Ryerson University

Department of Electrical and Computer Engineering

Final Examination, EES512 Electric Circuits

Duration: Bonus Test: 1.5 Hours, Optional Test: 2 Hours

April 2012

Student's Name:... **Student Number:** ....

NOTES:

- This is a **closed book, and closed course note examination.**
- **There are 9 questions. Answer all questions.** Marks for each question and its parts are mentioned. **Hand in this exam questionnaire with your solution booklet.**
- This exam is very clear in its contents and intentions. **NO QUESTIONS is allowed.** If doubt exists as to the interpretation of any question, you are urged to submit with the answer, a clear statement of any logical assumptions made.

Question	Mark	Mark obtained
Q1	10	
Q2	10	
Q3	10	
Q4	5	
Q5	3	
Q6	12	
Q7	10	
Q8	10	
Q9	10	
Total	80	

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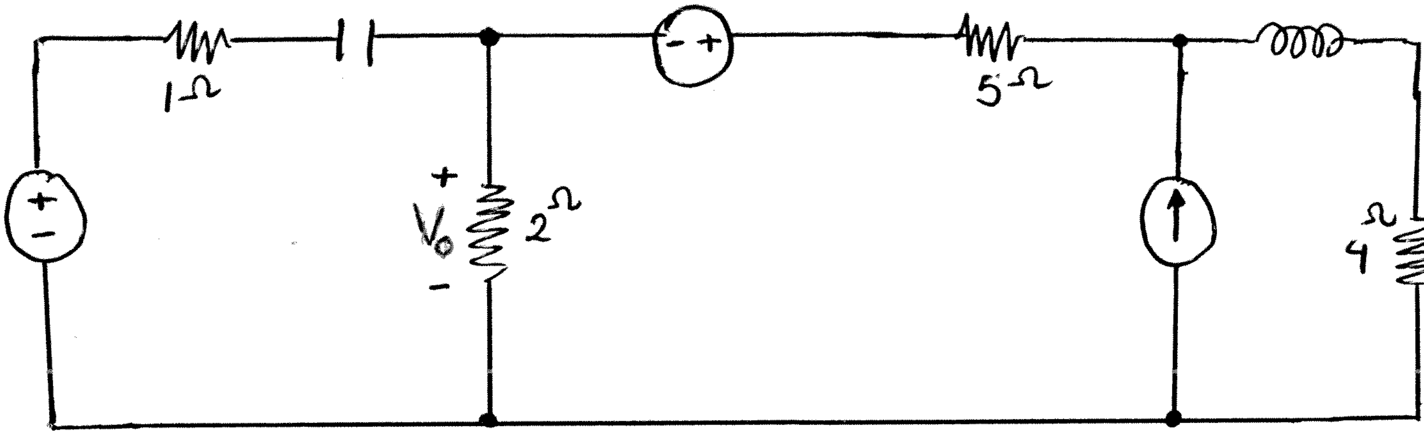
Student's Name:...

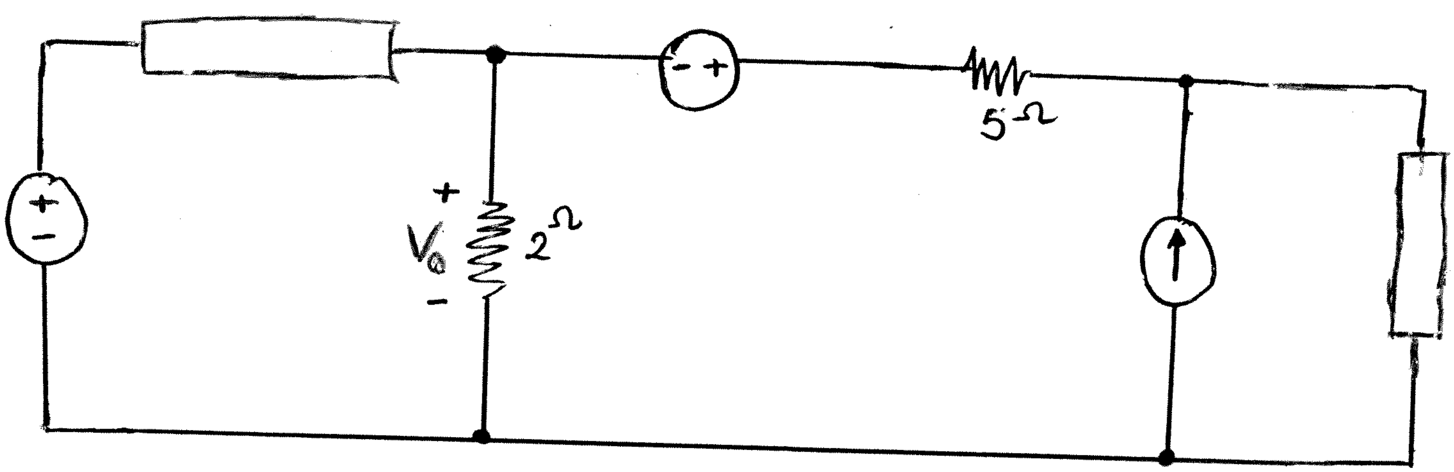
Student Number:... **Signature:**......................

Name: I.D.:

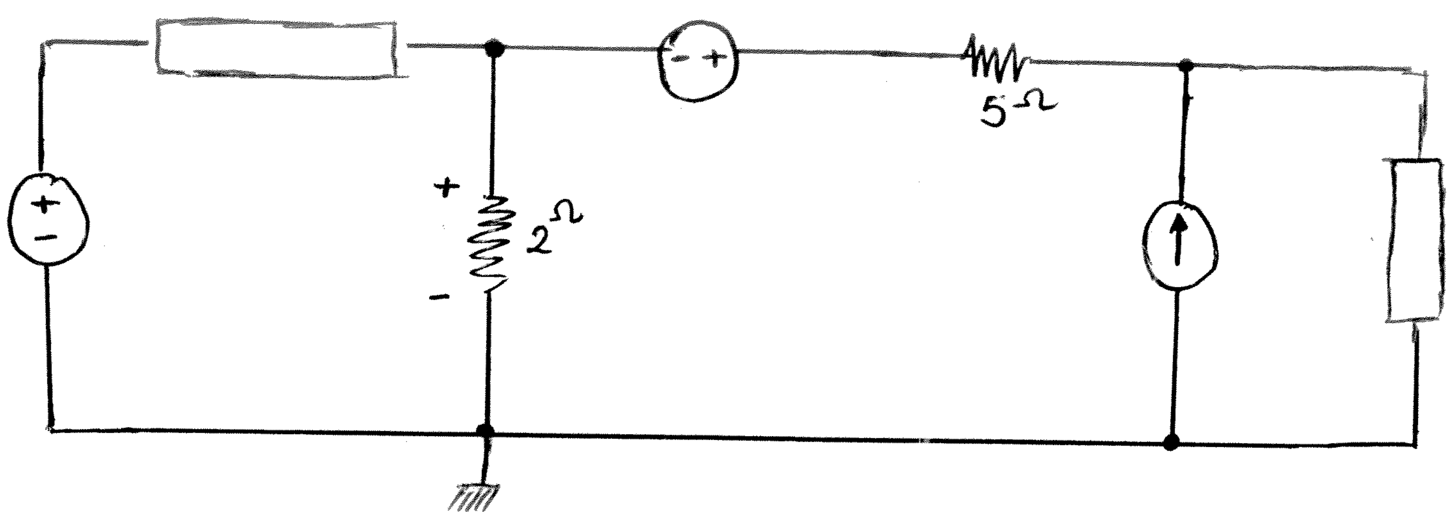
for time to frequency Conversion



for Mesh or Super mesh:



for Nodal or Super node:



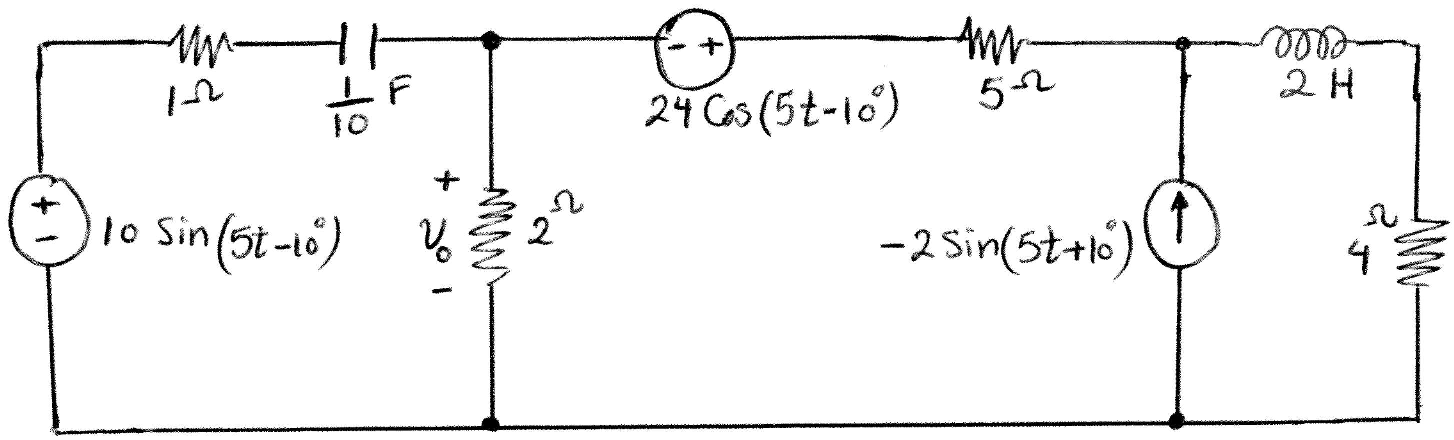


Fig. 1: (Mesh/Superposition/Nodal/Source Transformation)

Note: Find the answers to questions 1 to 4 in frequency domain (complex values).

Q1: **(Super-Mesh):** Find complex value of V_o in Fig. 1 using Mesh technique.

A. Write the mesh equations for the loops. Assign the loops **clockwise**. **(3 marks)**

B. Simplify the equations you wrote in part (A). **(2 marks)**

C. Solve the simplified system in part (B). Provide numerical answers. **(2 marks)**

D. Provide numerical complex value for V_o. **(3 marks)**

Q2: **(Superposition):** Find complex value of V_o in Fig. 1 using superposition technique.

A. We have 3 independent sources. Draw the 3 circuit configurations required for superposition. **(2 marks)**

B. Find V_{o1} due to the current source only. **(2 marks)**

C. Find V_{o2} due to the voltage source at the left only. **(2 marks)**

D. Find V_{o3} due to the voltage source on top only. **(2 marks)**

E. What is the total V_o due to both independent sources? **(2 marks)**

Q3: **(Super-Node):** Find V_o in Fig. 1 using Nodal technique. **The reference node is at the bottom of the circuit.**

A. Write the nodal equations for the non-reference nodes. **(3 marks)**

B. Simplify the equations you wrote in part (A). **(2 marks)**

C. Solve the simplified system in part (B). Provide numerical answers. **(2 marks)**

D. Express V_o (in complex form) in terms of node voltages. **(3 marks)**

Q4: **(Source Transformation):** Find complex value of V_o in Fig. 1 using source transformation.

