks

In a food processing and packaging plant, there are, on average, 2 packaging machine breakdowns per week.

- (a) (2 marks) Define X and its distribution?

Let X be the packaging machine breakdowns per week, $X \sim pois(\mu = 2)$

- (b) (2½ marks) Calculate the probability that there are no more than two machine breakdowns in a given week.

Solution:

$$P(X \leq 2) = \frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} = 0.1353 + 0.2707 + 0.2707 = 0.6767$$

- (c) (3 marks) What is the probability that there are no machine breakdowns in a given week?

Solution:

$$P(X = 0) = \frac{2^0 e^{-2}}{0!} = 0.1353 \approx 0.14$$

- (d) (2½ marks) In a random sample of 26 machines, what is the probability that there are no machine breakdowns in at least one of them in a given week?

Solution:

Let Y be the number of machines without breakdowns, $Y \sim Bin(26, P(X = 0) = 0.14)$

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - C_0^{26} (0.14)^0 (1 - 0.14)^{26-0} = 1 - 0.0198 = 0.9802$$

Question 10) 10 marks

Heidi prepares for an exam by studying a list of 15 problems. She can solve 9 of them. For the exam, the instructor selects 7 questions at random from the list of 15. Let X be the number problems that can be solved by Heidi.

- (a) (6 marks) If you sample the problems without replacement, what is the probability distribution of X ? What is the probability that Heidi can solve all 7 problems on the exam?

Solution:

$$X \sim HG(N = 15, M = 9, n = 7) \Rightarrow P(X = 7) = \frac{C_7^9 C_8^6}{C_{15}^7} = 0.0056$$

- (b) (4 marks) If you sample the problems with replacement, what is the probability distribution of X ? What is the probability that Heidi can solve all 7 problems on the exam?

Solution:

$$X \sim Bin(n = 7, p = \frac{M}{N} = 0.6) \Rightarrow P(X = 7) = C_7^7 (0.6)^7 (0.4)^0 = 0.028$$

Question 11) 10 marks

During off hours, cars arrive at a highway service station at an average rate of 5 cars per 10 minutes.

- (a) (1 mark) Define the r.v X and its distribution

Solution:

Let X be the number of cars arriving at the service station, $X \sim pois(\mu)$

- (b) Find the probability that during the:

- i. (1 mark) next minute three cars will arrive?

Solution:

$$\mu = 5/10 = 0.5 \Rightarrow P(X = 3) = \frac{0.5^3}{3!} e^{-0.5} = 0.012$$

- ii. (1 mark) next five minutes three cars will arrive?

Solution:

$$\mu = \frac{5}{10}(5) = 2.5 \Rightarrow P(X = 3) = \frac{2.5^3}{3!} e^{-2.5} = 0.214$$

- iii. (2 marks) next five minutes at least three cars will arrive?

Solution:

$$\mu = \frac{5}{10}(5) = 2.5 \Rightarrow P(X \geq 3) = \sum_{k=3}^{\infty} \frac{2.5^k}{k!} e^{-2.5} = 1 - \sum_{k=0}^2 \frac{2.5^k}{k!} e^{-2.5} = 1 - 0.544 = 0.456$$

- iv. (2 marks) next five minutes at most three cars will arrive?

Solution:

$$\mu = \frac{5}{10}(5) = 2.5 \Rightarrow P(X \leq 3) = \sum_{k=0}^3 \frac{2.5^k}{k!} e^{-2.5} = 0.758$$

- v. (1 mark) next five minutes less than three cars will arrive?

Solution:

$$\mu = \frac{5}{10}(5) = 2.5 \Rightarrow P(X < 3) = \sum_{k=0}^2 \frac{2.5^k}{k!} e^{-2.5} = 0.544$$

- vi. (2 marks) next five minutes more than three cars will arrive?

Solution:

$$\mu = \frac{5}{10}(5) = 2.5 \Rightarrow P(X > 3) = \sum_{k=4}^{\infty} \frac{2.5^k}{k!} e^{-2.5} = 1 - \sum_{k=0}^3 \frac{2.5^k}{k!} e^{-2.5} = 1 - 0.758 = 0.242$$