**Concordia University**
**Department of Economics**
**ECON 221 – Sections C, D, CC**
**Instructors: S. Elliston, I. Huq, G. Tsoublekas**
**Winter 2016 – ASSIGNMENT 3 - Due Date: Wednesday, April 6, before 3:00 pm**

| **Name:** | **I.D:** | **Section:** |
|---|---|---|

**Points Total: 95 points**

## Question 1 (15 marks)

$n=500$

a) From a random sample of 500 Montrealers, 400 indicated that they were in favour of the proposed policy to ban single-use plastic bags on the island. Test at the 10% significance level a politician's statement that "those who are in favour of this policy represent more than 85% of the population."

(i) Formulate the null and alternative hypotheses. [2 marks]

$H_0: P_0 \leq 0.85$

$H_1: P_0 > 0.85$

(ii) State the decision rule for this test. [2 marks]

We reject $H_0$ if $\frac{\hat{p}-P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} > Z_{\alpha/2}$.

(iii) Calculate the appropriate statistic for testing the null hypothesis. [2 marks]

$\hat{p} = \frac{400}{500} = 0.8$

$Z_{\alpha/2} = 1.28$

$$\frac{\hat{p}-P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.8-0.85}{\sqrt{\frac{0.85(1-0.85)}{500}}} \approx -3.13$$

(iv) Should the null hypothesis be rejected or not rejected? [1 mark]

Since $-3.13 < 1.28$, we cannot reject null hypothesis

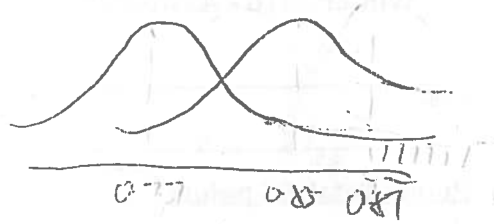
(v) What is your conclusion regarding the politician's statement? [3 marks]

From this sample, there is no sufficient evidence to conclude that people who are in favour of this policy is more than 85% of the population.

b) Suppose that the true proportion of those who favour this policy is 0.77. Calculate the power of the test conducted in (a). [5 marks]

$$\frac{\hat{p}-0.85}{\sqrt{\frac{0.85(1-0.85)}{500}}}=1.28$$

$\hat{p}=0.87$




The power of test:

$$P\left(Z>\frac{0.87-0.77}{\sqrt{\frac{0.77(1-0.77)}{500}}}\right)=0\%$$

**Question 2 (25 marks)**

At the beginning of the course of Statistical Methods I, the instructor recommended that students devote at least 3 hours per week for the duration of the 13-week semester, for a total of at least 39 hours. It is known that the times spent on studying statistics follow a normal distribution. Throughout the course's duration, students were claiming that they were following the instructor's recommendation. At the course's completion, a random sample of 9 students enrolled in the course was drawn where each student was asked how many hours he or she spent doing homework in statistics. The data are listed below:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 42 | 36 | 35 | 38 | 41 | 36 | 41 | 36 | 40 |

a) **[3 marks]** Calculate the sample mean and sample standard deviation.

$$\bar{X}=\frac{42+36+35+38+41+36+41+36+40}{9}$$

$\approx 38.33$

$$s^2=\frac{(42-38.33)^2+(36-38.33)^2+(35-38.33)^2+(38-38.33)^2+(41-38.33)^2+(36-38.33)^2+(41-38.33)^2+(36-38.33)^2+(40-38.3}{9-1}$$

$= 7.28$

$s\approx 2.69$

b)

i. **[2 marks]** Formulate a suitable null and alternative hypothesis to support the students' claim that they followed their instructor's recommendation at a 5% level of significance.

$H_0: \mu \geq 39$

$H_1: \mu < 39$

ii. **[2 marks]** What is the decision rule for this test?

We reject $H_0$ if $\dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} <= t_{n-1, \alpha}$

iii. **[2 marks]** Calculate the test statistic.

$-t_{8, 0.05} = -1.86$

$$t = \frac{38.33 - 39}{\frac{2.69}{\sqrt{9}}} \approx -0.75$$

iv. **[2 marks]** What is your conclusion regarding the students' average study time?

Since $-0.75 > -1.86$, we cannot reject null hypothesis.

From this sample, we can conclude that there is an ~~suff~~ fient evident that students devote at least 39 hours on this course.

c) **[5 marks] If the sample results had been obtained from a random sample of 16 students, would the conclusion about the students' claim be different from in part (b)? Explain.**

We are given that

$\bar{x}=38.33$ $S=2.69$ $n=16$

We ject Ho if $\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} < -t_{15,0.05}$

$-t_{15,0.05}=-1.753$

$t=\frac{38.33-39}{\frac{2.69}{\sqrt{16}}}=-1$

Since $-1>-1.753$, We cannot reject Ho.