A. Use source transformation at least once and create a simple circuit. **(3 marks)**

B. Use Voltage and/or Current divider or Ohm's laws to get V_o. **(2 marks)**

Q5: (Complex domain to Time domain conversion)

A. Find the value of v_o of Fig. 1 in time domain. The answers to Q1 to 4 are supposed to be the same. If you have not found similar answers, you may use any of the solutions you have found in Questions 1 to 4 to perform this task. **(3 marks)**

Q6: (Maximum Power Transfer):

A. Find the Thevenin equivalent circuit (V_{th} and R_{th}) seen from the load perspective in Fig 2.

(8 marks)

B. Calculate the value of R_L for maximum power transfer. Determine the maximum power absorbed by R_L. **(4 marks)**

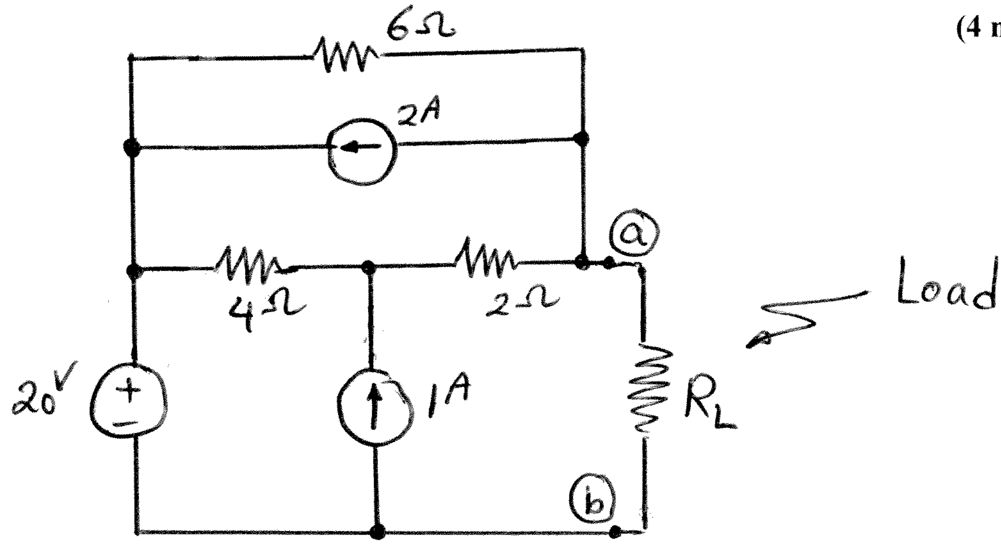


Fig. 2: (Maximum Power Transfer)

Q7: (Wheatstone Bridge/Norton): Find the Norton equivalent seen from terminals a-b in Fig. 3. **(10 marks)**

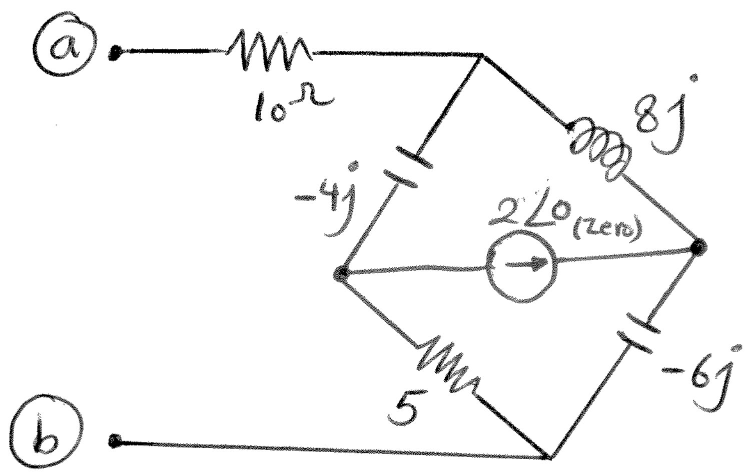


Fig. 3: (Wheatstone Bridge/Norton)

Q8: (RC): The switch in Fig. 4 has been in position A for a long time. At t=0, the switch moves from position A to B. Find $v(t)$. **(10 marks)**

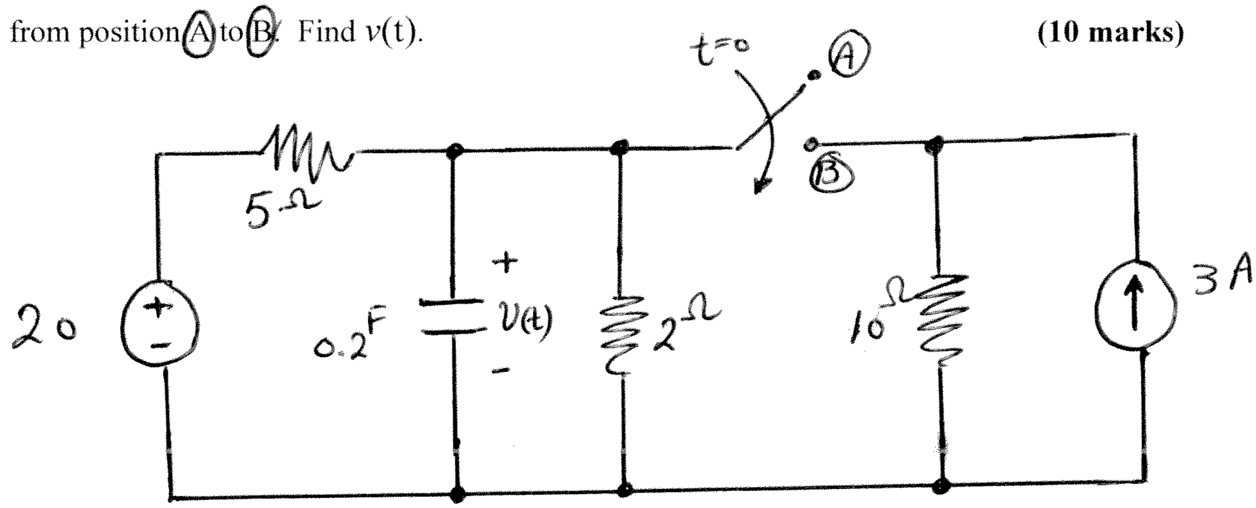


Fig. 4: (RC Circuit with DC excitation)

Q9: (RL): The switch in Fig. 5 has been in position A for a long time. At t=0, the switch moves from position A to B. Find $i(t)$. **(10 marks)**

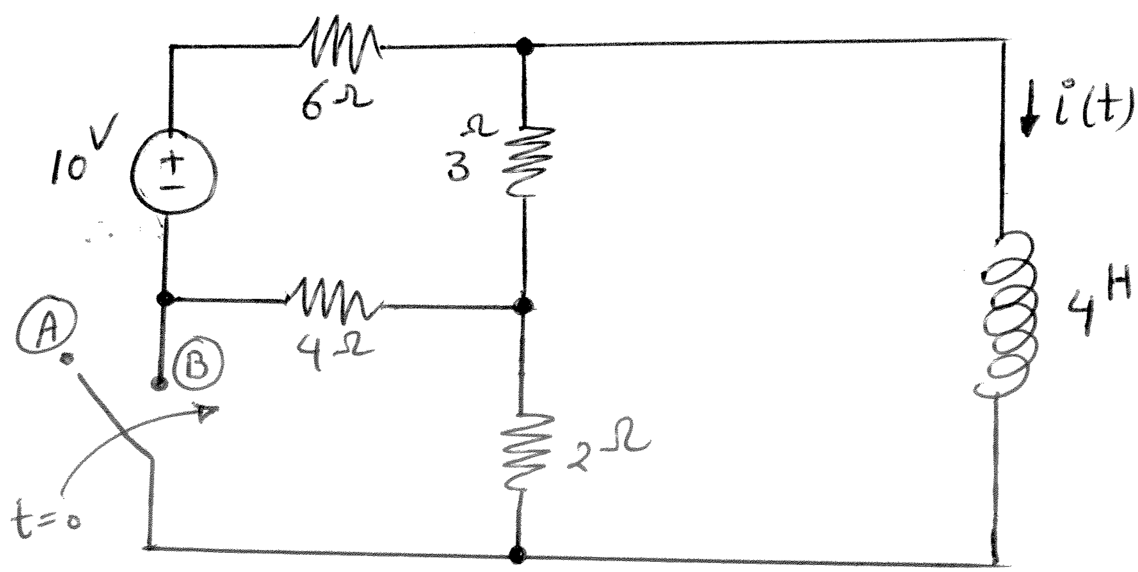
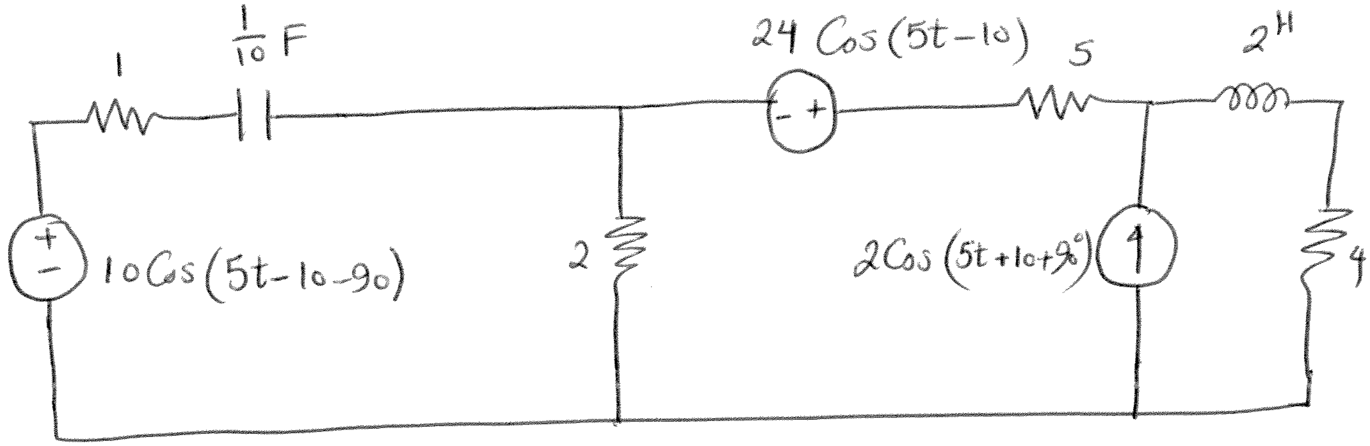


Fig. 5: (RL Circuit with DC excitation)

