# MATH3705 D – Test 2: Friday, Feb. 14, 14:35–15:25

**Name and Student Number:**

**Total points: 15. No partial marks for Questions 1-4.**

**Closed book! Non-programmer calculators are allowed!**

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1. (1 point) Find the general solution of $x^2y'' - 4xy' + 6y = 0$ for $x \neq 0$.

   (a) $c_1|x|^{-2} + c_2|x|^3$ (b) $c_1|x|^{-2} + c_2|x|^{-3}$ (c) $c_1|x|^{-1} + c_2|x|^{-5}$
   (d) $|x|^2[c_1\cos(\sqrt{2}\ln|x|) + c_2\sin(\sqrt{2}\ln|x|)]$ (e) $c_1|x|^2 + c_2|x|^3$

2. (1 point) Find the general solution of $x^2y'' + 5xy' + 4y = 0$ for $x \neq 0$.

   (a) $c_1|x|^2 + c_2|x|^{-2}\ln|x|$ (b) $c_1|x|^{-2} + c_2|x|^2\ln|x|$ (c) $c_1|x|^2 + c_2|x|^2\ln|x|$
   (d) $c_1|x|^2 + c_2|x|^{-2}$ (e) $c_1|x|^{-2} + c_2|x|^{-2}\ln|x|$

3. (1 point) Which of the following is a solution of $x^2y'' + 5xy' + 8y = 0$ for $x > 0$?

   (a) $x^2$ (b) $x^2\sin(4\ln x)$ (c) $x^{-2}$ (d) $x^{-2}\sin(\ln x)$ (e) $x^{-2}\sin(2\ln x)$

4. (1 point) Which of the following is a solution of $x^2y''(x) + xy'(x) + (x^2 - 2.25)y(x) = 0$, for $x > 0$?

   (a) $J_{-1.5}(x)$ (b) $J_1(1.5x)$ (c) $Y_{2.25}(x)$ (d) $Y_1(2.25x)$ (e) $Y_1(1.5x)$

5. (5 points) Let $y = \sum_{n=0}^{\infty} a_n x^n$ be the series solution of $(x^2 + 1)y'' + xy' + y = 0$. Find the relation between $a_{n+2}$ and $a_n$.

6. (6 points) Let $y = \sum_{n=0}^{\infty} c_n(r)x^{n+r},\ c_0(r) = 1$ be the solution of the following DE:

$$xy'' + (x - 0.5)y' - 0.5y = 0$$

for $x > 0$ near $x_0 = 0$. The recursive relation is: $c_{n+1}(r) = \frac{-1}{n+r+1}c_n(r), n \geq 0$.

(i) (2 points) Write down the indicial equation and solve it to determine $r_1$ and $r_2$, $r_1 \geq r_2$.