In this case, there is no diffrent about the conclusion

d) **[9 marks] Using the original data, test at the 5% significance level whether the population standard deviation of studying times is greater than 3 hours.**

$H_0: \sigma_0 \leq 3$

$H_1: \sigma_0 > 3$

we reject Ho if $\frac{(n-1)s^2}{\sigma_0^2} > \chi^2_{8,0.05}$

$\chi^2_{8,0.05}=15.507$

$\therefore \frac{(n-1)s^2}{\sigma_0^2}=\frac{8\times 7.28}{9}\approx 6.47$

Since $6.47<15.507$, we cannot reject Ho.

Therefore, from this Sample there is no suffient evident to show that Studying is greater than 3 hours.

## Question 3 (10 marks)

Thomas is a shift manager at a local fast food place, and is responsible for quality management. Thomas wants to ensure that all the frozen hamburger patties that get delivered by the supplier weigh at least four ounces on average. Assume that the standard deviation of the weight of hamburger patties is known to be 0.1 ounce. Thomas tells one of his employees that as a shipment arrives, select 25 patties at random and find the average weight for the 25 patties.

a) **[3 marks]** Formulate a suitable null and alternative hypothesis for Thomas to do the quality check.

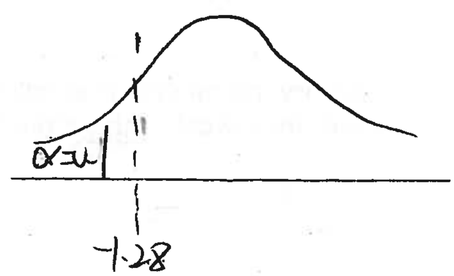
$H_0: \mu_0 \geq 4$

$H_1: \mu_0 < 4$

b) **[7 marks]** For what average weight would you tell the employee to reject the shipment, if you wanted the probability of a Type I error to be 0.10 or less?

We want $\alpha \leq 0.1$

We reject $H_0$ if $\frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} < -Z_{\alpha/2}$




$-Z_{\alpha/2} = -1.28$

$$\frac{\frac{\bar{x} - 4}{0.1}}{\sqrt{25}} \leq -1.28$$

$\bar{X} \leq 3.97$.

The average weight should be 3.97 in order to reject the shipment.

**Question 4 [10 marks]**

A new weight-loss technique, consisting of a liquid protein diet, is currently undergoing tests by Health Canada before its introduction into the market. A typical test performed by the Health Canada is the following: The weights of a random sample of five people are recorded before they are introduced to the liquid protein diet. The five individuals are then instructed to follow the liquid protein diet for 3 weeks. At the end of this period, their weights (in pounds) are again recorded. The results are listed in the table. Let $\mu_1$ be the true mean weight of individuals before starting the diet and let $\mu_2$ be the true mean weight of individuals after 3 weeks on the diet.

| Person | Weight Before Diet | Weight After Diet |
|---|---|---|
| 1 | 156 | 149 |
| 2 | 201 | 196 |
| 3 | 194 | 191 |
| 4 | 203 | 197 |
| 5 | 210 | 206 |

Before - after

Summary information is as follows: $\bar{d} = 5, s_d = 1.58$. Conduct a test at the 10% level of significance to determine if the diet is effective at reducing weight.

$H_0: \mu_1 - \mu_2 \leq 0$

$H_1: \mu_1 - \mu_2 > 0$

We reject Ho if $\dfrac{\bar{d}}{\frac{s_d}{\sqrt{n}}} > t_{n-1,\alpha}$

$t_{4,0.1} = 1.533$

$t = \dfrac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = \dfrac{5}{\frac{1.58}{\sqrt{5}}} \approx 7.08$

Since $7.08 > 1.533$, we reject Ho

From this sample, we can conclude that the diet is effective at reducing weight

**Question 5 (15 marks)**

The following data represent weights (in pounds) for two independent random samples of men of approximately 5 feet 10 inches tall and of medium build. The only difference is that the first group is comprised of athletic persons and the second of non-athletic ones.

10

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Athletic men:** | 152, | 148, | 156, | 155, | 157, | 162, | 159, | 168, | 150, | 173 |
| **Non-athletic men:** | 155, | 157, | 169, | 170, | 171, | 161, | 181, | 165, | 183 | 9 |

a) **[4 marks]** Calculate the means and variances of the two samples.

