

[6] 1. (a) Find the following limits

$$(i) \lim_{x \rightarrow 2} \frac{3x^2 + 2x - 1}{x^2 + 3x + 2} = \frac{3(2)^2 + 2(2) - 1}{2^2 + 3(2) + 2} = \frac{12 + 4 - 1}{4 + 6 + 2} = \frac{15}{12} = \frac{5}{4}$$

$$(ii) \lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x^2 + x - 6} = \frac{0}{0}$$

(b) Where is the function $n(x) = \frac{x-2}{(x-3)(x+1)}$ continuous? $x \in \mathbb{R} \setminus \{-1, 3\}$

$$ii) \lim_{x \rightarrow 2} \frac{(2x+1)(x-2)}{(x+3)(x-2)} = \lim_{x \rightarrow 2} \frac{2x+1}{x+3} = \frac{2(2)+1}{2+3} = \frac{5}{5} = 1$$

[5] 2. Find the derivative $f'(x)$ of the functions $f(x)$: (Do not simplify)

$$(a) f(x) = 3x^4 - 4x^3 + x - 2 \quad f'(x) = 12x^3 - 12x^2 + 1$$

$$(b) f(x) = \frac{x^{-7}}{7} + \sqrt[3]{x} = \frac{1}{7}x^{-7} + x^{1/3} \quad f'(x) = -x^{-8} + \frac{1}{3}x^{-2/3}$$

[9] 3. Find $\frac{dy}{dx}$ (do not simplify):

$$(a) y = \frac{e^{2x}}{x^2 - 4} \quad u = e^{2x} \quad v = x^2 - 4 \quad f'(x) = \frac{2e^{2x}(x^2 - 4) - e^{2x}(2x)}{(x^2 - 4)^2}$$

$$u' = 2e^{2x} \quad v' = 2x$$

$$(b) y = \frac{1}{3x^2 + 5} \quad f'(x) = \frac{1}{3x^2 + 5} \cdot 6x$$

$$(c) y = (2x^2 + 1)^3 (4x + 6)^2 \quad u = (2x^2 + 1)^3 \quad v = (4x + 6)^2$$

$$u' = 3(2x^2 + 1)^2 \cdot 4x \quad v' = 2(4x + 6) \cdot 4$$

$$(d) y = (4 + x^2 \ln x)^3 \quad f'(x) = 3(2x^2 + 1)^2 4x (4x + 6)^2 + (2x^2 + 1)^3 2(4x + 6) 4$$

$$u = x^2 \quad v = \ln x$$

$$u' = 2x \quad v' = \frac{1}{x}$$

$$f'(x) = 3(4 + x^2 \ln x)^2 \cdot \left(2x \ln x + x^2 \left(\frac{1}{x} \right) \right)$$

[7] 4. Let $f(x) = 3x^4 - 6x^2 - 7$ a) $f'(x) = 12x^3 - 12x$ $f'(2) = 12(2)^3 - 12(2) = 72$

(a) Find the slope of the tangent line to the curve when $x = 2$

(b) Find the equation of the tangent line to the curve when $x = 2$

$$f(2) = 3(2)^4 - 6(2)^2 - 7 = 17 \quad (2, 17)$$

$$y = mx + b$$

$$17 = 72(2) + b$$

$$-127 = b$$

$$y = 72x - 127$$

[13] 5. Let $f(x) = (x-2)(x^2 - 4x - 8) = x^3 - 4x^2 - 8x - 2x^2 + 8x + 16$
 $= x^3 - 6x^2 + 16$

Find

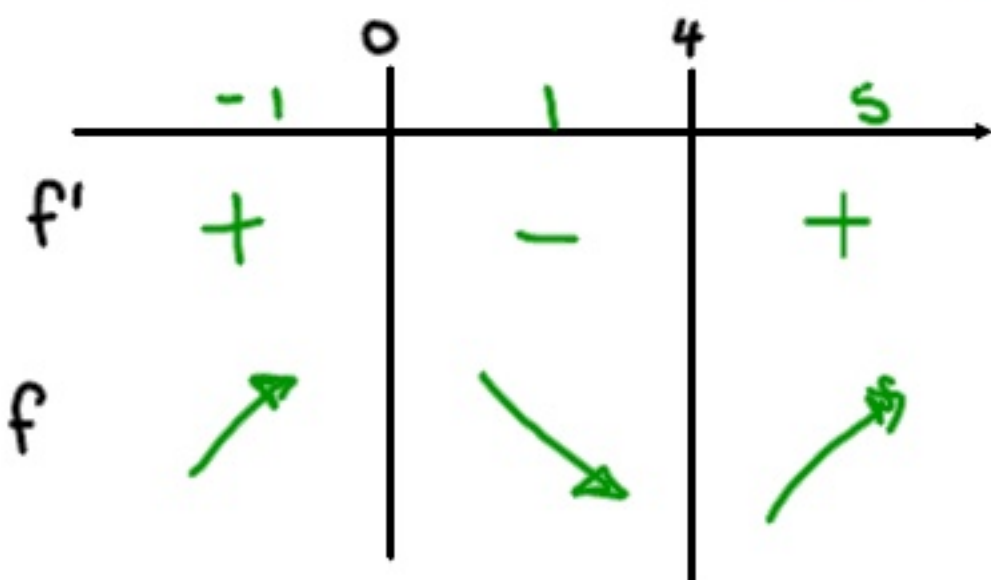
- (a) the critical and inflection points of $f(x)$
- (b) the intervals where $f(x)$ is increasing and where it is decreasing
- (c) the intervals on which $f(x)$ is concave up and on which it is concave down
- (d) use the above to sketch the graph

