

Last Name, First Name
Student ID
Tutorial Number

Term Test 2. You have 90 minutes. Answer the questions in the booklets provided. Show all your work, you will not receive any points for unsubstantiated answers.

(9 marks) 1. A firm has production function $Q = L^{1/2}K^{1/2}$. The wage rate is \$1 and the rental rate on capital is \$4.

- Is the marginal product of labour diminishing? Show mathematically.
- Derive the conditional input demand functions for L and K. Show your work.
- Find the combination of inputs that minimizes the cost of producing 1000 units of output. Graph your result on a isoquant-isocost diagram.
- Assume that the firm intending to produce 1000 units of output, has already purchased the amount of capital that you found in part (c). An unexpected increase in demand has made it desirable to produce 1500 units instead. Graphically and mathematically show the firm's short-run and long-run adjustments to the change in demand conditions using your diagram above. Make sure to label the isocosts associated with producing 1500 units of output in the short-run and the long-run.

2. Answer true or false and explain your answer with a isocost-isoquant diagram (in both part (a) and (b), assume the firm has convex isoquants and is using positive amounts of two inputs, labour and capital).

(4 marks) a. At the current bundle that the firm is using to produce a given level of output, if the marginal rate of technical substitution is not equal to the ratio of factor prices, then it is possible to reduce the cost of producing that level of output.

(5 marks) b. If the price of capital increases, the firm will use less capital and the cost of producing a given level of output may increase or decrease.

(10 marks) 3. A plant's production function is given by $Q = L + K$ that is inputs are perfect substitutes. The prices of inputs respectively are; $w=2$ and $r=3$.

- Does this production function exhibit increasing, constant or decreasing returns to scale? Show mathematically.
- What are the firm's conditional input demand functions at the input prices given above? What is the optimal bundle to produce 10 units of output? Show the optimal bundle on a isoquant-isocost diagram. Explain how you found the optimal bundle diagrammatically.
- Find the firm's output expansion path and draw it on adiagram. Explain in words what an output expansion path is.
- Derive the firm's long-run cost function at the input prices given above.
- Consider this firm in the short-run where it is currently employing 5 units of K (L is variable). Derive the firm's short-run cost function. Draw it together with long-run cost curve
- Now suppose instead that the firm in the short-run is currently employing 5 units of L (K is variable). Derive the firm's short-run cost function. Draw it together with long-run cost curve.

(8 marks) 4. George consumes leisure (L hours per day) and other goods (Y units per day). His preferences over L and Y can be described by convex indifference curves. The hourly wage rate is w and the price of Y is 1. There are 24 hours in a day to allocate between leisure and work. Currently he pays no tax.

- State and draw George's budget constraint with L on the horizontal axis and Y on the vertical axis.
- Suppose the government decides to impose an income tax of 20% on George's income (that is; his after-tax hourly wage rate becomes $0.8w$). Show the effect of this tax on the budget constraint on your diagram above.
- If leisure is a normal good for George, will the tax necessarily result in a decline in work time for George? Explain using your diagram above. Make sure that on your diagram you identify the income effect and the substitution effect for this change in the tax rate and also describe what the income effect and the substitution effect measure in this case.