

Student ID: _____

ENGG 201 – WINTER 2016 - QUIZ (/60) [60 min.]

- Closed book, closed notes, Schulich Calculator Only.
- Show all work, and box final answer.
- Use the back of pages if needed and indicate with an arrow if you do so

1. The Froude number is used to determine the nature of flow in open channels (e.g., flow of water in a river):

$$Fr = \frac{V}{\sqrt{gy}}$$

The flow depth y at any distance x is determined from the following dimensionally homogeneous equation

$$\frac{dy}{dx} = \frac{S}{1 - Fr^2}$$

where, Fr = Froude number

V = flow velocity

g = gravitational acceleration

y = flow depth (height of the liquid)

x = distance

S = difference between the bottom slope and friction slope of the channel

(i) Use the dimensions of V , g , and y , to show that the Froude number is dimensionless (/5)

$$Fr = \frac{V}{\sqrt{gy}}$$

$$[Fr] = \frac{[V]}{[\sqrt{gy}]} = \frac{[\frac{L}{t}]}{[\sqrt{\frac{L}{t^2} \cdot L}]} = \frac{[\frac{L}{t}]}{[\frac{L}{t}]} = 1$$

⇒ Fr is dimensionless.

(ii) Show that the quantity S in the second equation is also dimensionless. (/5)

$$\frac{dy}{dx} = \frac{S}{1 - Fr^2}$$

$$\Rightarrow S = \frac{dy}{dx} (1 - Fr^2)$$

Fr is dimensionless. Then,

$$[S] = \left[\frac{dy}{dx} \right] \times 1 = \frac{[L]}{[L]} \times 1 = 1$$

So, S is dimensionless.

(iii) Find the unit of the constant c appearing in the following dimensionally homogeneous equation:

$$Q = cby^{\frac{5}{3}}\sqrt{S}$$

where, Q is the volumetric flow rate (m^3/s) in the channel, b is the breadth (m) of the channel, and y is the height of liquid (m). (/5)

$$c = \frac{Q}{by^{\frac{5}{3}}\sqrt{S}}$$

We showed that S is dimensionless.

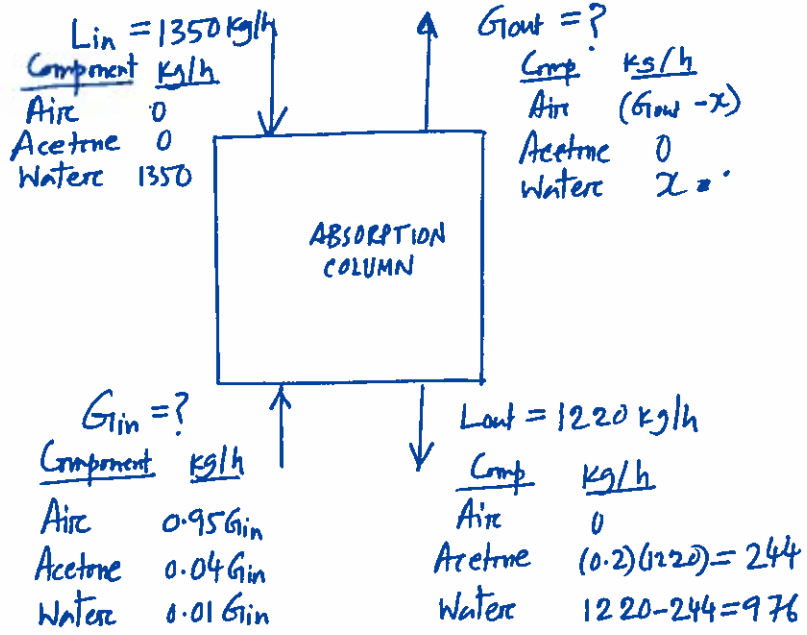
Then, units of c

$$= \frac{\frac{m^3}{s}}{m \cdot m^{\frac{5}{3}}} = \frac{m^{\frac{3}{s}}}{m^{\frac{8}{3}}}$$

$$= \frac{m^3}{s} \cdot m^{-\frac{8}{3}} = \frac{m^{3-\frac{8}{3}}}{s} = \boxed{\frac{m^{\frac{1}{3}}}{s}} \quad \underline{\underline{Ans}}$$

2. A stream of pure water (flow rate = 1350 kg/h) and a stream of a gas mixture enter into an absorption column. The entering gas stream ('G_{in}') has the following composition: 95 mass percent air, 4 mass percent acetone, and 1 mass percent water vapor. These two streams mix inside the column. A liquid stream and a gas stream leave the column. The leaving liquid stream (flow rate = 1220 kg/h) contains no air and has the following composition: 20 mass percent acetone and 80 mass percent water. The leaving gas stream ('G_{out}') has no acetone, and contains only air and moisture (water vapor). Assume the absorption column operates at steady-state and air is a single component.

- Calculate the flow rate (kg/h) of gas mixture entering the column. (/5)
- Calculate the flow rate (kg/h) of gas mixture exiting the column. (/5)
- Calculate percent mass of water in the gas stream leaving the column. (/5)



Let, $x =$ amount (kg/h) of water in G_{out} .
 3 unknowns: G_{in} , G_{out} & x . 3 equations may be required.
 No reactions, steady-state (IN=OUT) operation.
 Acetone appears in fewer places. So, write an acetone balance first.

