## PART A (35 marks)

**NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE CODED ON THE SCANTRON SHEET. ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.**

*1 mark* A1. Among a certain group of students at a local college, students eat different kinds of pizzas:

- 10 eat cheese pizzas
- 7 eat pepperoni pizzas
- 3 eat neither cheese nor pepperoni pizzas
- 8 don't eat vegetarian pizzas
- 3 eat pepperoni pizzas but don't eat vegetarian pizzas
- 4 eat neither cheese nor vegetarian pizzas
- 3 eat none of these kinds of pizzas
- 3 eat cheese and vegetarian pizzas, but not pepperoni pizzas

How many students eat all 3 types of pizzas?

| | | | | |
|---|---|---|---|---|
| A: 1 | B: 2 | C: 3 | D: 6 | E: 11 |

*1 mark* A2. Let $A$ and $B$ be subsets of a universal set $U$ with $n(U) = 50$, $n(A) = 25$, and $n(A \cup B) = 45$. Find $n(A \cup B^c)$.

| | | | | |
|---|---|---|---|---|
| A: 15 | B: 20 | C: 25 | D: 30 | E: 40 |

*1 mark* A3. How many subsets of the set $S = \{A, B, C, D, E, F, G, H\}$ contain $A$ and $B$ but not $G$?

| | | | | |
|---|---|---|---|---|
| A: $2^4 - 1$ | B: $2^4$ | C: $2^6$ | D: $2^7$ | E: $2^5$ |

**Use the following information for Questions A4 and A5.**

An auto dealer has 15 different cars. 4 of the cars are made by Toyota, 6 are made by Chevrolet and 5 are made by Ford. The cars are to be displayed in a line side-by-side in front of the sales office, with all facing the same direction. They are to be grouped by maker, with all Toyotas together, all Chevrolets together and all Fords together.

*1 mark* A4. In how many ways can the 15 cars be lined up?

| | | | | |
|---|---|---|---|---|
| A: 3!4!5!6! | B: 15! | C: 15!4!5!6! | D: 15! − 4!5!6! | E: 15! − 4!6!2! |

*1 mark* A5. In how many ways can the 15 cars be lined up if the Toyotas must be together at the left end of the line?

| | | | | |
|---|---|---|---|---|
| A: 2(4!5!6!) | B: 4!5!6! | C: 3!4!5!6! | D: 4(11!) | E: 4!11! |

*1 mark* A6. Find the number of distinct permutations of all of the letters of the word *STATISTICS*, if the 3 $S$'s must be grouped together.

| | | | | |
|---|---|---|---|---|
| A: 8! | B: 3!8! | C: $\frac{10!}{3!3!2!}$ | D: $\binom{10}{3}$ | E: $\frac{8!}{3!2!}$ |

*1 mark* A7. In how many ways can all of 10 **identical** candies be given to James, Andrew and Grace?

| | | | | |
|---|---|---|---|---|
| A: $\binom{13}{3}$ | B: $\binom{12}{9}$ | C: $\binom{12}{10}$ | D: $\binom{10}{3}$ | E: $\binom{13}{4}$ |

*1 mark* A8. Kyle has 2 nickels and 5 dimes in his pocket. He accidentally loses two of these coins when he pulls his car keys out of his pocket. What is the probability that the coins Kyle loses are one nickel and one dime?

| | | | | |
|---|---|---|---|---|
| A: $\frac{1}{7}$ | B: $\frac{2}{7}$ | C: $\frac{5}{21}$ | D: $\frac{10}{21}$ | E: $\frac{11}{21}$ |

*1 mark* A9. A fair die is tossed twice. What is the probability that the sum of the two numbers obtained is at least 10?

| | | | | |
|---|---|---|---|---|
| A: $\frac{1}{36}$ | B: $\frac{3}{36}$ | C: $\frac{6}{36}$ | D: $\frac{10}{36}$ | E: $\frac{3}{12}$ |

*1 mark* A10. What is the probability that a bridge hand (13 cards drawn at random from a standard deck of 52 cards) contains exactly 1 Ace and exactly 2 Kings?

| | | |
|---|---|---|
| A: $\dfrac{\binom{4}{1}\binom{4}{2}}{\binom{52}{5}}$ | B: $\dfrac{\binom{4}{1}\binom{4}{2}\binom{44}{10}}{\binom{52}{13}}$ | C: $\dfrac{\binom{4}{1}\binom{4}{2}}{\binom{52}{13}}$ |
| D: $\dfrac{\left\{\binom{4}{1}+\binom{4}{2}\right\}\times\binom{49}{10}}{\binom{52}{13}}$ | E: $\dfrac{\binom{4}{1}\binom{4}{2}\binom{49}{10}}{\binom{52}{13}}$ | |