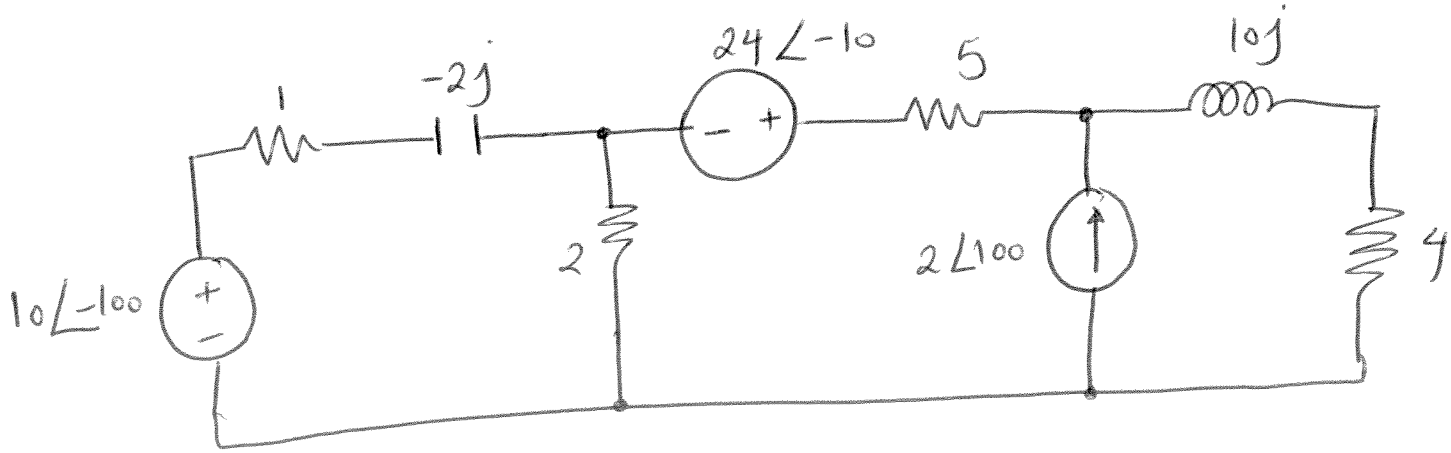
First Step: Time to Freq. Transformation

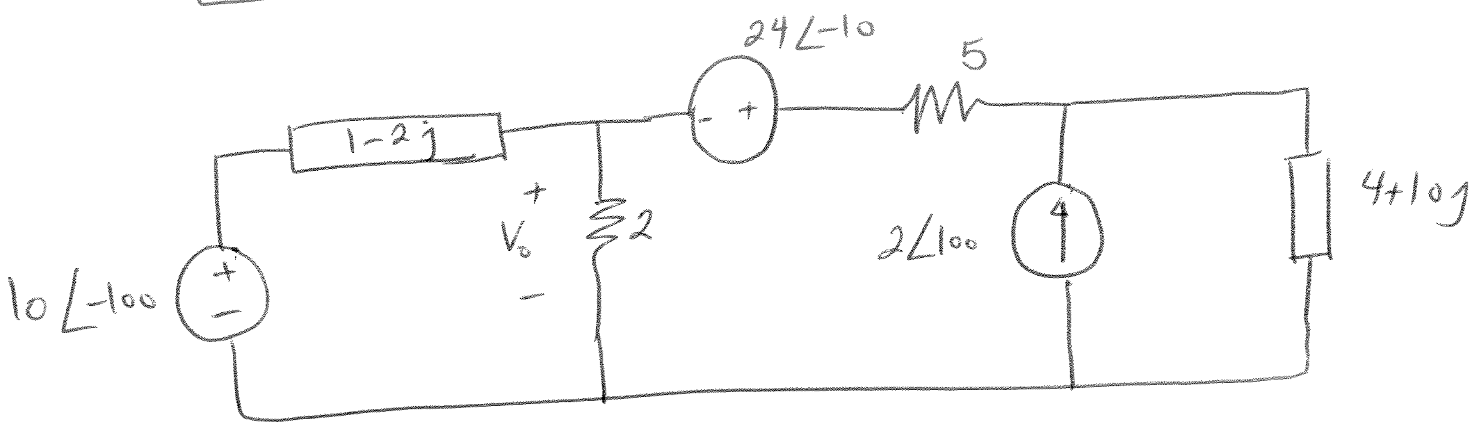


$\omega = 5$

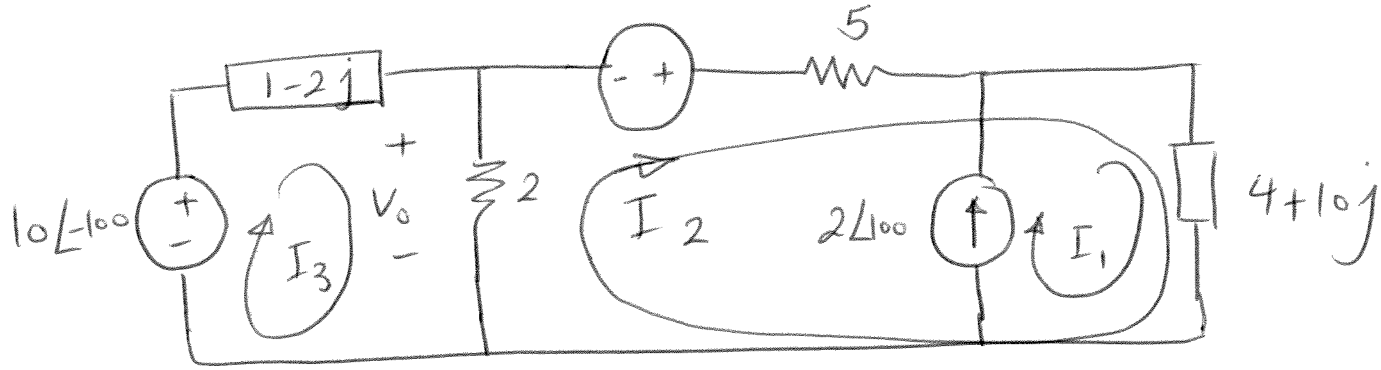
$$\frac{1}{10}F \longrightarrow \frac{1}{C\omega j} = \frac{1}{5\times\frac{1}{10}j} = \frac{1}{\frac{j}{2}} = \frac{2}{j} = -2j$$

$$2^H \longrightarrow L\omega j = 2\times 5j = 10j$$





① Super mesh: (one possible Solution)



$$\begin{cases} I_1 = 2\angle 100 \\ 2(I_2 - I_3) - 24\angle -10 + 5I_2 + (4+10j)(2\angle 100 + I_2) = 0 \\ 2(I_3 - I_2) - 10\angle -100 + (1-2j)(I_3) = 0 \end{cases}$$

$$\begin{cases} (2+5+4+10j)\,I_2 - 2I_3 = (24\angle -10) - (4+10j)(2\angle 100) \\ (-2)\,I_2 + (2+1-2j)\,I_3 = 10\angle -100 \end{cases}$$

$$\begin{cases} (11+10j)\,I_2 - 2I_3 = 44.721 - 8.5731j \\ -2I_2 + (3-2j)\,I_3 = 10\angle -100 \end{cases}$$

$$\Delta = \begin{vmatrix} 11+10j & -2 \\ -2 & 3-2j \end{vmatrix} = (11+10j)(3-2j) - (-2)(-2)$$
$$= 49 + 8j$$

$$\Delta_{I_2} = \begin{vmatrix} 44.721 - 8.5731j & -2 \\ 10\angle -100 & 3-2j \end{vmatrix}$$

$$= (44.721 - 8.5731j)(3-2j) - (-2)(10\angle -100)$$

$$= 113.54 - 134.85j$$

$$\Delta_{I_3} = \begin{vmatrix} 11+10j & 44.721 - 8.5731j \\ -2 & 10\angle -100 \end{vmatrix}$$

$$= (11+10j)(10\angle -100) + 2(44.721 - 8.5731j)$$

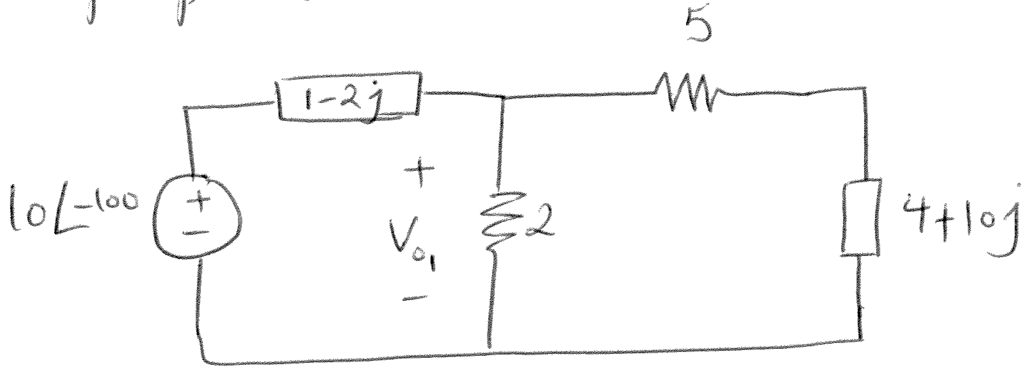
$$= 168.82 - 142.84j$$

$$I_2 = \frac{\Delta_{I_2}}{\Delta} = \frac{(113.54 - 134.85j)}{(49+8j)} = 1.82 - 3.049j$$

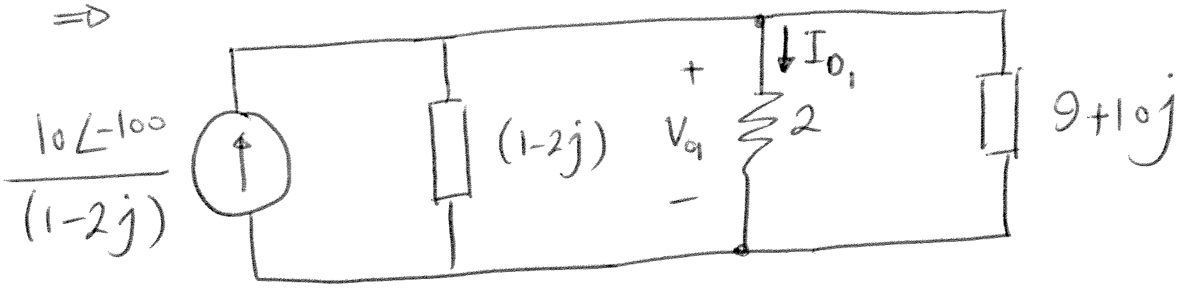
$$I_3 = \frac{\Delta_{I_3}}{\Delta} = \frac{(168.82 - 142.84j)}{(49+8j)} = 2.892 - 3.38j$$

$$V_0 = 2(I_3 - I_2) = 2(1.07266 - 0.338j) = 2.145 - 0.6758j$$

Superposition:



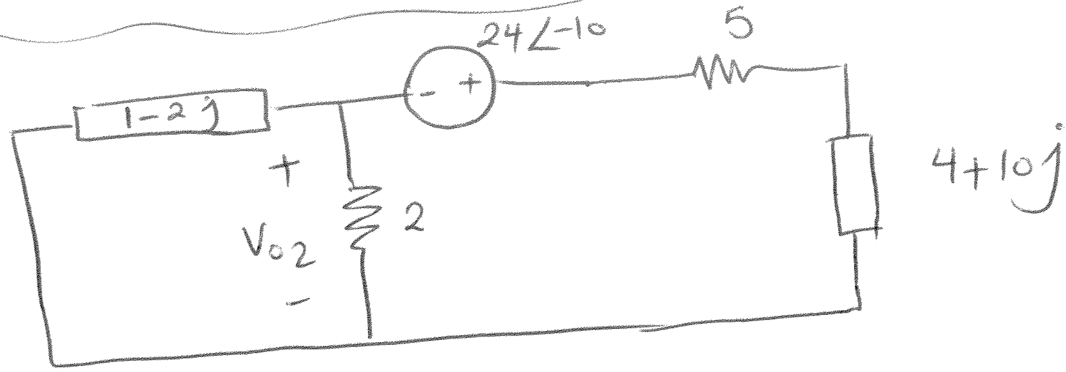
$\Rightarrow$

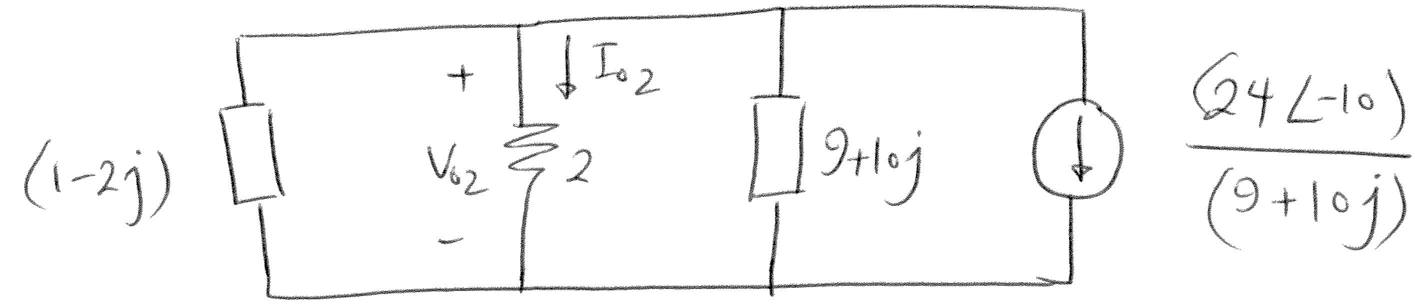


$$I_{o_1} = \frac{\frac{1}{2}}{\left(\frac{1}{2} + \frac{1}{(1-2j)} + \frac{1}{(9+10j)}\right)} \times \frac{(10\angle -100)}{(1-2j)}$$

$$= 1.303 - 2.376j$$

$$V_{o_1} = 2 I_{o_1} = 2.606 - 4.752j$$

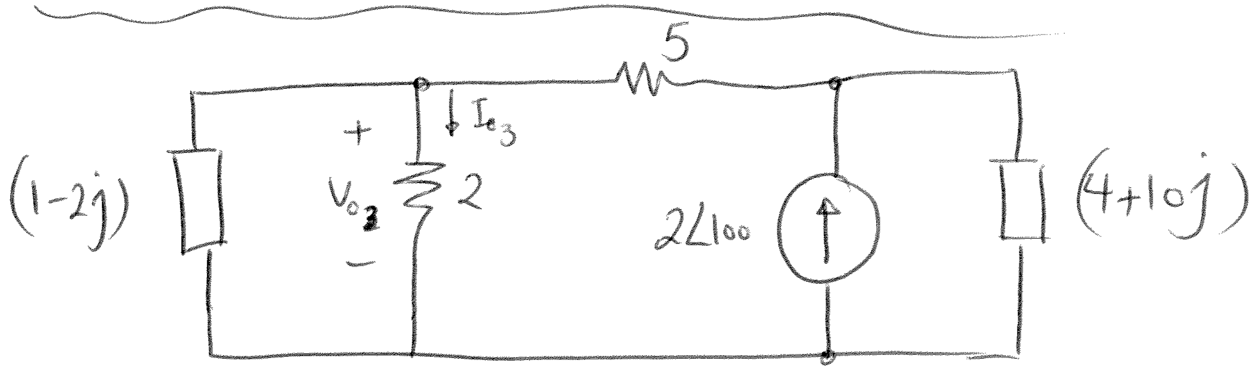


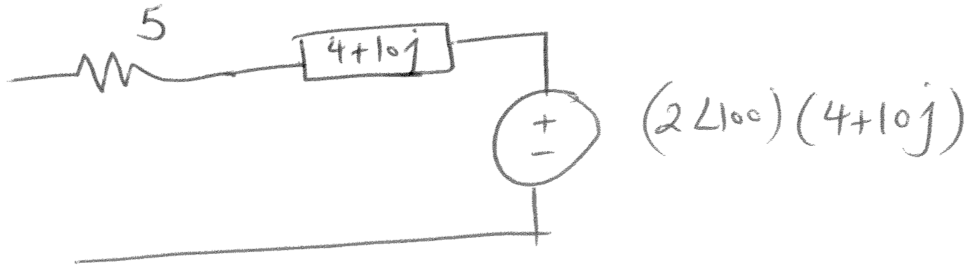


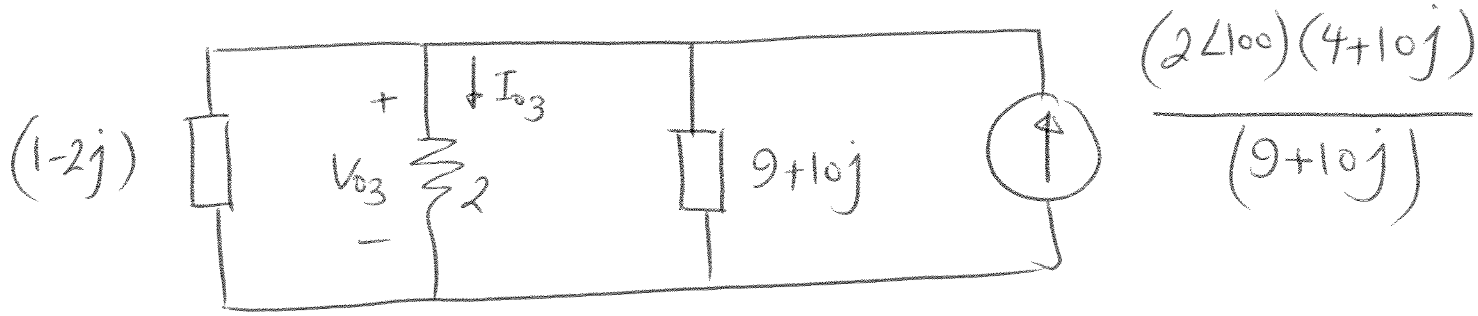
$$-I_{o2} = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{1-2j}+\frac{1}{9+10j}} \times \left(\frac{(24\angle -10)}{(9+10j)}\right)$$

$$= 0.1372 - 1.072j$$

$$V_{o2} = 2\,I_{o2} = 2\left(-0.1372 + 1.072j\right) = -0.27441 + 2.1443j$$







$$I_{o_3} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{(1-2j)} + \frac{1}{(9+10j)}} \times \frac{(2\angle 100)(4+10j)}{(9+10j)}$$

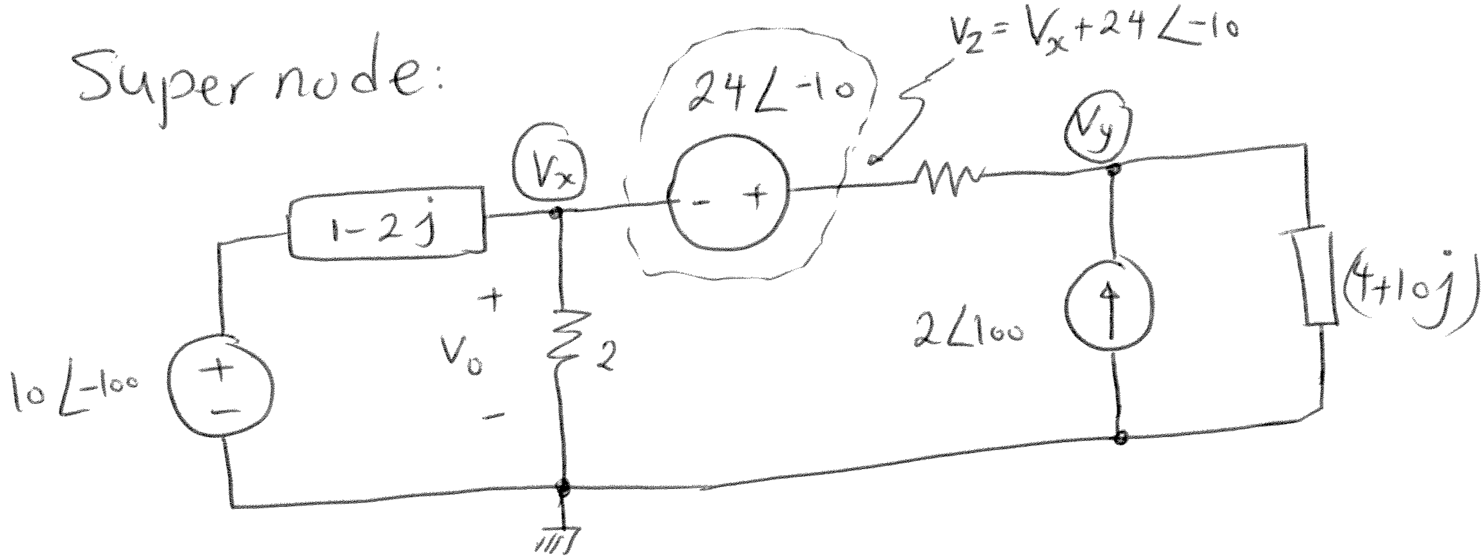
$$= -0.093 + 0.966j$$

$$V_{o_3} = 2 I_{o_3} = -0.1856 + 1.932j$$

$$V_{o_T} = V_{o_1} + V_{o_2} + V_{o_3} = (2.606 - 4.752j) + (-0.27441 + 2.1443j) + (-0.1856 + 1.932j)$$

$$= 2.146 - 0.6757j$$

Super node:



$$\begin{cases} \dfrac{V_x - (10\angle{-100})}{(1-2j)} + \dfrac{V_x - 0}{2} + \dfrac{(V_x + 24\angle{-10}) - V_y}{5} = 0 \\[2ex] \dfrac{V_y - (V_x + 24\angle{-10})}{5} + \dfrac{V_y - 0}{(4+10j)} - 2\angle 100 = 0 \end{cases}$$

$$\begin{cases} V_x\left(\dfrac{1}{1-2j} + \dfrac{1}{2} + \dfrac{1}{5}\right) + V_y\left(\dfrac{-1}{5}\right) = \dfrac{10\angle{-100}}{(1-2j)} - \dfrac{(24\angle{-10})}{5} \\[2ex] V_x\left(\dfrac{-1}{5}\right) + V_y\left(\dfrac{1}{5} + \dfrac{1}{4+10j}\right) = \dfrac{24\angle{-10}}{5} + 2\angle 100 \end{cases}$$

$$\Delta = \begin{vmatrix} \dfrac{1}{1-2j} + \dfrac{1}{2} + \dfrac{1}{5} & \dfrac{-1}{5} \\[3ex] -\dfrac{1}{5} & \dfrac{1}{5} + \dfrac{1}{4+10j} \end{vmatrix} = \cancel{\left(\left(\dfrac{1}{1-2j} + \dfrac{1}{2} + \dfrac{1}{5}\right)\right)}$$