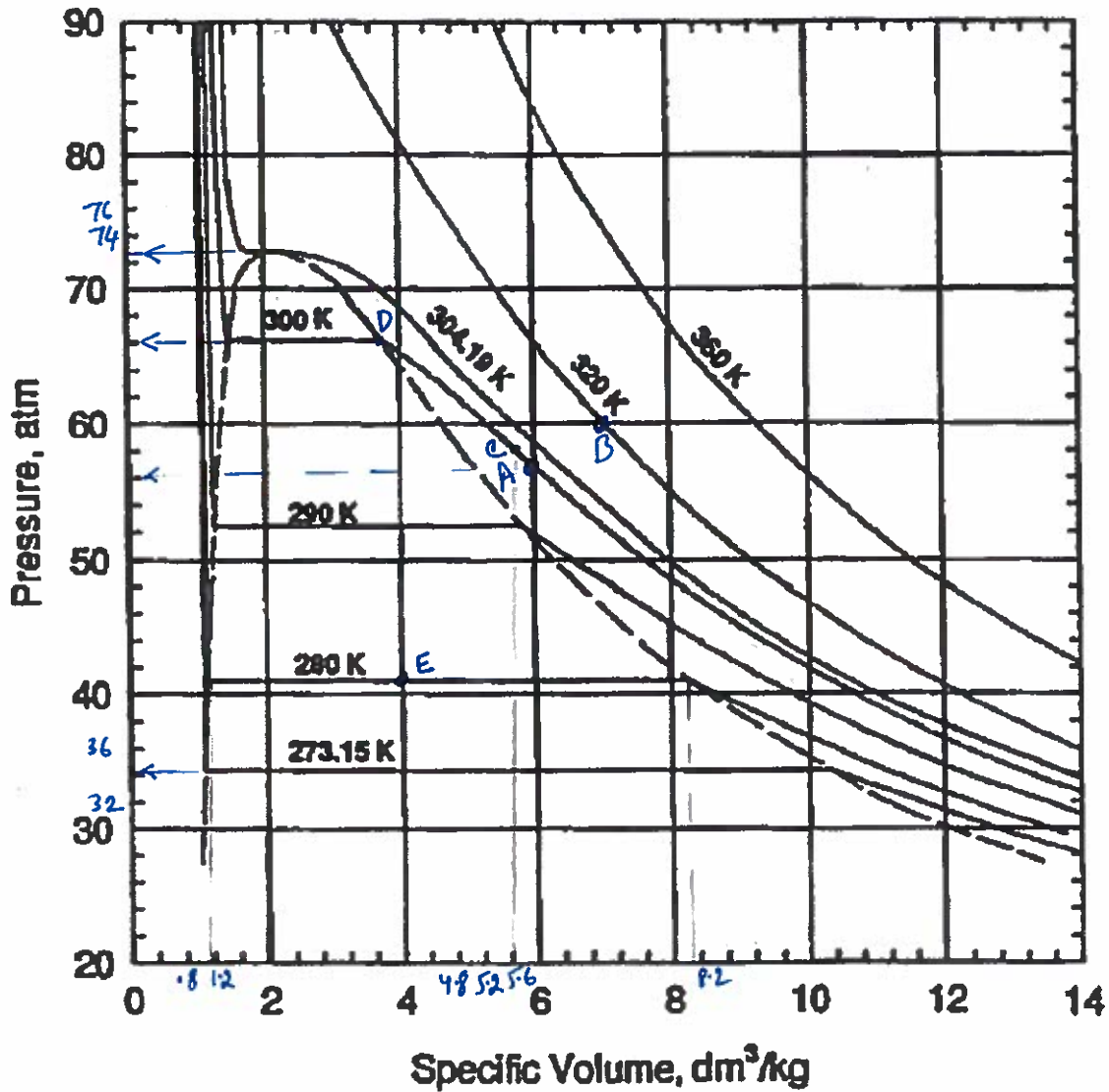
Acetone Balance: $IN = OUT$
 $\Rightarrow 0.04 G_{in} = 244 \Rightarrow G_{in} = \boxed{6100 \text{ kg/h}}$ ← (part 'a')

Overall mass balance: $1350 + G_{in} = G_{out} + 1220$
 $\Rightarrow 1350 + 6100 = G_{out} + 1220 \Rightarrow G_{out} = \boxed{6230 \text{ kg/h}}$ ← part 'b'

Water balance: $1350 + 0.01 G_{in} = x + 976$
 $\Rightarrow 1350 + 0.01 \times 6100 = x + 976 \Rightarrow x = 435 \text{ kg/h}$

Then, % mass of water in $G_{out} = \frac{x}{G_{out}} \times 100\% = \frac{435}{6230} \times 100\% = \boxed{6.98\%}$ ← part 'c'

3. Use the phase diagram for the vapour-liquid region of CO₂ to answer the following questions. Show all your work for full marks.



