

Text Merlin by tomorrow to check our work (514) 568-3394

Practice for Final

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1. For each of the following differential equations
 - (a) Find equilibrium solutions. Determine whether they are stable or unstable.
 - (b) Find the solution, satisfying the initial condition $y(0) = y_0$. Sketch graphs of several different non-negative y_0 at the same plot.

a) $y'(t) = 2y(t) \left(1 - \frac{y(t)}{8}\right)$;

b) $y'(t) = y^2(t) - 4y(t) + 3$.

2. Solve the differential equations

a) $y'(t) = \frac{y e^{xy} \cos 2x - 2 e^{xy} \sin 2x + 2x}{3 - x e^{xy} \cos 2x}$;

b) $y'(t) = \frac{y e^y - 2x}{y}$.

3. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 96°C when freshly poured, and 5 minutes later cools to 71°C in a room at 21°C , determine how great will coffee cool in 5 minutes if the room temperature is changing as $(15 + 6 e^{-t/5})^\circ\text{C}$.

4. Solve the differential equation

$$t^2 y'' + 5t y' + 4y = \ln(t^t), \quad t > 0.$$

5. A certain series circuit has a capacitor of 0.8×10^{-6} F, a resistor of $R \Omega$, and an inductor of 0.2 H, thus the charge satisfying the equation

$$\frac{1}{5} Q'' + R Q' + 1250000 Q = 0.$$

Find the resistance that the system is critically damped and sketch the graph of the solution in this situation which satisfies the initial values problem where charge at $t = 0$ is 4 C and current is 0 A.

6. The Bessel equation of order zero is

$$x^2 y'' + x y' + x^2 y = 0.$$

- (a) Show that $x = 0$ is a regular singular point.
- (b) Find roots of the indicial equation and find one of the solutions as a power series about $x_0 = 0$.
- (c) Write down the general form of the second fundamental solution. Try to find it.

7. A certain spring-mass system satisfies the initial value problem

$$u'' + \frac{1}{4}u' + u = g(t), \quad u(0) = u'(0) = 0,$$

where $g(t) = 2\delta(t - 1) - \delta(t - 3)$.

- (a) Sketch the graph of $g(t)$. Observe that it is two impulses at times 1 and 3 and they are applied in controversial directions, besides first one is twice large.
- (b) Solve the initial value problem. Sketch the graph and describe its principal features.