

Use the space provided to present you solutions (you may use the opposite side of each page if needed. Adding extra pages, writing unintelligibly, not signing properly your paper, etc will all result in deductions) TOTAL 33p (28 + 5)

Assignment 1: Electric Current Resistance and Circuits

Assigned: Jan 22nd

Due: Jan 22nd 6:00PM

1. An electric current is given by the expression $I(t) = 100 \sin(120\pi t)$, where I is in amperes and t is in seconds. What is the total charge carried by the current from $t = 0$ to $t = (1/240)$ s? (Integrating will be needed)

$$Q = \int_{t=0}^{t=1/240} I(t) dt = \int_{t=0}^{t=1/240} 100 \sin(120\pi t) dt = -\frac{100}{120\pi} [\cos(120\pi t)]_0^{1/240} = -\frac{100}{120\pi} \left[\cos \frac{\pi}{2} - \cos 0 \right] = \frac{5}{6\pi} \text{ C}$$

2. 1 m long copper wire of 1mm diameter is stretched permanently to 1.5 m length. What is its new resistance ?

$$R_1 = \rho \frac{l_1}{A_1} \quad R_2 = \rho \frac{l_2}{A_2} \quad V_1 = V_2 \Rightarrow l_1 A_1 = l_2 A_2$$

$$R_1 = \rho \frac{l_1}{A_1} \quad R_2 = \rho \frac{1.5l_1}{\frac{A_1}{1.5}} = (1.5)^2 \rho \frac{l_1}{A_1} = 2.25R_1$$

3. A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?

$$I = \frac{P}{\Delta V} = \frac{600 \text{ W}}{120 \text{ V}} = \boxed{5.00 \text{ A}} \quad \text{and} \quad R = \frac{\Delta V}{I} = \frac{120 \text{ V}}{5.00 \text{ A}} = \boxed{24.0 \Omega} .$$

4. A toy battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R . (a) What is the value of R ? (b) What is the internal resistance of the battery?

$$\Delta V_{term} = \Delta V_{load} \Rightarrow P_R = I \Delta V_{term} \Rightarrow 20 \text{ W} = I(11.6 \text{ V}) \Rightarrow I = \frac{20}{11.6} \text{ (A)}$$

$$\Delta V_{load} = IR \Rightarrow R = \frac{\Delta V_{load}}{I} = \frac{(11.6)^2}{20}$$

$$rI = \varepsilon - \Delta V_{term} \Rightarrow r = \frac{\varepsilon - \Delta V_{term}}{I} = \frac{3.4}{20/11.6} = \frac{(3.4)(11.6)}{20}$$

5. How much energy is dissipated as heat during a two-minute time interval by a 1.5-k Ω resistor which has a constant 20-V potential difference across its leads?

- a) 58 J b) 46 J c) 32 J d) 72 J e) 16 J

Show your work below!

$$P_R = \frac{(\Delta V_{load})^2}{R} ; \quad E = P_R \Delta t = \frac{(\Delta V_{load})^2}{R} \Delta t = \frac{20^2}{1500} 120 = \frac{480}{15} = 32 \text{ J}$$

ASSIGNMENT 1: CONT.

6 Calculate the power delivered to each resistor in the circuit shown in Figure (right):

$$R_p = \left(\frac{1}{3.00} + \frac{1}{1.00} \right)^{-1} = 0.750 \Omega$$

$$R_s = (2.00 + 0.750 + 4.00) \Omega = 6.75 \Omega$$

$$I_{\text{battery}} = \frac{\Delta V}{R_s} = \frac{18.0 \text{ V}}{6.75 \Omega} = 2.67 \text{ A}$$

CIRCUIT REDUCTION

$$P = I^2 R : P_2 = (2.67 \text{ A})^2 (2.00 \Omega) \quad P_2 = \boxed{14.2 \text{ W}} \text{ in } 2.00 \Omega$$

$$P_4 = (2.67 \text{ A})^2 (4.00 \Omega) = \boxed{28.4 \text{ W}} \text{ in } 4.00 \Omega$$

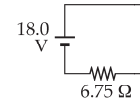
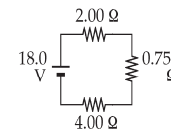
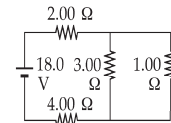
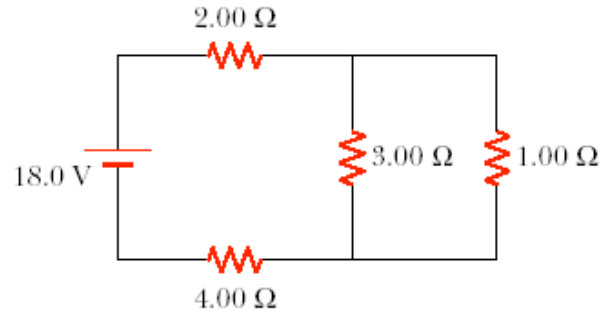
$$\Delta V_2 = (2.67 \text{ A})(2.00 \Omega) = 5.33 \text{ V},$$

$$\Delta V_4 = (2.67 \text{ A})(4.00 \Omega) = 10.67 \text{ V}$$

$$\Delta V_p = 18.0 \text{ V} - \Delta V_2 - \Delta V_4 = 2.00 \text{ V} (= \Delta V_3 = \Delta V_1)$$

$$P_3 = \frac{(\Delta V_3)^2}{R_3} = \frac{(2.00 \text{ V})^2}{3.00 \Omega} = \boxed{1.33 \text{ W}} \text{ in } 3.00 \Omega$$

$$P_1 = \frac{(\Delta V_1)^2}{R_1} = \frac{(2.00 \text{ V})^2}{1.00 \Omega} = \boxed{4.00 \text{ W}} \text{ in } 1.00 \Omega$$



7 Determine the current in each branch of the circuit shown.

From Kirchhoff's current rule, $I_3 - I_1 - I_2 = 0$.

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3 ,

$$12.0 \text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00 \text{ V} = 0$$

$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2 ,

$$-(6.00)I_2 - 4.00 \text{ V} + (8.00)I_1 = 0 \quad (8.00)I_1 = 4.00 + (6.00)I_2.$$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \text{ or } \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases} \text{ and to the single equation } 8 = 4I_1 + 13.3I_1 - 6.67$$

$$I_1 = \frac{14.7 \text{ V}}{17.3 \Omega} = 0.846 \text{ A}. \text{ Then } I_2 = 1.33(0.846 \text{ A}) - 0.667 \text{ and } I_3 = I_1 + I_2 \text{ give}$$

$$\boxed{I_1 = 846 \text{ mA}, I_2 = 462 \text{ mA}, I_3 = 1.31 \text{ A}}.$$

All currents are in the directions indicated by the arrows in the circuit diagram.