Let $\bar{X}$ denotes mean weight of Athletic men and $\bar{Y}$ denotes Non-athletic men.

$$\bar{X} = \frac{152+148+156+155+157+162+159+168+150+173}{10} = 158$$

$$\bar{Y} = \frac{155+157+169+170+171+161+181+165+183}{9} = 168$$

$$S_{\bar{X}} = \frac{(152-158)^2+(148-158)^2+(156-158)^2+(155-158)^2+(157-158)^2+(162-158)^2+(159-158)^2+(168-158)^2+(150-158)^2+(173-158)^2}{10-1}$$

$\approx 7.86$

$$S_{\bar{Y}} = \frac{(155-168)^2+(157-168)^2+(169-168)^2+(170-168)^2+(171-168)^2+(161-168)^2+(181-168)^2+(165-168)^2+(183-168)^2}{9-1}$$

$\approx 9.72$

b) **[11 marks]** Assuming that the variances of the two populations are equal, test, at the 10% level of significance, whether the mean weight differs between the two groups.

$H_0: \mu_X - \mu_Y = 0$

$H_1: \mu_X - \mu_Y \neq 0$

We reject $H_0$ if $\dfrac{\bar{X}-\bar{Y}}{\sqrt{\frac{S_p^2}{n_X}+\frac{S_p^2}{n_Y}}} > t_{n_X+n_Y-2,\alpha/2}$ or $\dfrac{\bar{X}-\bar{Y}}{\sqrt{\frac{S_p^2}{n_X}+\frac{S_p^2}{n_Y}}} < -t_{n_X+n_Y-2,\alpha/2}$

$\alpha/2 = 0.05$ $\quad t_{10+9-2,0.05} = 1.703$ $\quad -t_{10+9-2,0.05} = -1.703$

$$S_p^2 = \frac{(n_X-1)S_X^2+(n_Y-1)^2S_Y^2}{n_X+n_Y-2} = \frac{(10-1)\times 7.86+(9-1)\times 9.72}{10+9-2} = 5.5$$

$$t = \frac{\bar{X}-\bar{Y}}{\sqrt{\frac{S_p^2}{n_X}+\frac{S_p^2}{n_Y}}} = \frac{158-168}{\sqrt{\frac{5.5}{10}+\frac{5.5}{9}}} \approx -9.28$$

Since $-9.28 < -1.703$, we reject $H_0$.

In this sample, we have suffient evidence to conlude that the mean weight is different.

**Question 6 (10 marks)**

Random samples of 900 people in the United states and in Canada indicated that 60% of the people in the United States were positive about the future economy, whereas 66% of the people in Canada were positive about the future economy. Does this provide strong evidence that people in Canada are more optimistic about the future? Test at the 5% level of significance using the p-value approach.

Let $P_x$ denotes percentage of people in Canada, who are positive about the future economy and $P_y$ denotes United states

$H_0: P_x - P_y \le 0$

$H_1: P_x - P_y > 0$

We reject $H_0$ if $\dfrac{\hat{P}_x - \hat{P}_y}{\sqrt{\dfrac{\hat{P}_0(1-\hat{P}_0)}{n_x} + \dfrac{\hat{P}_0(1-\hat{P}_0)}{n_y}}} > Z_\alpha$

$Z_{0.05} = 1.645$

$\hat{P}_x = 0.66 \quad \hat{P}_y = 0.6$

$$\hat{P}_0 = \frac{\hat{P}_x n_x + \hat{P}_y n_y}{n_x + n_y} = \frac{0.66 \times 900 + 0.6 \times 900}{900 + 900} = 0.63$$

$$Z = \frac{0.66 - 0.6}{\sqrt{\dfrac{0.63(1-0.63)}{900} + \dfrac{0.63(1-0.63)}{900}}} \approx 2.64$$

$$\begin{aligned} P\text{-value} &= 1 - F(Z = 2.64) \\ &= 1 - 0.9959 \\ &= 0.0041 \end{aligned}$$

Since $\alpha = 0.05$ which is bigger than 0.0041, we reject $H_0$.

In this sample, we have sufficient evidence to conclude that Canada are more optimistic about the future than United states

**Question 7 [10 marks]**

The federal government is interested in determining whether salary discrepancies exist between men and women in the private sector. Suppose random samples of 15 women and 21 men are drawn from the population of first-level managers in the private sector. The information is summarized in the table below:

| | Women | Men |
|---|---|---|
| Sample mean | \$28,400 | \$30,300 |
| Sample standard deviation | \$4,600 | \$5,206 |

Test at the 5% level of significance, whether the true variance in the salaries of men is higher.

Let Sx be the sample standard deviation of salary earned by men and Sy be women.

$H_0: \sigma_x^2 - \sigma_y^2 \leq 0$

$H_1: \sigma_x^2 - \sigma_y^2 > 0$

We reject Ho if $\frac{S_x^2}{S_y^2} > F_{n_x-1, n_y-1, \alpha}$

$F_{20, 14, 0.05} = 2.388$

$F = \frac{(5206)^2}{(4600)^2} \approx 1.28$

Since $1.28 < 2.388$, we cannot reject Ho.

In this sample, there is no sufficient evidence that true variance in the salaries of men is higher.