*1 mark* A11. Let $G$ and $H$ be events with $Pr[G] = 0.5$, $Pr[H] = 0.4$ and $Pr[G \cup H] = 0.8$. What is $Pr[G^c \cap H]$?

| | | | | |
|---|---|---|---|---|
| A: 0.4 | B: 0.3 | C: 0.5 | D: 0.6 | E: 0.7 |

*1 mark* A12. Let $E$ and $F$ be events in the same sample space with $Pr[E] = 0.2$, $Pr[F] = 0.5$, and $Pr[E \cup F] = 0.6$. What is $Pr[F \mid E]$?

| | | | | |
|---|---|---|---|---|
| A: 0.1 | B: 0.2 | C: 0.3 | D: 0.4 | E: 0.5 |

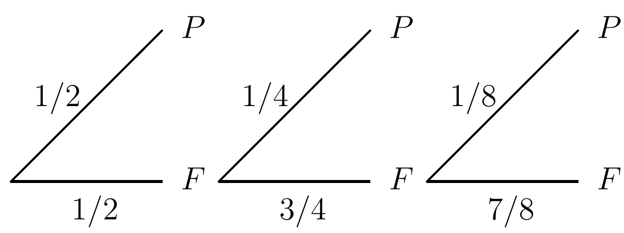
*1 mark* A13. Let $E$ and $F$ be events in the same sample space with $Pr[E] = 0.4$, and $Pr[E \cap F] = 0.3$. If $E$ and $F$ are independent events, what is $Pr[E \cup F]$?

| | | | | |
|---|---|---|---|---|
| A: 0.7 | B: 0.6 | C: 0.85 | D: 0.4 | E: Cannot be determined. |

*1 mark* A14. If $A$ and $B$ are events such that $Pr[A] = 0.3$, $Pr[B] = 0.25$ and $Pr[A \mid B] = 0.4$, what is $Pr[A \cap B]$?

| | | | | |
|---|---|---|---|---|
| A: 0.05 | B: 0.1 | C: 0.15 | D: 0.2 | E: 0.45 |

*1 mark* A15. Seymour will have three attempts at passing the final exam in a certain course. That is, if Seymour passes, he is given credit for the course, but if he fails he can try again, writing a different final exam, but with no more than three attempts to pass an exam allowed, so if he fails 3 exams, he fails the course. Seymour has a 50% chance of passing the exam on his first attempt. However, Seymour becomes discouraged when he fails. Any time Seymour fails the exam, he is only one half as likely to pass on his next attempt as he was on the attempt he just failed.
The probability tree shown here models this stochastic process, where $P$ denotes passing an exam and $F$ denotes failing an exam:




Find the probability that Seymour does not pass the course.

| | | | | |
|---|---|---|---|---|
| A: $\frac{43}{64}$ | B: $\frac{1}{2}$ | C: $\frac{40}{64}$ | D: $\frac{21}{64}$ | E: $\frac{41}{64}$ |

*1 mark* A16. Let $E$ and $F$ be events in a sample space with $Pr[E] = 0.4$ and $Pr[F] = 0.5$. If $E$ and $F$ are mutually exclusive events, what is $Pr[E \cap F]$?

| | | | | |
|---|---|---|---|---|
| A: 0.2 | B: 0 | C: 0.9 | D: 0.8 | E: Cannot be determined. |

*1 mark* A17. Customers arrive at random at the Burger King drive-through window. After serving millions of such customers, management has determined that (on average) 40% of all customers order onion rings in the lunch period. On Tuesday at noon, four customers are lined up at the drive-through window. What is the probability that all four customers order onion rings, given that at least one of the four does?

| | | | | |
|---|---|---|---|---|
| A: $\frac{(0.6)^4}{1-(0.4)^4}$ | B: $(0.4)^4$ | C: $\frac{(0.4)^4}{1-(0.6)^4}$ | D: $\frac{(0.4)^4}{(0.6)^4}$ | E: $(0.6)^3$ |

*1 mark* A18. $X$ is a discrete random variable whose probability distribution function (pdf) is given here:

| $x$ | $\Pr[X = x]$ |
|---|---|
| 12 | 1/6 |
| 9 | 1/3 |
| $-4$ | 1/2 |

What is the value of $E(X)$?

| | | | | |
|---|---|---|---|---|
| A: 25 | B: 17 | C: 3 | D: 7 | E: $-2$ |

*1 mark* A19. $X$ is a discrete random variable whose probability distribution function (pdf) is given here:

| $x$ | $\Pr[X = x]$ |
|---|---|
| $-1$ | .3 |
| 0 | .5 |
| 1 | .2 |

The mean of $X$ is $\mu(X) = -0.1$. Find the variance, $V(X)$.

