

Unit 5: Trigonometric Functions

1. a) $360 - 92 = 262$

b) $\frac{-\pi}{3} \times \frac{180}{\pi} = \frac{-180\pi}{3\pi} = 60^\circ$

$460 - 60 = 300$

2. $\cot\theta = 1.6$

$$\frac{1}{\tan\theta} = \frac{1}{1.6} = 0.625$$

$$RA = 32^\circ$$

Quad #1: $0 + 32 = 32$

Quad #2: $180 + 32 = 212$

3. a) $\sin 90^\circ$

$x=0$

$y=1$

$r=1$

$$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$$

b) $\cos \pi$

$x=-1$

$y=0$

$r=1$

$$\cos \pi = \frac{x}{r} = \frac{-1}{1} = -1$$

c) $\csc\left(\frac{3\pi}{2}\right)$

$x=0$

$y=-1$

$r=1$

$$\csc\left(\frac{3\pi}{2}\right) = \frac{1}{\sin\theta} = \frac{r}{y} = \frac{1}{-1} = -1$$

4. a) $\frac{220}{1} \times \frac{\pi}{180} = \frac{11\pi}{9}$

$$\text{b) } \frac{48}{1} \times \frac{\pi}{180} = \frac{4\pi}{15}$$

$$\text{5. a) } \frac{3\pi}{4} \times \frac{180}{\pi} = \frac{3\pi \times 180}{4\pi} = \frac{540\pi}{4\pi}$$
$$\frac{540\pi}{9\pi} = 60^\circ = 135^\circ$$

$$\text{b) } 1.5 \times \frac{180}{\pi} \frac{270}{\pi} = 86^\circ$$

$$\text{6. } \frac{7\pi}{12} \times \frac{180}{\pi} = 105^\circ$$

$$\text{Quad\#2: } 180^\circ - 103^\circ = 75^\circ$$

$$\text{7. a) } \tan\left(\frac{5\pi}{6}\right)$$
$$\frac{-1}{3}$$

$$\text{b) } \cos\left(\frac{7\pi}{4}\right)$$
$$\frac{1}{2}$$

$$\text{8. } y = 3\cos(x - 5) + 11$$

$$(0,1) \rightarrow \left(\frac{1}{k}(0) + 5, 3(1) + 11\right)$$

(5,4) this is a cosine curve

$$\text{9. } \tan\theta = \left(\frac{4}{x}\right)$$

$$\tan\theta = \left(\frac{-6}{-8}\right)$$

$$\theta = \tan^{-1}\left(\frac{-6}{-8}\right)$$

$$\theta = 37^\circ$$

Feature ($y = \tan\theta$)	Value
Period	π
$\frac{1}{4}$ Wave	$\frac{\pi}{4}$
Vertical Asymptotes	$\frac{-3\pi}{2}, -2\pi, \frac{3\pi}{2}, 2\pi$
Domain	$\{x \in \mathbb{R} \mid x \neq \frac{-3\pi}{2}, -2\pi, \frac{3\pi}{2}, 2\pi\}$
Range	$\{y \in \mathbb{R}\}$

Feature ($y = \cot\theta$)	Value
Period	π
$\frac{1}{4}$ Wave	$\frac{\pi}{4}$
Vertical Asymptotes	$-2\pi, -\pi, 0, \pi, 2\pi$
Range	$\{y \in \mathbb{R}\}$

$$\begin{aligned}
11. \quad \frac{\cos 11\pi}{6} &= \frac{\cos(12-1)\pi}{6} = \cos\left(2\pi - \frac{\pi}{6}\right) \\
&= \frac{\cos\pi}{6} \\
&= \cos 30 \\
&= \frac{\sqrt{3}}{2}
\end{aligned}$$

$$\begin{aligned}
12. \quad &\left(x + \frac{\pi}{2}\right) \\
&\cos(x)\cos\left(\frac{\pi}{2}\right) - \sin(x)\sin\left(\frac{\pi}{2}\right) \\
&- \sin(x)\sin\left(\frac{\pi}{2}\right) + 0 \\
&- \sin(x)
\end{aligned}$$

13. $\cos x = \frac{12}{13}, \sin(2x)$

$$\cos = \frac{12}{13}$$

$$12^2 + b^2 = 13^2$$

$$b^2 = 13^2 - 12^2$$

$$b^2 = \sqrt{25}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\frac{120}{169} \vee 0.71006$$

14. $2\cos x - 2 = \sin^2 x$

$$2\cos x - 2 - (1 - \cos^2 x) = 0$$

$$\cos^2 x + 2\cos x - 1 - 2$$

$$\cos^2 x + 2\cos x - 3$$

$$-3 + \cos^2 x + \cos x = 0$$

$$\cos x = 1, \cos x = 3$$

$$x = 0 \vee x = 2\pi$$

15.