See next page.
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$$= \left(\frac{1}{1-2j} + \frac{1}{2} + \frac{1}{5}\right)\left(\frac{1}{5} + \frac{1}{4+10j}\right) - \left(\frac{-1}{5}\right)\left(\frac{-1}{5}\right)$$

$$= 0.2055 + 0.01621j$$

$$\Delta V_x = \begin{vmatrix} \dfrac{10\angle -100}{(1-2j)} - \dfrac{24\angle -10}{5} & \dfrac{-1}{5} \\ \dfrac{24\angle -10}{5} + 2\angle 100 & \dfrac{1}{5} + \dfrac{1}{(4+10j)} \end{vmatrix}$$

$$-1.135 - 1.83j$$

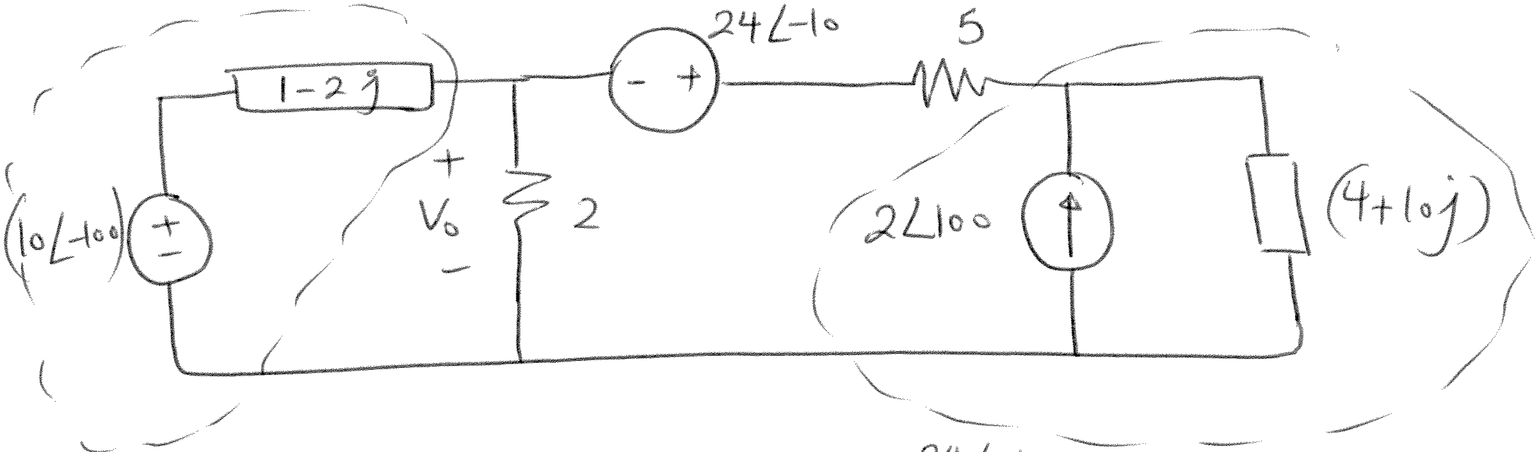
$$-0.424 - 0.33j$$

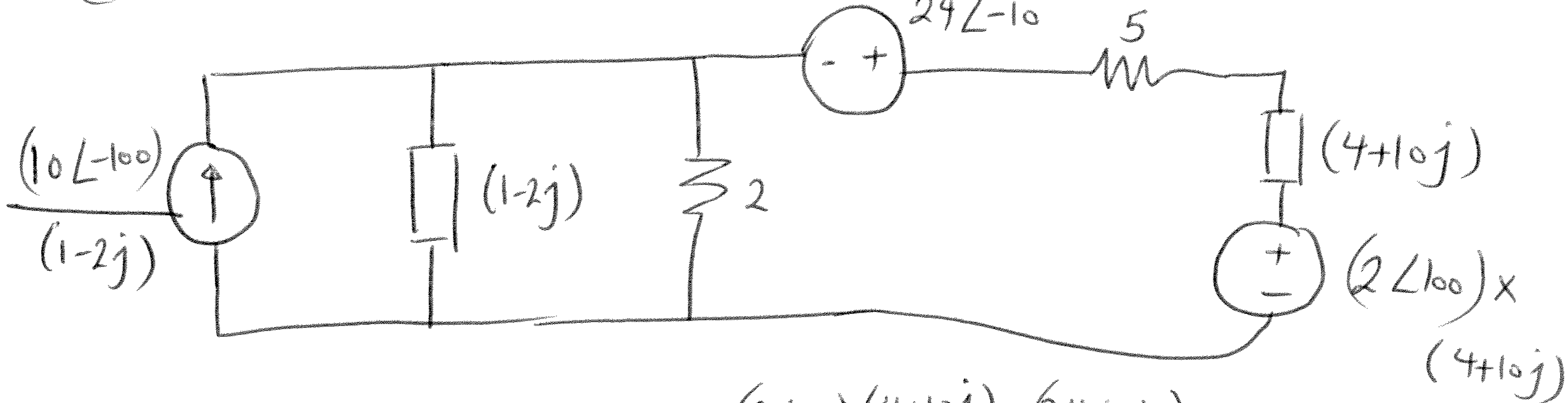
$$4.3798 + 1.136j$$

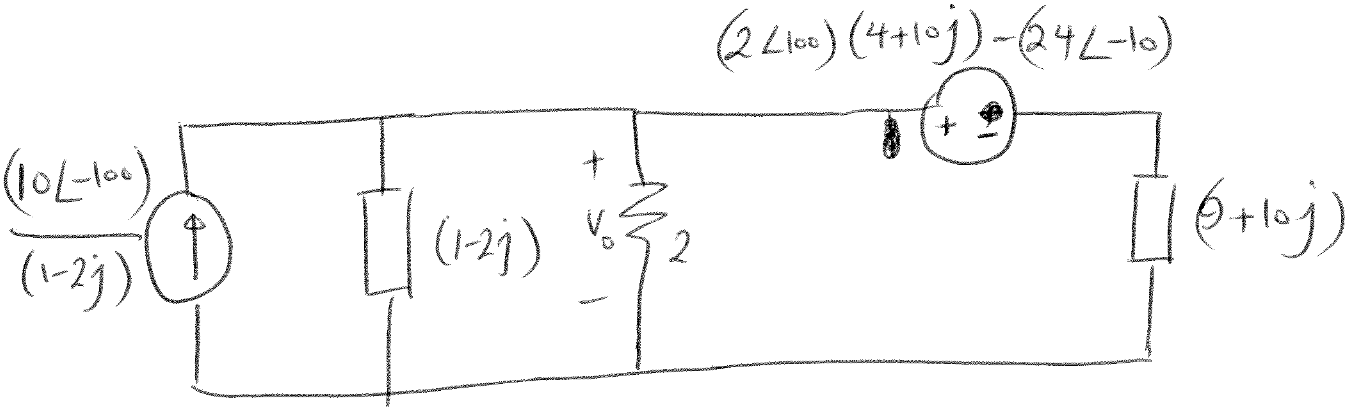
$$\Delta_{V_x} = 0.452 - 0.1042j$$

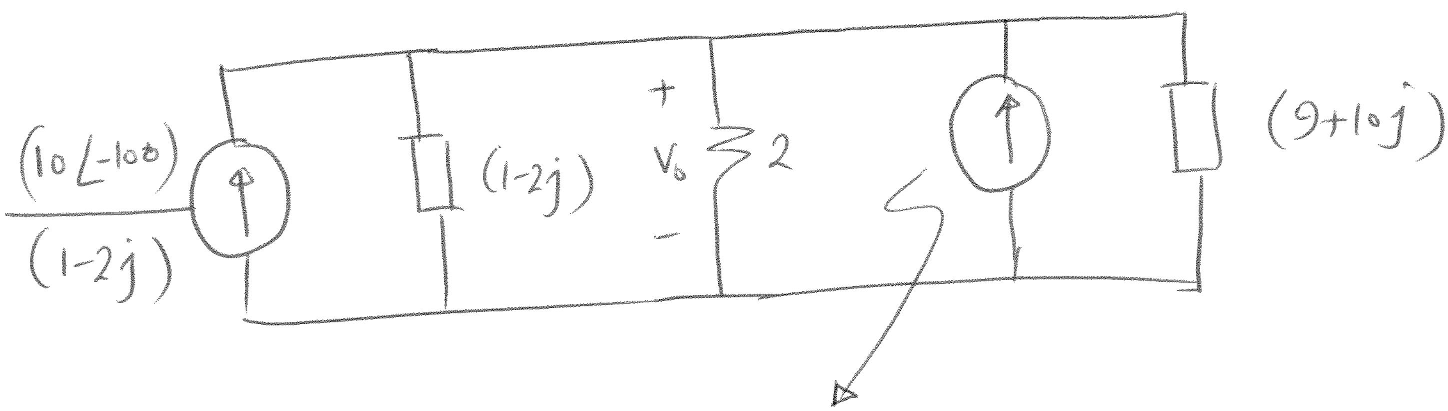
$$V_x = \frac{\Delta_{V_x}}{\Delta} = \frac{0.452 - 0.1042j}{0.2055 + 0.01621j} = 2.146 - 0.67j$$