a) $f' = 3x^2 - 12x = 0$
 $3x(x-4) = 0$
 $x=0 \quad x=4$

$f(0) = (0)^3 - 6(0)^2 + 16 = 16$

$f(4) = (4)^3 - 6(4)^2 + 16 = -16$

C.POINTS: $(0, 16)$ & $(4, -16)$



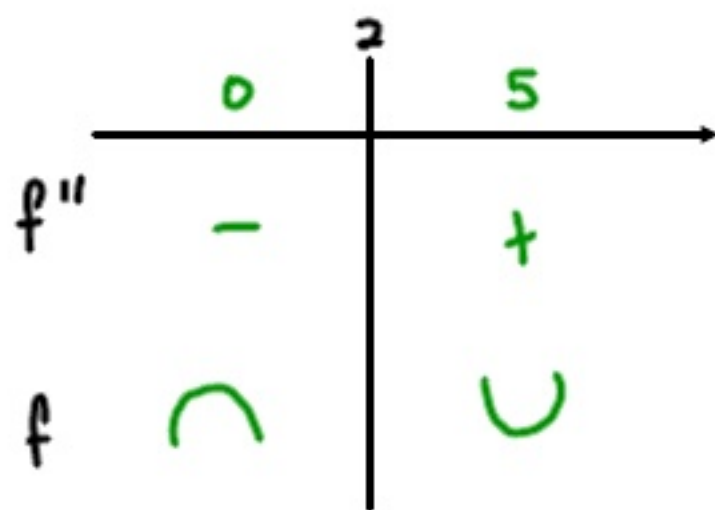
INT OF \uparrow : $x \in (-\infty, 0) \cup (4, \infty)$