| | | | | |
|---|---|---|---|---|
| A: 0.50 | B: 0.51 | C: 0.40 | D: 0.60 | E: 0.49 |

*1 mark* A20. $X$ is a discrete random variable whose probability distribution function (pdf) is given here:

| $x$ | $\Pr[X = x]$ |
|---|---|
| 0 | .5 |
| $10 + a$ | .3 |
| $10 - a$ | .2 |

If the expected value of $X$ is $E(X) = 6$, find the value of $a$.

| A: 11 | B: 0 | C: 10 | D: 2 | E: 1 |
|---|---|---|---|---|

*1 mark* A21. Alanna tosses an ordinary six-sided die once. If she rolls a 3, she wins \$8; otherwise she wins \$2. What is the expected value of her winnings?

| A: \$10 | B: \$2 | C: \$3 | D: \$6 | E: \$18 |
|---|---|---|---|---|

*1 mark* A22. $X$ is a discrete random variable with mean $\mu(X) = 5$ and standard deviation $\sigma(X) = 3$. Find $E(X^2)$.

| A: 34 | B: 14 | C: 16 | D: 2 | E: 8 |
|---|---|---|---|---|

*1 mark* A23. $X$ is a discrete random variable with variance $V(X) = 25$. Find $\sigma(-X + 1)$.

| A: $-5$ | B: $-4$ | C: 4 | D: 5 | E: 16 |
|---|---|---|---|---|

*1 mark* A24. $X$ is a discrete random variable with variance $V(X) = 2$. Find $V(2X - 3)$.

| A: 1 | B: 16 | C: 8 | D: 17 | E: 3 |
|---|---|---|---|---|

*1 mark* A25. $X$ and $Y$ are independent random variables such that $E(X) = 4$ and $E(Y) = 3$. Which of the following **must always be true**?

$$\begin{aligned} &(i) & E(X-Y) &= 1 \\ &(ii) & E(XY) &= 12 \\ &(iii) & E(3X+1) &= 13 \end{aligned}$$

| A: all of them | B: none of them | C: (ii) and (iii) only |
|---|---|---|
| D: (i) and (ii) only | E: (i) and (iii) only | |

*1 mark* A26. $X$ and $Y$ are independent random variables such that $V(X) = 10$ and $V(Y) = 6$. Which of the following **must always be true**?

$$\begin{aligned} &(i) & V(X+Y) &= 16 \\ &(ii) & V(X-Y) &= 4 \\ &(iii) & \sigma(X-Y) &= 4 \end{aligned}$$

| A: all of them | B: none of them | C: (ii) and (iii) only |
|---|---|---|
| D: (i) and (ii) only | E: (i) and (iii) only | |

*1 mark* A27. $X$ and $Y$ are independent random variables such that $V(X) = 1$ and $V(Y) = 4$. Find $\sigma(3X + 2Y)$.

| | | | | |
|---|---|---|---|---|
| A: 11 | B: $\sqrt{11}$ | C: 73 | D: 5 | E: $\sqrt{3} + 2\sqrt{2}$ |

*1 mark* A28. The probability density function $f(x)$ of a continuous random variable $X$ is defined by $f(x) = kx + 2$ if $0 \le x \le 1$ and $f(x) = 0$ otherwise. Find the value of $k$.

| | | | | |
|---|---|---|---|---|
| A: 2 | B: 1 | C: $-2$ | D: $\frac{1}{2}$ | E: $-\frac{1}{2}$ |

*1 mark* A29. $X$ is a discrete random variable whose possible values are 9, 12, 15, 18 and 21. Continuous random variable $Y$ is a good approximation for $X$. Which one of the following gives the value of $Pr[X = 15]$?

| | | |
|---|---|---|
| A: $Pr[12 < Y < 18]$ | B: $Pr[13.5 < Y < 16.5]$ | C: $Pr[Y = 15]$ |
| D: $Pr[14.5 < Y < 15.5]$ | E: $Pr[11.5 < Y < 19.5]$ | |

In the following questions, $Z$ is the standard normal random variable. Use the table provided at the back of the exam paper for questions A30 through A35.