Source Transformation

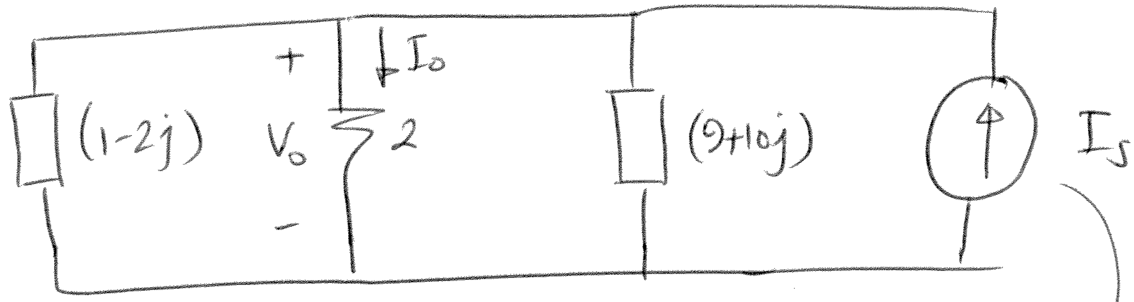








$$\frac{(2\angle 100)(4+10j)-(24\angle -10)}{(9+10j)}$$



$$I_s = \frac{(2\angle 100)(4+10j) - (24\angle -10)}{(9+10j)} + \frac{(10\angle -100)}{(1-2j)} =$$

$$I_s = 1.842 + 0.233j$$

$$I_o = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{(1-2j)} + \frac{1}{(9+10j)}} \times I_s$$

$$= 1.073 - 0.338j$$

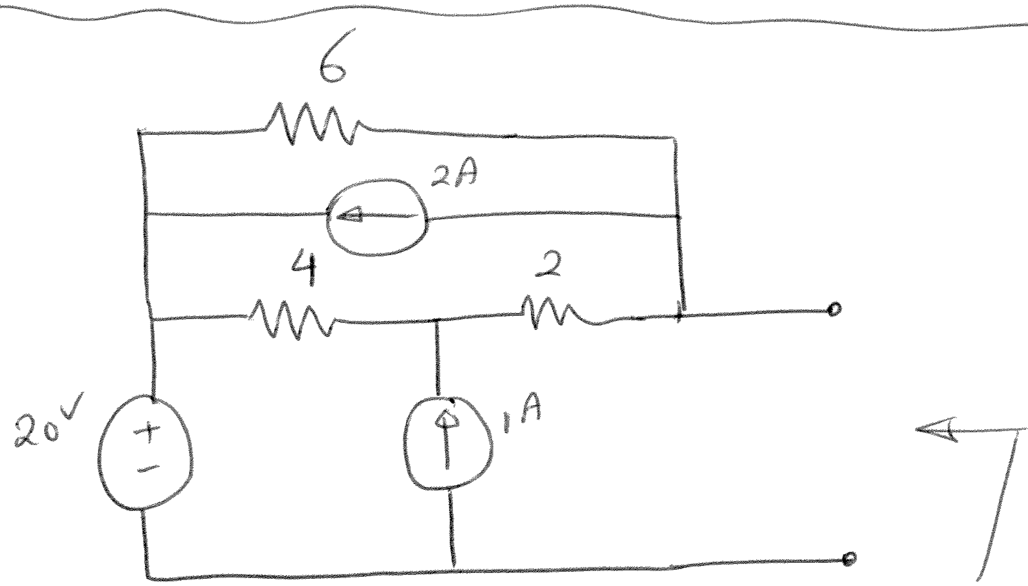
$$V_o = 2 I_o = 2.1458 - 0.676j$$

Q5 Complex to Time domain:

$$2.146 - 0.67j = 2.25 \angle -17.34^\circ$$

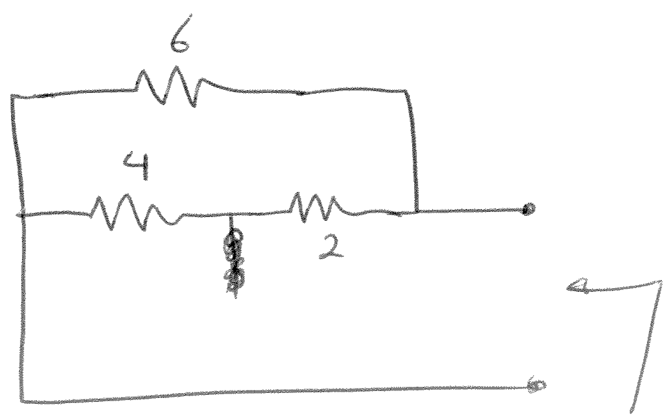
$$= 2.25 \cos(5t - 17.34^\circ)$$

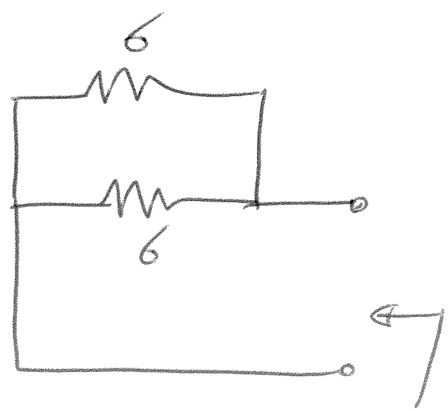
Q6:

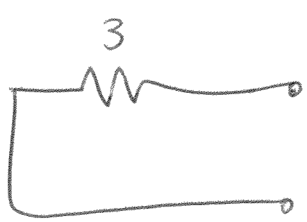


Thevenin ?

R_{th}: Set independent sources to zero.



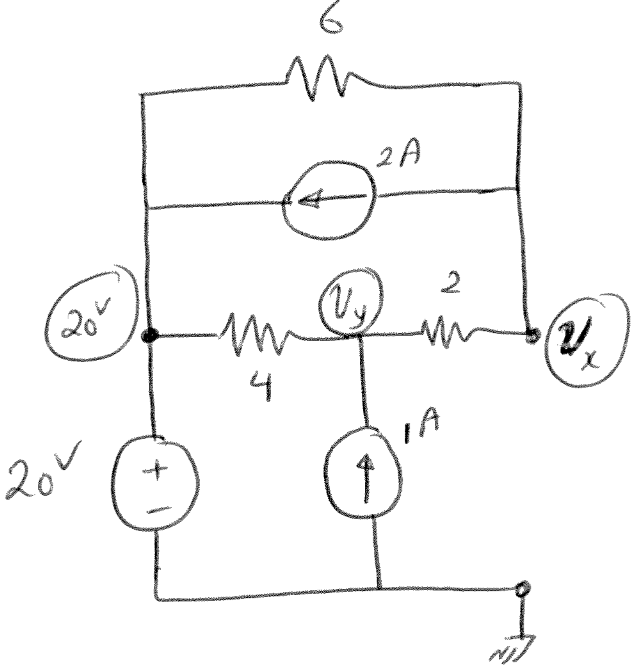




$\Rightarrow R_{th} = 3\,\Omega$

Find $V_{open} = V_{th}$:

Using Nodal:



Node (x):

$$\frac{V_x - 20}{6} + 2 + \frac{V_x - V_y}{2} = 0$$

Node (y)

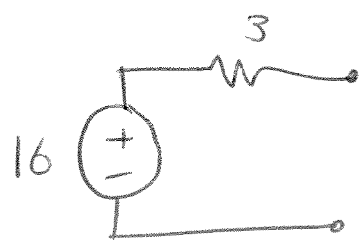
$$\frac{V_y - 20}{4} + \frac{V_y - V_x}{2} - 1 = 0$$

$$\begin{cases} V_x\left(\frac{1}{2}+\frac{1}{6}\right) + V_y\left(\frac{-1}{2}\right) = \frac{20}{6} - 2 \\ V_x\left(\frac{-1}{2}\right) + V_y\left(\frac{1}{2}+\frac{1}{4}\right) = \frac{20}{4} + 1 \end{cases}$$

Calculator

$\Rightarrow$ $V_x = 16$, $V_y = 18.667$

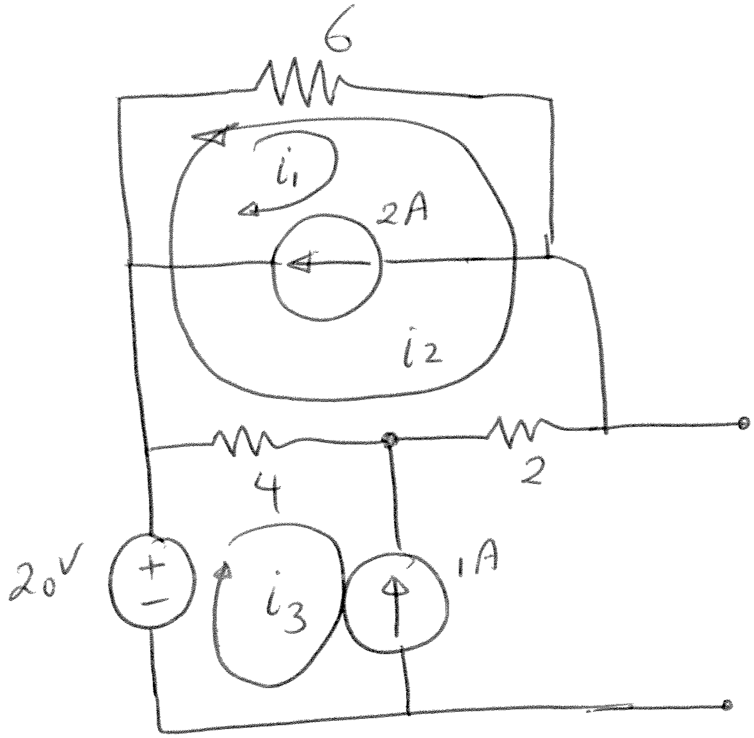
$V_{th} = V_{open} = V_x$



$\Rightarrow R_{th} \xlongequal[\text{Power Transfer}]{\text{for Max}} R_L \Rightarrow R_L = 3\,\Omega$

$$(P_L)_{max} = \frac{(V_{th})^2}{4R_L} = \frac{(16)^2}{4 \times 3} = \frac{(16)^2}{12}$$

We can also find V_{th} Using mesh tech. as shown next.



$i_1 = 2^A$

$i_3 = -1^A$

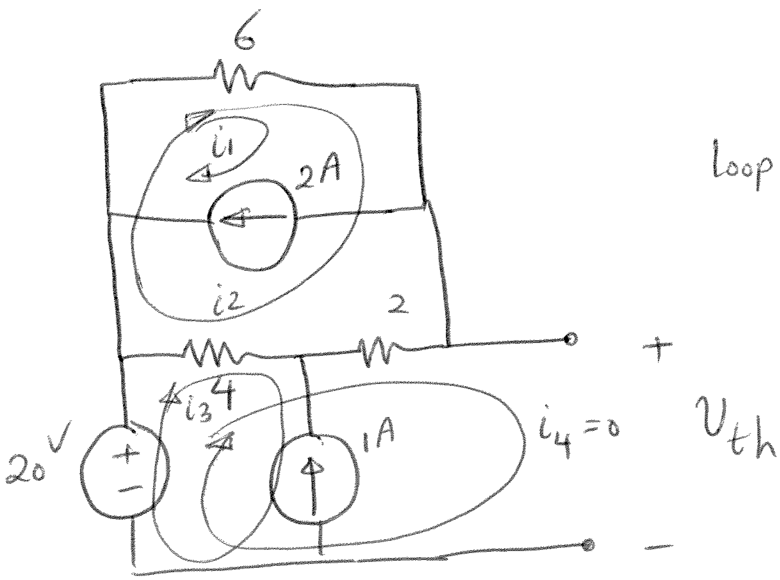
$6(i_1 + i_2) + 2\,i_2 +$

$4\,(i_2 - i_3) = 0$

$\Rightarrow 6\,(i_2 + 2) + 2 i_2 +$

$4\,(i_2 - (-1)) = 0$

$\Rightarrow \quad i_2 = -\frac{16}{12} = \frac{-4}{3}$



Loop (4):

$+v_{th} - 20 + 4\,(0 + i_3 - i_2) +$

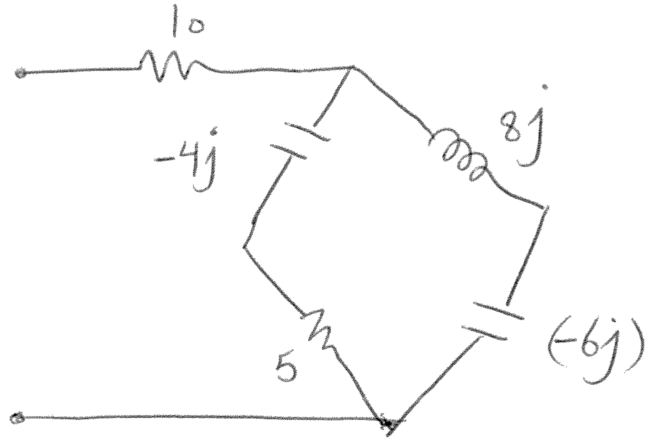
$2\,(0 - i_2) = 0$

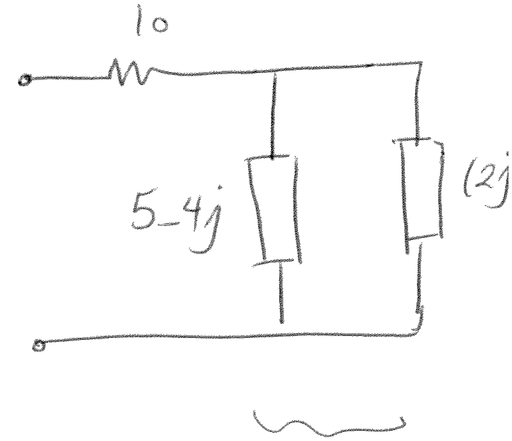
$V_{th} - 20 + 4\left(-1 + \frac{4}{3}\right) + 2\left(\frac{4}{3}\right) =$

$V_{th} = 16$ ✓

Q7: Find Thevenin first, then transform it for Norton.

