

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	203	All
Examination	Date	Pages
Final	April 2009	3
Instructors	Course Examiner	
D. Dryanov, A. Laurin, J.R. Perez-Buendia, N. Rossokhata	H. Proppe	
Special Instructions		
▷ Only Sharp EL 531 or Casio FX 300 MS calculators are allowed.		

MARKS

- [10] 1. (a) Sketch the graph of the function $f(x) = |1 - (x - 2)^2|$ starting from the graph of the standard parabola and using appropriate transformations.
- (b) Suppose $f(x) = e^x - 1$. Find its domain and range, and then find $f^{-1}(x)$ and its domain and range.
- (c) Suppose $f(x) = \sqrt{x}$, $g(x) = x/4$ and $h(x) = 4x - 8$. Find formulas for $h \circ g \circ f(x)$ and $f \circ g \circ h(x)$.

- [9] 2. Evaluate the limits:

(a) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$ (b) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$ (c) $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + 1}}{3x - 5}$

Do not use l'Hôpital's rule.

- [10] 3. (a) Let $f(x) = \frac{|x^2 - 1|}{x^2 - 1}$.

Calculate both one-sided limits at the two points where the function is undefined.

- (b) Find the numbers a and b that make the function

$$f(x) = \begin{cases} \sqrt{4 - x^2} & \text{if } -2 \leq x < 0 \\ ax + b, & \text{if } 0 \leq x < 2 \\ 0, & \text{if } x \geq 2 \end{cases}$$

continuous on its whole domain. Sketch the graph of this function.

[15] 4. Find derivatives of the functions (do not simplify the answer):

(a) $f(x) = \frac{x^3 + 3x\sqrt{x} + x^{-1/2}}{x^2}$;

(b) $f(x) = e^{x^2}(x^2 + 1) + x^4(2^x + 1)$;

(c) $f(x) = \frac{\ln(x^2 e^{2x})}{x + \ln x}$;

(d) $f(x) = \sin(3 \tan(x^2))$;

(e) $f(x) = (\cos x)^x$ (use logarithmic differentiation).

[12] 5. (a) Given that $f(x) = \sin x$, find the linearization $L(x)$ of $f(x)$ at $a = 1$ and use $L(x)$ to estimate $\sin(1.1)$. You may use the following data: $\sin(1) = 0.84$ and $\cos(1) = 0.54$ (**Do not** use a calculator to answer this question).

(b) Answer part (a) using differentials, that is, identify dx and calculate df . (**Do not** use a calculator to answer this question).

(c) If $g(x) = \frac{1}{\sqrt{2x+1}}$, use the definition of the derivative to find $g'(1)$.

(d) Use the appropriate differentiation rule(s) to verify your answer to part (c).

[18] 6. (a) The equation of a curve defined implicitly is $x^3 y^2 = -3xy$. Verify that the point $(-1, -3)$ belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) Show that $f(x) = x^3 - x^2 - x + 1$ satisfies the conditions of the Mean Value Theorem on $[0, 1]$. Then find a number c that satisfies the conclusions of the Mean Value Theorem for the function $f(x)$ on $[0, 1]$.

(c) Use l'Hôpital's rule to evaluate $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$.

[10] 7. (a) Let $f(x) = e^x \sin x$. Find $f''(x)$, $f^{(4)}(x)$ and $f^{(8)}(x)$.

(b) A forest fire spreads in a circle with radius increasing at a rate of 2 m/minute. When the radius reaches 200 m, at what rate is the area of the burning region increasing?

[16] 8. Given the function $f(x) = \frac{x^2 - 3}{x^3}$,

- (a) Find the domain and check for symmetry. Find all asymptotes.
- (b) Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if any).
- (c) Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward and inflection point(s) (if any).
- (d) Sketch the graph of the function, and label the local extrema and inflection point(s) you have found (if any) in parts (b) and (c).

[5] **Bonus Question**

Two runners start a race at time 0. At some time $t = a > 0$ one runner has pulled ahead, but later, at time $t = b$, the other runner has taken the lead. Show that at some instant $t = c$ during the race they were running at exactly the same speed. [Hint: Let $g(t)$ be the distance the first runner covers in time t and let $h(t)$ be the distance the second runner covers in time t . Now put $f(t) = g(t) - h(t)$ and notice that $f(0) = 0$, $f(a) > 0$ and $f(b) < 0$; explain how Rolle's Theorem applies to this situation and you will see the solution].