Transformations	Value
Vertical Stretch	a=4 Stretch by a factor of 4
Horizontal Stretch	$k = \frac{1}{2}$ Period is 2
Vertical Shift	c=-7 -7 units downwards
Horizontal Shift	$d = \frac{\pi}{4}$ Shifts to the right

16. a) $\frac{\sin^2 \theta}{1 \cos \theta} = 1 - \cos \theta$

$$\frac{(\sin^2(x)(1 - \cos x))}{(1 + \cos x)(1 - \cos x)}$$

$$\frac{(\sin^2(x)(1 - \cos x))}{1 + \cos^2(x)}$$

$$\frac{(\sin^2(x)(1 - \cos x))}{1 + \cos^2 x(1 - \cos^2(x))}$$

$$\frac{\sin^2 x - \sin^2(x)\cos x}{\sin^2 x}$$

$$\frac{\sin^2(x)}{\sin^2(x) - \sin^2(x)\cos(x)}$$

$$1 - \cos\theta$$

$$\text{b) } \frac{\cos(2x) + 1}{\sin(2x)} = \cot x$$

$$\frac{\cos^2 x - \sin^2 x + 1}{\sin(2x)}$$

$$\frac{2\cos^2(x)}{\sin(2x)}$$

$$\frac{2\cos^2 x}{2\sin(x)\cos(x)}$$

$$\frac{\cos x}{\sin x}$$

$$\cot x$$

17.

Transformations	Value
Vertical Stretch	a=2

	Vertical stretch factor of 2
Horizontal Stretch	$k=\frac{2}{3}$ Period is 3π
Vertical Shift	$c=5$ Axis $y=5$ units upwards
Horizontal Shift	$d = \pi$
Domain	{XER}
Range	{YER}

18. Step#1: the y-axis is where the side curve begins at the point of the graph (0, -1)

Step#2: Amplitude of Graph (max=2, min=-4)

$$a = \frac{\mathit{max} - \mathit{min}}{2}$$

$$a = \frac{2 - (-4)}{2}$$

$$a = 3$$

Step#3: Period of the Graph (k=2)

$$K = \frac{2\pi}{P}$$

$$K = \frac{2\pi}{\pi}$$

$$K = 2$$

Step#4: Vertical Shift (max =2 min =-4, c=-1)

$$y = \frac{\mathit{max} + \mathit{min}}{2}$$

$$y = \frac{2 + (-4)}{2}$$

Y-axis = -1, graph shifted 1 unit downward

Step#5: Phase Shift of the Graph (d=0)

Transformation	Value
Vertical Stretch	a=3

Horizontal Stretch	k=2
Vertical Shift	c=-1
Horizontal Shift	d=0

Graph Equation: $[y = 3\sin(2x) - 1]$

19. a) $1.5m + 2m + 2m = 5.5m$

The maximum height above the ground is 5.5 meters.

1.5 = lowest point above the ground

2 = middle

2 = above the ground

c) Step#1: Side curve starts at the y-axis at the point of the graph

(0, 5.5)

Step#2: Amplitude of the Graph

max=5.5

min=1.5

$$a = \frac{\text{max} - \text{min}}{2}$$

$$a = \frac{5.5 - 1.5}{2}$$

$$a = 2$$

Step#3: Period of the Graph

$$P = \frac{2\pi}{K}$$

$$K = \frac{2\pi}{P}$$

$$K = \frac{2\pi}{8}$$

$$K = \frac{\pi}{4}$$

Step#4: Vertical Shift

$$y = \frac{\text{min} - \text{max}}{2}$$

$$y = \frac{(5.5) - (1.5)}{2}$$

$$y = \frac{7}{2}, c = \frac{7}{2} \vee 3.5$$

Step#5: No phase shift, therefore d=0

Transformation	Value
Vertical Stretch	a=2
Horizontal Stretch	$\frac{\pi}{4}$ k= $\frac{\pi}{4}$
Vertical Shift	c=3.5
Horizontal Shift	d=0

Graph Equation: $y = 2\cos\left(\frac{\pi}{4}x\right) + 4$

$$d) \quad y = 2\cos\left[\frac{\pi}{4}x\right] + 4, x = 7$$

$$y = 2\cos\left[\frac{\pi}{4}(7)\right] + 4$$

$$y = 2\left[\cos\left(\frac{7\pi}{4}\right)\right] + 4$$

$$y = 4 + \sqrt{2}$$

$$[y = 3.5 + \sqrt{2} = 4.9]$$

$$y = 5.4$$

$$e) \quad 4.5 = 2\cos\left[\frac{\pi}{4}x\right] + 4$$

$$4.5 = 4 = 2\cos\frac{\pi}{4}(x)$$

$$0.5 = 2\cos\frac{\pi}{4}(x)$$

$$\frac{0.5}{2} = \cos\frac{\pi}{4}(x)$$

$$\cos a = \frac{1}{4}$$

$$a = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$\text{We let } a = \frac{\pi}{4}(x) \text{ then } \frac{\pi}{4}(x) = \frac{3\pi}{4}, \vee \frac{\pi}{4}(x) = \frac{5\pi}{4}$$