Z_{th}:

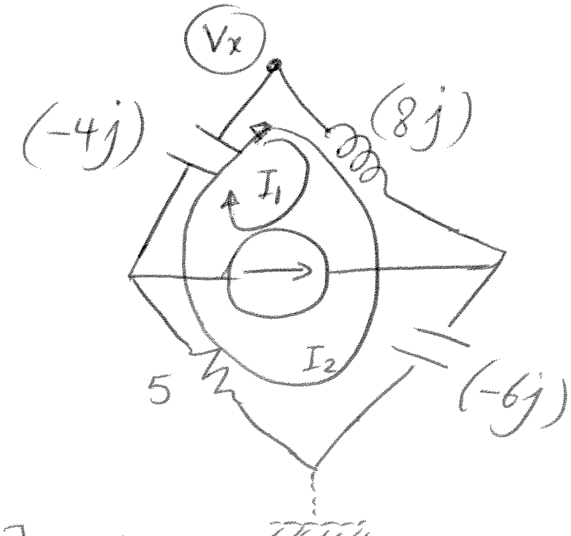




$$Z_{th} = 10.6896 + 2.2758j = 10 + \frac{(2j)(5-4j)}{(2j)+(5-4j)}$$

$\xleftarrow{\text{worked this way}}$

For V_{th}, Since the output is open, the current through 10Ω is zero.



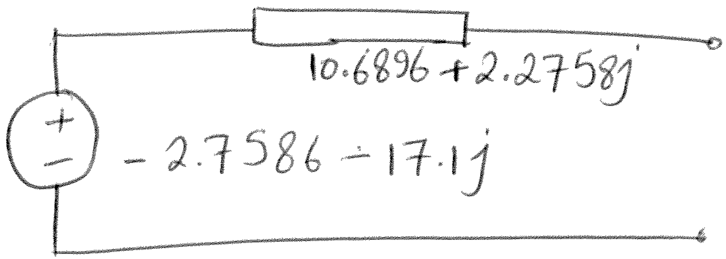
Using Mesh: $I_1 = -2\angle 0 = -2$

$$(8j)(I_1 + I_2) + (-6j)(I_2) + 5(I_2) + (-4j)(I_1 + I_2) = 0$$

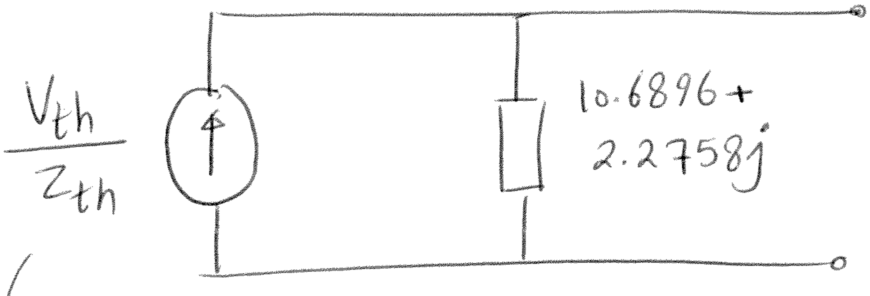
$$I_2(8j - 6j + 5 - 4j) + [(8j) + (-4j)](I_1) = 0$$

$$I_2(5-2j) = -(4j)(-2) = +8j \Rightarrow I_2 = -0.552 + 1.379j$$

$$V_{th} = V_x = (-4j)(-I_1 - I_2) + 5(-I_2) = -8j + I_2(-5+4j)$$
$$= -2.7586 - 17.10j$$



Thevenin

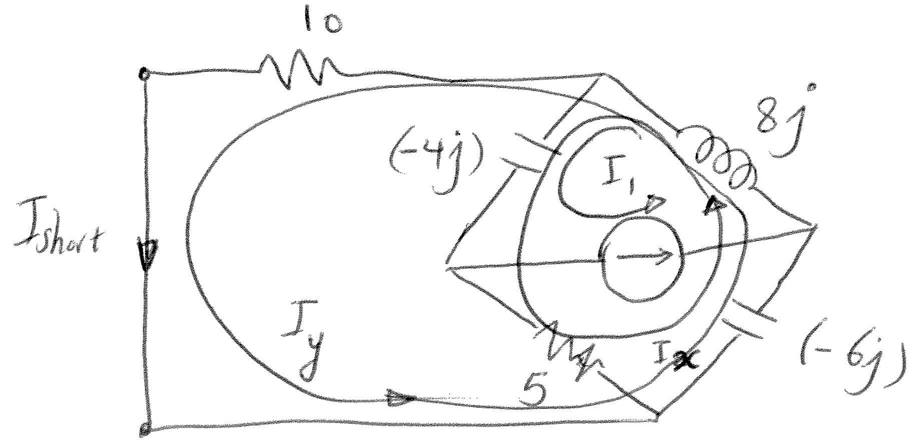


Norton

$\hookrightarrow$ $-0.573 - 1.4781j$

If you have gone to find I_{short}, here it is.

I choose Counter-clockwise to match I_{short} direction



$I_1 = 2\angle 0 = 2$

$$\begin{cases} (-4j)(2+I_x) + 5I_x + (-6j)(I_x+I_y) + (8j)(I_x+I_y+2) = 0 \\ 10\,I_y + (-6j)(I_x+I_y) + (8j)(I_x+I_y+2) = 0 \end{cases}$$

$$\begin{cases} I_x(-4j-6j+8j+5) + I_y(8j-6j) = 8j-16j \\ I_x(-6j+8j) + I_y(10-6j+8j) = -16j \end{cases}$$

$$\begin{cases} I_x(5-2j) + I_y(2j) = -8j \\ I_x(2j) + I_y(10+2j) = -16j \end{cases}$$

$$\Delta = \begin{vmatrix} 5-2j & 2j \\ 2j & 10+2j \end{vmatrix} = (5-2j)(10+2j)+4$$

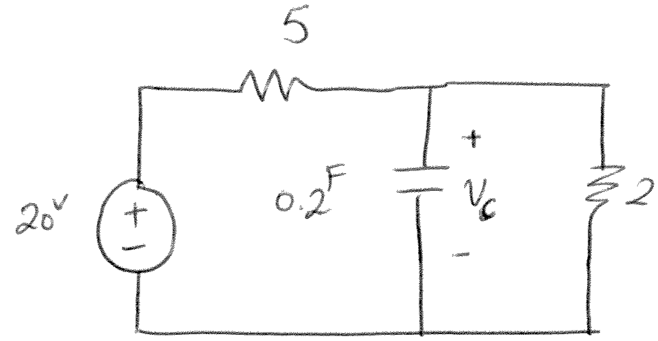
$$= 58-10j$$

$$\Delta_{I_y} = \begin{vmatrix} 5-2j & -8j \\ 2j & -16j \end{vmatrix} = -48-80j$$

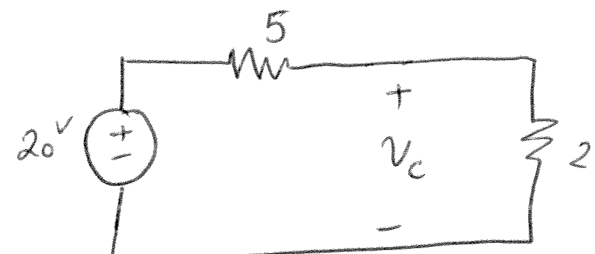
$$I_y = I_{short} = \frac{\Delta_{I_y}}{\Delta} = \frac{-48-80j}{58-10j} = -0.573 - 1.478j$$

Q8:

$t<0$

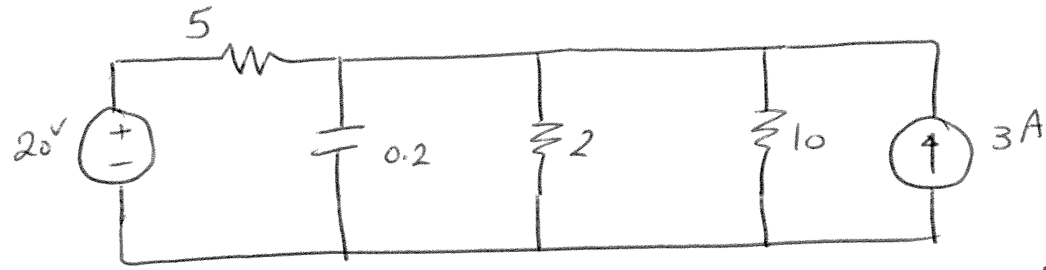


Long time: Capacitor acts like open.