INT OF \downarrow : $x \in (0, 4)$

$f'' = 6x - 12 = 0$
 $6x = 12$
 $x = 2$

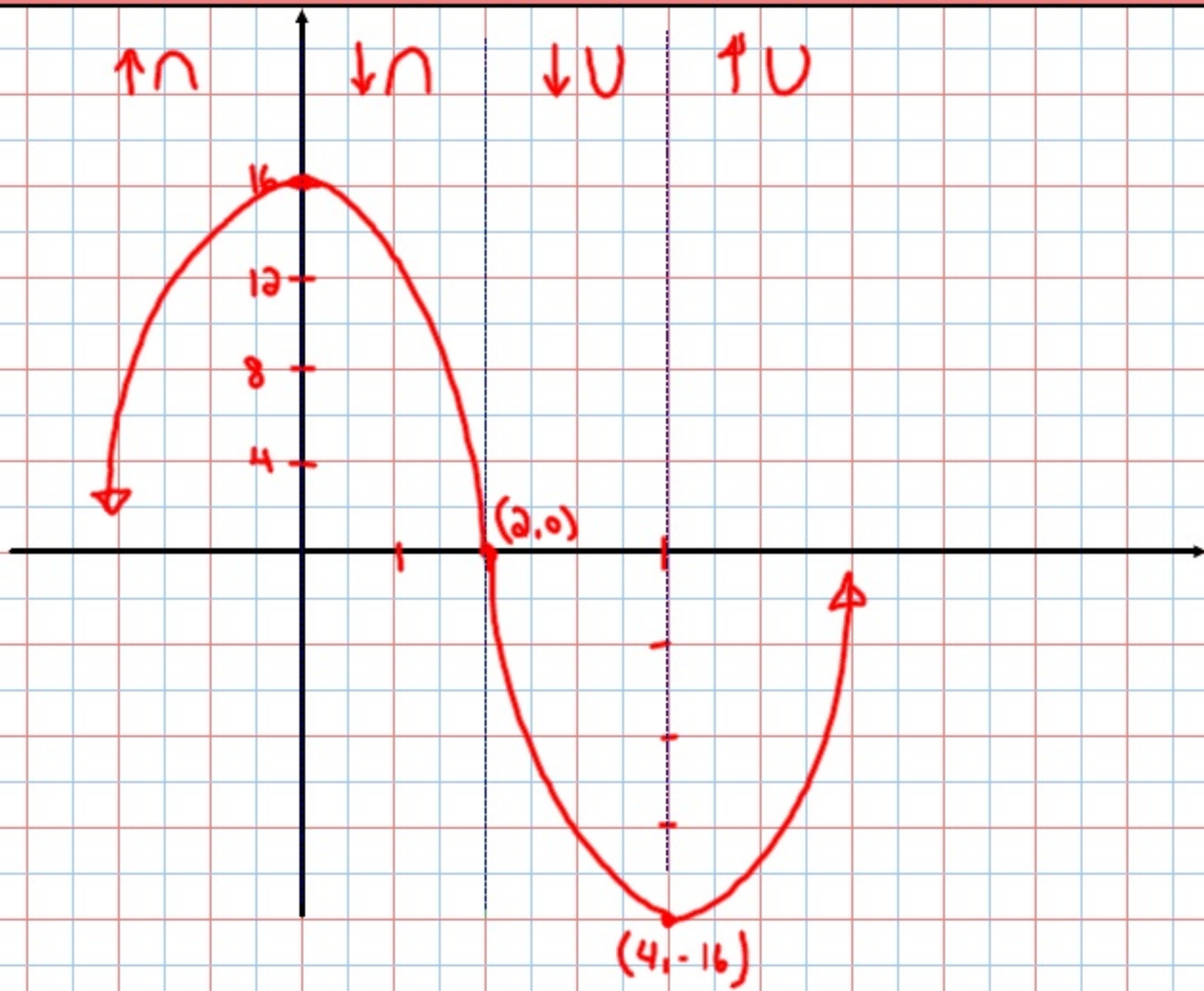
$f(2) = 2^3 - 6(2)^2 + 16 = 0$

I.POINTS: $(2, 0)$



CONCAVE UP ON: $x \in (2, \infty)$

CONCAVE DOWN: $x \in (-\infty, 2)$



- [8] 6. A student center sells 1600 cups of coffee per day at the price of \$2.40 per cup. A market survey shows that for every \$0.05 reduction in price per cup, 50 more cups of coffee will be sold.

How much should the student center charge for a cup of coffee in order to maximize revenue?

$$R = (1600 + 50x)(2.40 - 0.05x) = 3840 + 40x - 2.5x^2$$

$$R' = 0: \quad 40 - 5x = 0$$
$$x = 8$$

$$P = 2.40 - 0.05x = 2.40 - 0.05(8) = \$2.00$$

[7] 7. Find the absolute extrema of the function $f(x) = x^3 - 6x^2 + 9x - 6$ on the interval $[-1, 5]$.

$$\begin{aligned} f' = 0: \quad 3x^2 - 12x + 9 &= 0 & f(-1) &= (-1)^3 - 6(-1)^2 + 9(-1) - 6 = -22 \\ x^2 - 4x + 3 &= 0 & f(1) &= (1)^3 - 6(1)^2 + 9(1) - 6 = -2 \\ (x-3)(x-1) &= 0 & f(3) &= (3)^3 - 6(3)^2 + 9(3) - 6 = -6 \\ x = 3 \quad x = 1 & & f(5) &= (5)^3 - 6(5)^2 + 9(5) - 6 = 14 \end{aligned}$$

max of 14 at $x=5$, min of -22 at $x=-1$

- [3] 8. If interest is compounded continuously and the interest rate is 6.4%, how long will it take for money invested to double?

$$A = Pe^{rt}$$

$$2P = Pe^{0.064t}$$

$$\ln 2 = 0.064t \rightarrow t = 10.83 \text{ yrs}$$

- [10] 9. Find the equation(s) of the tangent line(s) to the graph of $y^2 - xy - 6 = 0$ at the point(s) with $x = 1$.

find point: plug-in $x=1$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0 \quad \text{OR} \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = 3 \quad y = -2$$

2 points! $(1, -2)$ & $(1, 3)$

find Slope:

$$y^2 - xy - 6 = 0$$

$$u = -x \quad v = y$$

$$u' = -1 \quad v = y'$$

$$2yy' - y - xy' = 0$$

plug-in $(1, -2) \rightarrow 2(-2)y' - (-2) - (1)y' = 0$ $-4y' + 2 - y' = 0$ $y' = \frac{2}{5}$

$(1, 3) \rightarrow 2(3)y' - 3 - (1)y' = 0$ $6y' - 3 - y' = 0$ $y' = \frac{3}{5}$