*1 mark* A30. Find $Pr[0 \le Z \le 2.21]$.

| | | | | |
|---|---|---|---|---|
| A: .9864 | B: .0136 | C: .5000 | D: .4864 | E: .5136 |

*1 mark* A31. Find $Pr[Z > -2.17]$.

| | | | | |
|---|---|---|---|---|
| A: .9850 | B: .6950 | C: .5150 | D: .4850 | E: .0150 |

*1 mark* A32. Find $k$ if $Pr[Z < k] = .9972$.

| | | | | |
|---|---|---|---|---|
| A: 0.84 | B: $-2.77$ | C: 2.77 | D: $-0.84$ | E: None of A, B, C or D. |

*1 mark* A33. Find $k$ if $Pr[k < Z < 0] = .4922$.

| | | | | |
|---|---|---|---|---|
| A: .7823 | B: .0078 | C: $-1.21$ | D: $-2.42$ | E: $-1.78$ |

*1 mark* A34. $X$ is a normal random variable with mean $\mu = 2$ and standard deviation $\sigma = 4$. Find $Pr[3 < X < 4]$.

| | | | | |
|---|---|---|---|---|
| A: .6915 | B: .5987 | C: .0010 | D: .3085 | E: .0928 |

*1 mark* A35. $X$ is a normal random variable with mean $\mu = 15$ and standard deviation $\sigma = 3$. Find $k$ if $Pr[X < k] = .8413$.

| | | | | |
|---|---|---|---|---|
| A: $-1$ | B: 1 | C: 6 | D: 12 | E: 18 |

## PART B (15 marks)

**SHOW YOUR WORK FOR ALL QUESTIONS IN PART B.**

*4 marks* B1. A box contains 2 coins. One is a regular fair coin and the other coin has a head on each side. A coin is selected at random from the box and that coin is flipped. Heads comes up.

(a) What is the probability that the fair coin was flipped?

(b) Suppose that the same coin is flipped again and heads appears again. What is the probability that the coin is the fair coin?

*3 marks* B2. The Coffee House is holding a Tear the Tab contest where customers who order coffee have a chance to win a prize by tearing off a tab on the cup. The contest is advertising that there is a 2 in 10 chance of winning a prize.

(a) If a customer orders a coffee once every day for a one-week (7 day) period during the contest, what is the probability that the customer will win at least one prize?
**DO NOT SIMPLIFY YOUR ANSWER.**

(b) What is the minimum number of times that a customer must order coffee in order to have at least a 40% chance of winning at least one prize?

*3 marks* B3. Discrete random variable $X$ assumes only integer values. Let $F(x)$ be the cumulative distribution function associated with $X$. You are given that $F(5) = 0.42$, $F(6) = 0.45$, $F(7) = 0.53$, $F(8) = 0.68$ and $F(9) = 0.81$.

(a) Find $Pr[X = 6]$.

(b) Find $Pr[7 < X < 9]$.

(c) Find $Pr[X \geq 8]$.

*5 marks* B4. 90% of the students at a very large university are against an increase in tuition fees. A sample of 100 students is selected at random from the student population at the university. Let $X$ be the number of students in the sample who are against an increase in tuition fees.

(a) Find the expected value, $E(X)$.

(b) Find the standard deviation, $\sigma(X)$.

(c) Use a normal approximation and the table provided at the back of the exam paper to find $Pr[92 \leq X \leq 97]$.

**The Cumulative Distribution Table for the Standard Normal Random Variable $Z$**

$Pr[Z < k]$

| $k$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

Instructor's Name (**Print**) ____________________

Student's Name (**Print**) ____________________

Student's Signature ____________________

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 1228B Final Examination

Thursday, April 21, 2011 **Code 111** 9:00 a.m. - 12:00 noon

INSTRUCTIONS

1. Fill in the top of this page **and also the next page** completely.
2. Fill in the top of the scantron card completely. **You MUST both print AND code** your Student Number, Section Number (below) and Exam Code (above).
3. CALCULATORS AND NOTES ARE NOT PERMITTED.
4. DO NOT UNSTAPLE THE BOOKLET. The 3 blank pages at the back may be removed and used for rough work.
5. There are two parts to this examination: PART A (35 marks) in multiple choice format and PART B (15 marks) in show your work format.
6. In Part A, **circle** the correct answer to each question **on this paper** AND fill in the appropriate box on the **scantron** card with an HB pencil. You **will not** be given extra time at the end of the exam to fill in your scantron card.
7. In Part B, show all your work in the space provided.
8. Questions are printed on both sides of the paper. They begin on Page 1 and continue to Page 9. Be sure that your booklet is complete.
9. The table showing the Cumulative Distribution for the Standard Normal Random Variable is given on Page 10 and should be used where needed.
10. You must hand in the question paper, scantron card, and all rough work sheets.
11. Circle your section in the list below.

| Name | Time/College | Section |
|---|---|---|
| Kossovskiy | 10:30 MWF | 001 |
| Olds | 12:30 MWF | 002 |
| Robertson | 2:30 MWF | 003 |
| Florence | Brescia | 530 |
| Kuzmin | Huron | 550 |
| Adcock | Kings | 570 |

12. TOTAL MARKS = 50.

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

| PAGE | MARK |
|---|---|
| 1–5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| TOTAL | |