Pressure - Volume Diagram for CO₂

(a) (i) What is the critical pressure of CO₂. (2)

$$P_{crit} \approx \boxed{72.5 \text{ atm}}$$

(ii) Estimate the vapor pressure of CO₂ at 0°C. (1/2)

$$0^\circ\text{C} = 273.15\text{K}$$

$$P_{vap} \approx \boxed{34.1 \text{ atm}}$$

(iii) What phase or phases exist at 300 K and at a specific volume of 6 dm³/kg? (/2)

$T = 300\text{K} < T_c$
Point A: $P < P_c$, state is VAPOUR

(iv) What phase or phases exist at 320 K and at 60 atm? (/2)

Point B: $T = 320\text{K} > T_c$
 $P < P_c$
 state is GAS

(v) Is the pressure of the system at the following conditions ($T = 300\text{K}$, specific volume = 6 dm³/kg) greater or less or equal to the vapor pressure of CO₂ at 300 K (/3)

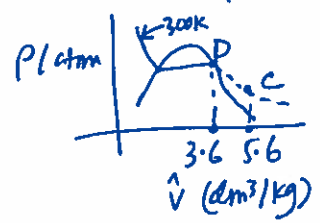
@ $T = 300\text{K}$, $\hat{v} = 6\text{ dm}^3/\text{kg} \rightarrow$ point A.
 $P_{\text{system}} \approx 56\text{ atm}$
 $P_{\text{vap@300K}} \approx 66\text{ atm}$

$P_{\text{system}} < P_{\text{vap}}$

(vi) 100 kg of pure CO₂ is contained in a rigid container of volume 560 dm³ at a temperature of 300K. Determine how much CO₂ (kg) would have to be added to the container at the same temperature so that the phase of CO₂ become saturated vapor? (/4)

100 kg CO₂ in 560 dm³ $\Rightarrow \hat{v} = \frac{560}{100} = 5.6 \frac{\text{dm}^3}{\text{kg}}$ @ $T = 300\text{K}$
 point C.

To be saturated, \hat{v} should be $\approx 3.6 \frac{\text{dm}^3}{\text{kg}}$ (point D)



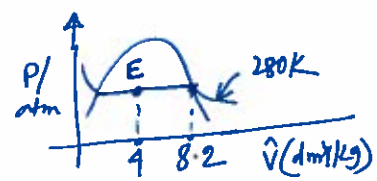
Then, $\frac{560}{m_{\text{new}}} = 3.6$
 $\Rightarrow m = 155.6\text{ kg}$
 Then $(155.6 - 100) = \boxed{55.6\text{ kg}}$ CO₂ should be added.

(b) A separate container of volume 400 dm³ contains 100 kg of CO₂ at a temperature of 280K.

$\hat{v} = 4\text{ dm}^3/\text{kg}$
 $T = 280\text{K}$ } \Rightarrow (L+v) at point E

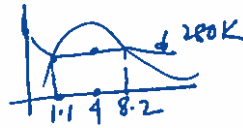
(i) What is the specific volume of the vapor present inside the container? (/2)

$\hat{v}_{\text{mixture}} = 4\text{ dm}^3/\text{kg}$
 $\hat{v}_{\text{vapor}} = \boxed{8.2\text{ dm}^3/\text{kg}}$



(ii) What is the specific volume of the liquid present inside the container? (/2)

$$\hat{V}_{liq} \approx 1.1 \frac{dm^3}{kg}$$



(iii) How much liquid is present? (/4)



$$\frac{m_L}{m_{tot}} = \frac{\hat{V}_{vap} - \hat{V}_{mix}}{\hat{V}_{vap} - \hat{V}_{liq}} = \frac{8.2 - 4}{8.2 - 1.1} \approx 0.59$$

$$\Rightarrow m_L = 0.59 \times m_{tot} = 0.59 \times 100 \text{ kg} = \boxed{59 \text{ kg}}$$

or, in terms of volume, $V_{liq} = (59 \text{ kg})(1.1 \frac{dm^3}{kg}) = \boxed{65 dm^3}$

Both are OK.

(iv) The temperature of the above container is raised to 290 K. What fraction of total mass is liquid? (/5)

Rise in temp changes the specific volumes of vap & liq.

$$\hat{V}_{liq} \approx 1 \quad \hat{V}_{mix} = 4 \quad \hat{V}_{vap} \approx 5.8$$

$$\frac{m_{liq}}{m_{tot}} = \frac{\hat{V}_{vap} - \hat{V}_{mix}}{\hat{V}_{vap} - \hat{V}_{liq}} = \frac{5.8 - 4}{5.8 - 1} = \boxed{0.375} \text{ Ans}$$

$$\Rightarrow m_{liq} = \boxed{37.5 \text{ kg}}$$

(v) Calculate the degree of freedom for the system when the temperature of the container is raised to 290 K as in part (iv). (/2)

$$F = 2 + C - P$$

$$\text{From part (iv)} P = 2$$

$$F = 2 + 1 - 2 = \boxed{1} \text{ Ans.}$$

Note: Since T is given, there is no intensive variable left to be specified.