2 lines: ① $(1, -2)$ $m = 2/5$

② $(1, 3)$ $m = 3/5$

$$y = mx + b$$

$$-2 = \frac{2}{5}(1) + b$$

$$-\frac{12}{5} = b$$

$$y = \frac{2}{5}x - \frac{12}{5}$$

$$y = mx + b$$

$$3 = \frac{3}{5}(1) + b$$

$$\frac{12}{5} = b$$

$$y = \frac{3}{5}x + \frac{12}{5}$$

[10] 10. Compute these antiderivatives:

$$(a) \int (3x^5 - 2x^3 - 7) dx = \frac{3x^6}{6} - \frac{2x^4}{4} - 7x + C = \frac{x^6}{2} - \frac{x^4}{2} - 7x + C$$

$$(b) \int \frac{e^{-7x}}{2 + e^{-7x}} dx \quad \begin{array}{l} u = 2 + e^{-7x} \\ dx = \frac{du}{-7e^{-7x}} \end{array} \int \frac{\cancel{e^{-7x}}}{u} \cdot \frac{du}{-7\cancel{e^{-7x}}} = -\frac{1}{7} \int \frac{1}{u} du = -\frac{1}{7} \ln|u| + C = -\frac{1}{7} \ln|2 + e^{-7x}| + C$$

$$(c) \int \frac{x^2}{\sqrt{x-3}} dx$$

[10] 11. Evaluate the integrals:

$$(a) \int_0^1 (x^3 - 4) dx = \left. \frac{x^4}{4} - 4x \right|_0^1 = \left(\frac{1}{4} - 4 \right) - (0 - 0) = -3.75$$

$$(b) \int_2^7 \frac{1}{x-5} dx \quad \begin{array}{l} u = x-5 \quad x=2 \quad u=-3 \\ dx = du \quad x=7 \quad u=2 \end{array} \int_{-3}^2 \frac{1}{u} du = \ln|u| \Big|_{-3}^2 = \ln 2 - \ln 3 = \ln \frac{2}{3}$$

$$(c) \int_4^6 \sqrt{x-3} dx \quad \begin{array}{l} u = x-3 \quad x=4 \quad u=1 \\ dx = du \quad x=6 \quad u=3 \end{array} = -0.40547$$

$$= \int_1^3 u^{1/2} du = \left. \frac{2u^{3/2}}{3} \right|_1^3 = \frac{2(3)^{3/2}}{3} - \frac{2}{3} = 2.7974$$

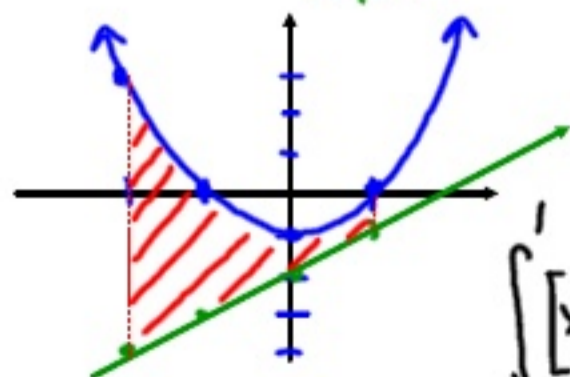
$$\int \frac{x^2}{\sqrt{x-3}} dx \quad \begin{array}{l} u = x-3 \quad x = u+3 \\ dx = du \end{array} \int \frac{x^2}{u^{1/2}} du = \int \frac{(u+3)^2}{u^{1/2}} du = \int \frac{(u+3)(u+3)}{u^{1/2}} du$$

$$= \int \left(\frac{u^2 + 6u + 9}{u^{1/2}} \right) du = \int \left(u^{3/2} + 6u^{1/2} + 9u^{-1/2} \right) du = \frac{2u^{5/2}}{5} + \frac{12u^{3/2}}{3} + 18u^{1/2} + C$$

$$= \frac{2}{5}(x-3)^{5/2} + 4(x-3)^{3/2} + 18(x-3)^{1/2} + C$$

[10] 12. Find the area bounded by the graphs of $f(x) = x^2 - 1$ and $y = x - 2$ over the interval $-2 \leq x \leq 1$.

Step 1



x	y
-2	3
-1	0
0	-1
1	0

x	y
-2	-4
-1	-3
0	-2
1	-1

$$\int_{-2}^1 [x^2 - 1 - (x - 2)] dx = \int_{-2}^1 (x^2 - 1 - x + 2) dx = \int_{-2}^1 (x^2 - x + 1) dx$$

$$= \left. \frac{x^3}{3} - \frac{x^2}{2} + x \right|_{-2}^1 = \left(\frac{1}{3} - \frac{1}{2} + 1 \right) - \left(-\frac{8}{3} - 2 - 2 \right)$$

$$= 7.5$$