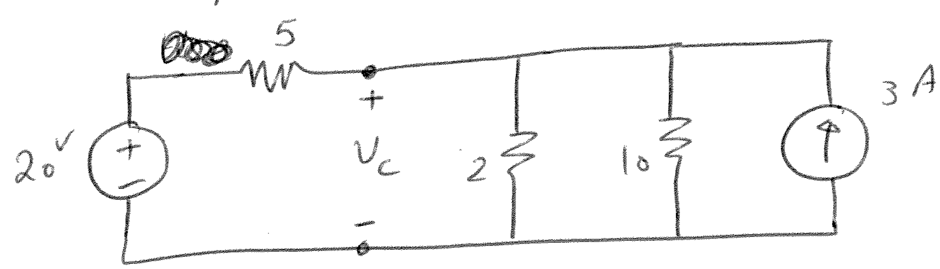


$$\Rightarrow v_c(0^-) = \frac{2}{2+5} \times 20^v = \frac{40}{7} = 5.71^v$$

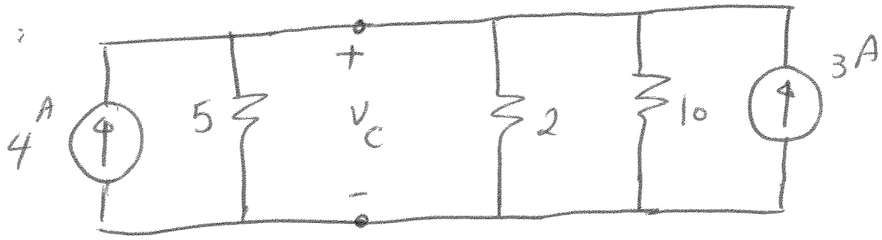
$t \geq 0$



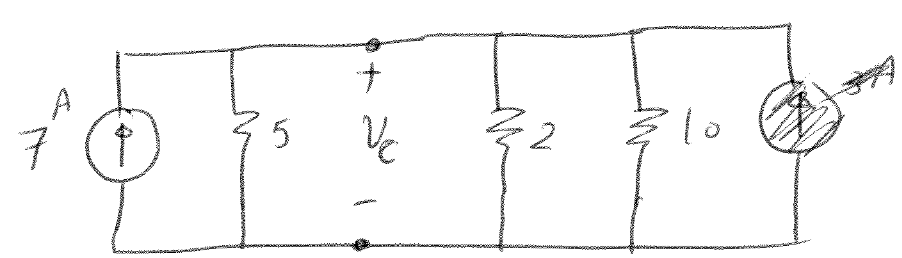
Long time: capacitor acts like an open line.

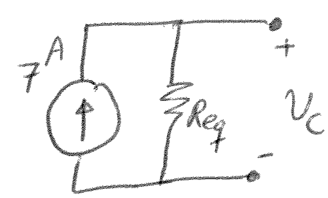


Source Transformation:



Combine sources:





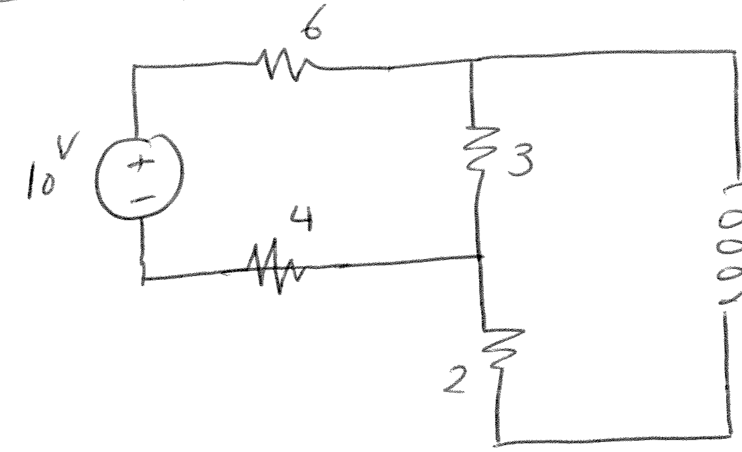
$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10} \Rightarrow R_{eq} = 1.25$$

$$V_C(\infty) = 7^A \times (1.25) = 8.75^V$$

$$\tau_C = R_{eq} \cdot C = (1.25)(0.2) = 0.25^{sec}$$

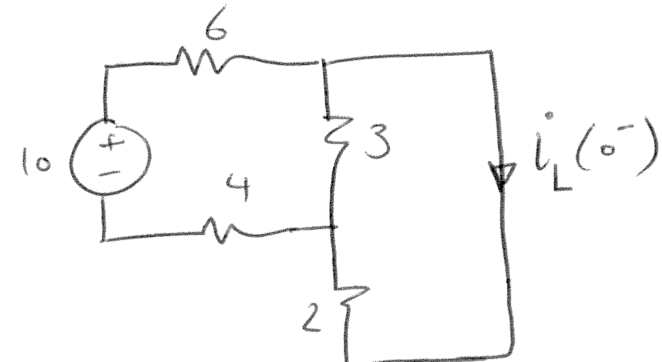
$$V_C(t) = 8.75 + (5.71 - 8.75)\, e^{-\frac{t}{0.25}}$$

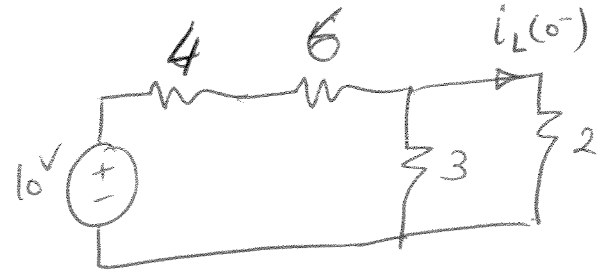
Q9 :



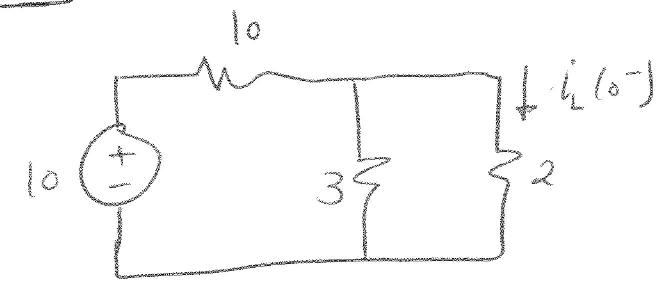
$t<0$

long time:
Inductor is short.

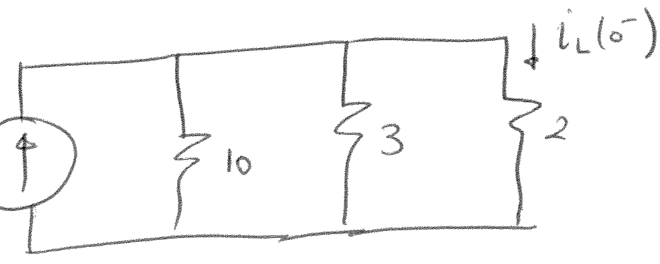




$\Rightarrow$



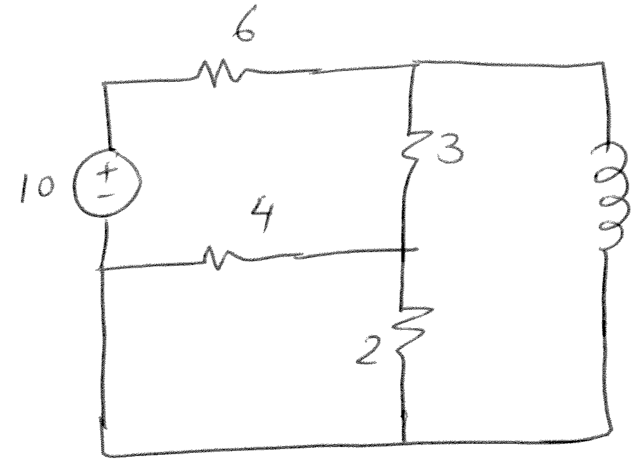
$\Rightarrow$ $1^A = \frac{10}{10}$



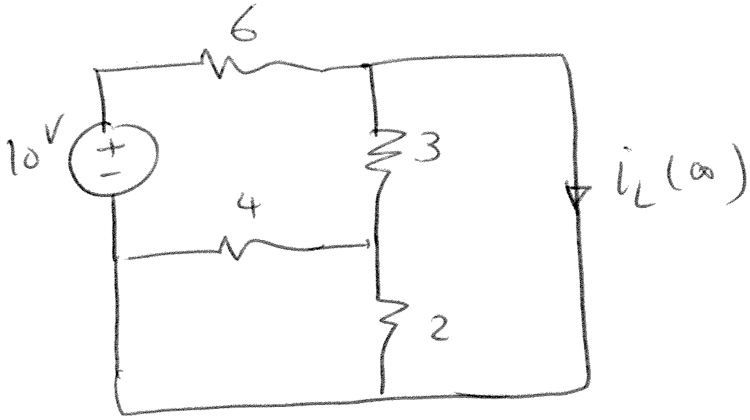
$$i_L(0^-) = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{10}} \times 1^A$$

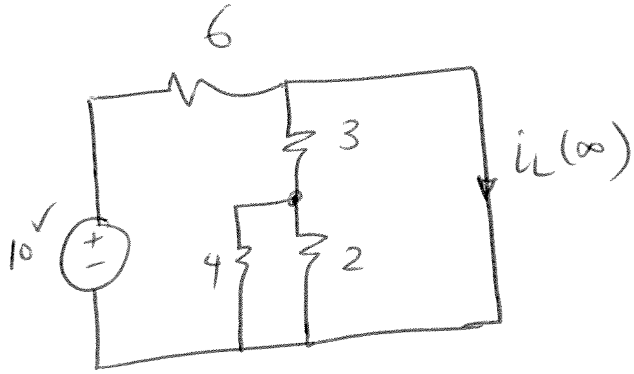
$$= 0.5357 \text{ Amp}$$

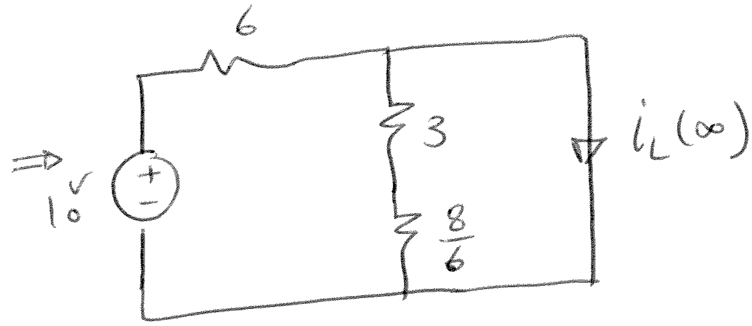
$t > 0$

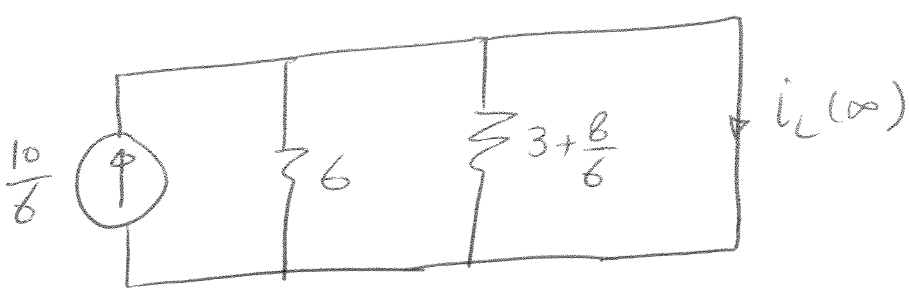


Long time:
Inductor Acts
like
a short line.









$i_L(\infty) = \frac{10}{6}$ Amp

$$R_{eq} = \frac{(3+\frac{8}{6})(6)}{(3+\frac{8}{6})+(6)} = 2.516$$

$$\tau = \frac{L}{R} = \frac{4}{R_{eq}} = \frac{4}{2.516} = 1.5897$$

$$i_L(t) = \frac{10}{6} + \left[0.5357 - \frac{10}{6}\right] e^{-\frac{t}{1.5897}}$$